SSRG International Journal of Civil Engineering
ISSN: 2348-8352/ https://doi.org/10.14445/23488352/IJCE-V11I3P103

Volume 11 Issue 3, 30-42, March 2024

Original Article

A Fuzzy Mixed Integer Linear Programming Approach for Reverse Logistics of Waste Plastic Recycling at Strategic Level

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Received: 07 January 2024 Revised: 06 February 2024 Accepted: 04 March 2024 Published: 31 March 2024

Abstract *- This study addresses the pressing issue of plastic waste management in India, where 3.47 million tons of plastic waste was generated in the fiscal year 2019-2020. Recognizing the complexities and uncertainties in waste plastic recycling, the research introduces a novel Fuzzy Mixed-Integer Linear Programming (Fuzzy MILP) model. The model aims to optimize the entire waste plastic recycling supply chain, considering the inherent uncertainties in recycling operations. Emphasizing the role of reverse logistics in waste management, the study builds upon established models, contributing to the field's knowledge. The proposed strategic-level reverse logistics model seeks to minimize total costs and determine the optimal number of recycling plants, addressing the limited infrastructure in developing countries. This research provides a valuable framework for policymakers and industry stakeholders, offering sustainable solutions to mitigate the environmental impact of plastic pollution in India and beyond.*

Keywords *- Waste management, Plastic recycling, Reverse logistics, Supply chain, Fuzzy programming.*

1. Introduction

1.1. Background

Due to their farfetched qualities, such as durability, low weight, and superior thermal and electrical insulating capabilities, plastics have established themselves as a global commodity. These features have created a wide range of application options at an affordable price. The United Nations Environment Programme [1] estimates that more than 400 million metric tons of plastic waste are produced worldwide yearly. It is estimated that 6.3 billion tons of plastic waste were produced worldwide between 1950 and 2015. Still, only 9% of this enormous waste stream was recycled, and a staggering 80 per cent of it was carelessly dumped in landfills or, worse, made its way into natural ecosystems.

India's total waste plastic generation is approximately 3.47 million tons per annum for the fiscal year 2019-2020 [2]. In India, 5.5 million metric tons of plastic waste are reprocessed or recycled annually, accounting for 60% of the nation's total plastic waste production. Of this waste, 70% is reprocessed in registered or formal facilities, the informal sector handles 20%, and the remaining 10% is recycled at the household level [3]. The remaining 40% of waste plastic is ultimately left uncollected or discarded, which leads to more pollution (of the land and water) and clogging of drains. Large quantities of plastic waste are generated in hospitals, of which only 9.8% of the recyclable waste generated is separated, and 35% of this waste is made of plastic [4]. Plastic waste not only spreads disease but also seriously harms the ecosystem by clogging drains and affecting aquatic life.

1.2. Problem Statement

Plastics need to be recycled due to their non-biodegradable nature. Plastics can exist in the ecosystem for a very long time without decomposing. Manufacturing industries do not invest in recycling waste plastic because of the complexity of the recycling system. Government legislation, infrastructure development, stakeholder participation, and the inherent technological difficulties of the procedure all further influence the waste plastic recycling landscape. These elements greatly impact how plastic waste is collected, sorted, processed, and recycled.

Several countries are shifting from a linear economy to a circular economy in response to these problems, which aims to recover some value from used plastic by recycling it. In a circular economy that promotes recovery, reuse, and recycling cycles for the flow of secondary resources, Reverse Logistics (RL) is essential [5]. Transportation and distribution logistics play a crucial role in the sustainability of recycling operations. Distance, route optimization, and environmental effects are some of the factors that influence the transportation process.

Waste plastic is normally transported from collection points to recycling facilities and potentially further to landfills for disposal of non-recyclable residues.

The main logistical cost is transportation, which occurs at various stages across the channel and accounts for more than 25% of total recycling costs incurred before intermediate processing. Inefficient municipal solid waste management can often be attributed to poor logistics in collection routes. Researchers have increasingly turned to reverse logistics as a viable strategy to improve the efficiency and sustainability of solid waste management practices. As demonstrated in the work of Montoya et al. [6], reverse logistics offers a well-suited approach to address the complexities of handling solid waste while simultaneously promoting environmental sustainability.

Additionally, it can be difficult for plastic recycling operations to balance supply and demand dynamics since recycling plants' processing capacity must match waste plastics' availability at collection centres. Failure to achieve this equilibrium can result in inefficiencies and increased transportation costs. The reverse logistics of recycling waste plastic poses challenging logistical hurdles and uncertainties, underscoring the need for advanced optimization techniques.

Fuzzy mixed-integer linear programming emerges as a promising solution to navigate the complexities of this process. Unlike traditional supply chain planning research, which relies on probability distributions derived from historical data, it is crucial to recognize that stochastic models may not always be suitable, especially in situations where statistical data is inaccurate or unavailable. To address the uncertainties inherent in supply chain dynamics, alternatives such as fuzzy set theory and possibility theory present simpler and less data-demanding options than traditional probability theory [7].

This study aims to create a unique fuzzy mixed-integer linear programming (Fuzzy MILP) model that is customized to the complex dynamics of recycling waste plastic and to demonstrate how it may be used practically in the realm of sustainable reverse logistics in the Indian context. This study aims to address the urgent need for effective waste plastic recycling solutions that might lessen the growing environmental burden caused by plastic pollution. The study seeks to provide a thorough framework that optimizes the complete waste plastic recycling supply chain, which includes the stages of collection, transportation, sorting, processing, and disposal to add a level of adaptability and robustness to the MILP model that takes into consideration the inherent uncertainties and variability in recycling operations.

1.3. Research Gaps and Objective

With the growing importance of reverse logistics in the context of waste plastic recycling, existing literature predominantly focuses on deterministic optimization models. There is a noticeable scarcity of research that effectively integrates fuzzy with mixed integer linear programming (MILP) to address the essential uncertainties and ambiguity associated with the reverse logistics processes in waste plastic recycling.

The existing models often fail to capture the dynamic and uncertain nature of factors such as collection, transportation, sorting, processing, and disposal costs. The strategic-level decision-making process in waste plastic recycling remains largely unexplored, with a limited number of studies offering inclusive optimization solutions for strategic planning. The primary objective of the research paper is to develop a fuzzy MILP model for reverse logistics of waste plastic recycling in the city of Patna, India, for cost minimization and facility allocation of recycling plants.

2. Literature Review

The mitigation of plastic pollution is a pressing concern, underscoring the significance of plastic recycling. However, the efficacy of interferences aimed at mitigating plastic pollution requires thorough estimation and evaluation. This necessitates the modelling of frameworks and flows within the global plastic system, coupled with the implementation of practical interventions. Furthermore, a nuanced approach is essential, considering both the financial and social costs associated with implementing mitigation strategies and potential drawbacks, such as those related to waste-to-energy processing methods.

With an understanding of the environmental impacts of plastic waste and systematically evaluating the effectiveness of various interventions, one can formulate a comprehensive guide for plastic recycling practices. Diverging from the conventional path of the supply chain, Fleischmann et al. [8] characterize reverse logistics as the strategic planning of the inbound flow and storage of secondary goods and related information. This systematic approach aims to efficiently and effectively recover value from products while ensuring environmentally responsible disposal practices. It outlines five fundamental processes for managing end-of-life products, encompassing collection, inspection or separation, reprocessing, disposal, and redistribution.

Reverse logistics can play a pivotal role in resource recovery, as defined by the series of activities involved in collecting used products for purposes such as reuse, repair, remanufacturing, recycling, or disposal. The previous study put forward a MILP model for cost minimization for reverse logistics of construction and demolition wastes. Different researchers have put forward reverse logistics of various materials like C&D wastes, sand and aluminium. Its application in handling plastic waste can mitigate the adverse environmental impact of uncontrolled plastic disposal. Despite its potential benefits, the adoption of reverse logistics for plastic waste management remains limited in several countries. A key contributing factor to this limited implementation is the lack of understanding and awareness regarding the significance of reverse logistics in waste management. However, reverse logistics is becoming more widely recognized as a workable method for managing waste plastic as a result of the rising volume of waste plastic and the growing awareness of the need for proper waste management [9].

While reverse logistics is widely recognized as a dynamic system characterized by numerous unknown variables, such as return rates, prices, processing fees, and logistic providers, the predominant focus in studies has been on deterministic reverse logistics models. To overcome the limitations of deterministic models, stochastic models have been formulated [10].

However, employing a stochastic technique for simulating the reverse supply chain network faces severe constraints, primarily stemming from the substantial computational costs associated with the multitude of scenarios needed to capture the intricate nature of uncertainty accurately. It has become extremely difficult to reach a compromise between the need for numerous scenarios to characterize uncertainty and the constraints imposed by computational resources fully. The fuzzy-based models have emerged as a potential solution to this problem. Their need comes from its ability to efficiently handle a wide range of uncertainties, offering a pacification that takes into account both the requirement for scenario variety and the capacity to characterize ambiguity in a manageable and computationally efficient manner.

The different applications of fuzzy-based models can be studied in [11]. The ambiguity surrounding factors like return rate, discarded product quality, and recovery alternatives, coupled with the lack of historical data and interdependency among variables, makes them subjects of uncertainty in research [12]. Addressing this challenge, Bing et al. [13] proposed a Mixed Integer Linear Programming (MILP) model that effectively minimizes both transportation costs and environmental impact in the context of plastic waste network design in the Netherlands.

Fleischmann et al. [14] developed a generic MILP network model for designing product recovery networks, exemplifying its application in a case study focused on paper recycling. In the present study, a generalized modelling approach was applied specifically to the domain of waste plastic management. Recognizing the real-world decision problems characterized by ambiguity and imprecise information, various studies have contributed to managing uncertain parameters within the reverse logistics network design framework. Dhouib [15] employed a categorical-based evaluation technique to accommodate linguistic evaluations from decision-makers, particularly in analyzing alternatives for reverse logistics in recycling used vehicle tyre waste. Additionally, Govindan et al. [16] introduced a multi-objective fuzzy mathematical programming model for reverse logistics network design, aiming to minimize the present value of costs while considering environmental effects and social responsibility.

Gao and Cao [17] introduced a multi-objective scenario-based optimization model, concentrating on the sustainable redesign of Reverse Logistics (RL) networks. This model uniquely considers both the quantity of used products and the uncertainty associated with demand. Das and Chowdhury [18] devised a comprehensive recycling and logistics model specifically created to enhance the efficient management of electronic product waste streams. Their model's primary aim was to minimize the overall processing costs across the entire recycling network. This model was structured by encompassing four crucial recycling phases, namely, collection, separation, recycling, and repair.

Furthermore, their research extended its scope to consider the logistical aspects of recycling by incorporating key locations within the system, including a dumping point, primary market, and secondary market. The key finding was that transportation costs emerged as a substantial component of the overall recycling costs. Pishvaee et al. [19] introduced an advanced mathematical programming model tailored for optimizing a multi-period logistics network. The primary focus of their study was directed towards the reduction of both fixed costs and transportation costs. Customer locations, collection points, quality inspection facilities, recycling centres, and disposal sites were just a few of the crucial parts of this extensive logistical network. The researchers used a simulated annealing process as their main solution method to deal with this problem's complexity.

Diabat et al. [20] used several sequential stages to address the complicated Reverse Logistics (RL) network problem. Their research aimed to precisely ascertain the optimal number and strategic placement of primary collecting sites, centralized return facilities, and the maximum allowable retention time for small volumes of returned goods. The researchers developed a thorough modelling strategy to address the broad problem of lowering costs related to RL network design, including costs related to inventory management, preparation, transportation, and more. They used the computational power of genetic and artificial immune system algorithms to successfully tackle this complex task.

3. Problem Description

The accelerating environmental concerns and escalating global plastic waste crisis necessitate a comprehensive understanding and optimization of the reverse logistics processes involved in recycling waste plastics. The motivation behind this research necessitates the urgency to minimize the environmental impact of plastic waste, reduce landfill sites, and promote a more sustainable approach to plastic consumption. The novelty of the work lies in its thorough analysis of existing reverse logistics systems for waste plastic recycling, identifying shortcomings and proposing innovative strategies to enhance efficiency and sustainability.

This paper presents a strategic-level reverse logistics model, particularly relevant in developing countries with limited infrastructure. The objective is to minimize the overall cost and determine the optimal number of waste plastic recycling plants needed, considering the uncertainties in both supply and demand conditions. Figure 1 illustrates the proposed network's structure, which comprises four parts: municipalities or sources, collection centres, recycling plants, and landfills.

Table 1. Different applications of recycled waste plastics [21]

Plastic-Type	Product Identification Code (SPI)	Applications
PET	PETE	Drink Bottles, Detergent Bottles
PVC	PVC	Food Packaging, Textile and Medical Materials
HDPE	HDPE	Detergent Bottles and Mobile Components
PP	PP	Compost Bins and Curbside Recycling Crates
PS	PS	Disposable Cutlery
LDPE	LDPE	Bottles, Plastic Tubes and Food Packaging

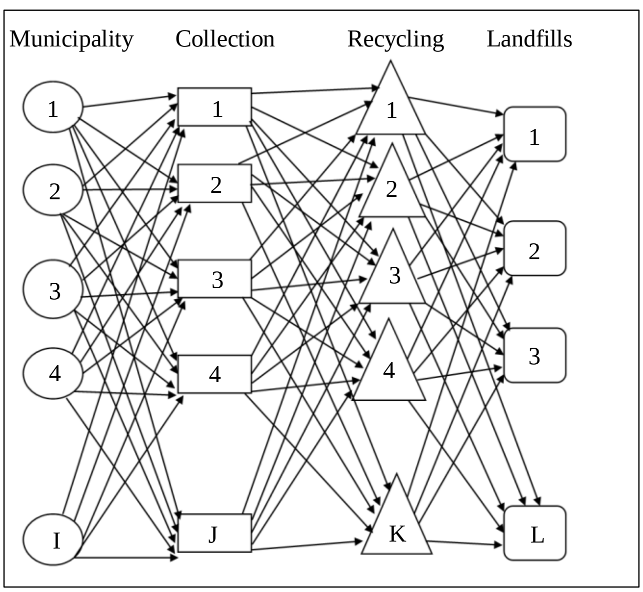

Fig. 1 Reverse logistics network of waste plastic recycling

In the first stage, waste plastics mixed with other solid wastes are collected from different households in the municipality or source. The collected wastes from the sources are then sorted into different plastic types (Table 1), dirt and other materials present are also removed in a designated collection centre located before the recycling plants. The sorted and shredded waste plastics received from the collection centres are further processed in the recycling plants to turn into new recycled products.

The wastes left from the recycling plants are sent directly to the landfills for disposal. The potential locations of recycling plants are known. The cost structures associated with various processes in the reverse logistics of plastic waste, such as transportation, sorting, processing, and disposal, are well-defined, as are the capacities of recycling plants. Additionally, the locations and capacities of collection centres are known factors in the overall reverse logistics network.

3.1. Formulation of Fuzzy Model

This work emphasizes the foundation laid by Peidro et al. [22] in their proposal of the Fuzzy Mixed-Integer Linear Programming (FMILP) model for strategic supply chain planning. The different stakeholders involved in the reverse logistics network are assumed to be sources=15, collection centers=20, recycling plants=30 and landfills=5, respectively.

Index sets and parameters applied in the FMILP problem:
$i \in I$ - Set of sources or Municipalities
$j \in J$ - Set of collection centres
$k \in K$ - Set of recycling plants
$l \in L$ - Set of landfills
F_k = Fixed cost of establishing a recycling plant
q_k = A binary variable indicating whether the recycling plant is open or closed.
0, when the recycling plant is not open
1, when the recycling plant is open
T^f_{ijk} = Fuzzy transportation costs from source i to collecting centre j for recycling plant k
S^f_{ijk} = fuzzy sorting cost at collection centre j for recycling plant k
P^f_k = fuzzy processing cost at recycling plant k
D^f_{kl} = fuzzy disposal cost from recycling plant k to landfill l
v_{ij} = amount of waste plastic from source i sent to collection centre j
w_{jk} = The amount of waste plastic from collection centre j sent to recycling facility k
z_{kl} = The amount of waste plastic from the recycling plant k transferred to the landfill
S_j = crisp capacity of the collection centre j
G_k = crisp capacity of the recycling plant k
D_l = crisp capacity of landfill l
Objective Function:-
Minimize (∑ Fixed cost + ∑ Transportation cost + ∑ Sorting cost + ∑ Processing cost +∑ Disposal cost)

Minimize

$$\sum_{k=1}^{30} F_k\ q_k + \sum_{i=1}^{15}\sum_{j=1}^{20} T_{ijk}^{f}\ v_{ij} + \sum_{j=1}^{20} S_{ijk}^{f}\ w_{jk} + \sum_{j=1}^{20} P_k^{f}\ w_{jk} + \sum_{k=1}^{30}\sum_{l=1}^{5} D_{kl}^{f}\ Z_{kl} \quad (1)$$

Subject to:-

$$\sum_{j=1}^{20} v_{ij} \leq 1 \quad \text{for } i = 1, 2, 3, 4\ldots\ldots..\ 15 \quad (2)$$

$$\sum_{k=1}^{30} w_{jk} \leq 1 \text{ for } j = 1, 2, 3, 4\ldots\ldots..20 \quad (3)$$

$$\sum_{l=1}^{5} z_{kl} \leq 1 \text{ for } k = 1, 2, 3, 4\ldots\ldots\ldots..30 \quad (4)$$

$$\sum_{k=1}^{30} w_{jk} \geq R_k q_k, \text{ for } j =1, 2, 3, 4\ldots20 \quad (5)$$

$$\sum_{j=1}^{20} v_{ij} = \sum_{j=1}^{20} w_{jk} + \sum_{l=1}^{5} z_{kl}\ , \text{ for } i = 1, 2\ldots\ldots.15,\ k = 1, 2\ldots\ldots30 \quad (6)$$

Fuzzy capacity constraints for sorting costs at collection centres:- µ (sorting capacity feasibility$_{ijk}$) or fuzzy membership function of sorting capacity,

$$\mu \text{ (sorting capacity feasibility}_{ijk}) = 1 - \frac{\sum_{j=1}^{20} S_{ijk}^{f}}{S_j} \quad (7)$$

Fuzzy capacity constraints for processing costs at recycling plants:- µ (processing capacity feasibility$_k$) or fuzzy membership function of processing capacity,

$$\mu \text{ (processing capacity feasibility}_{k}) = 1 - \frac{\sum_{k=1}^{30} P_k^{f}}{G_k} \quad (8)$$

Fuzzy capacity constraints for disposal costs at landfills:- µ (disposal capacity feasibility$_{kl}$) or fuzzy membership function of disposal capacity,

$$\mu \text{ (disposal capacity feasibility}_{kl}) = 1 - \frac{\sum_{k=1}^{30} D_{kl}^{f}}{D_l} \quad (9)$$

In the objective function, the initial term signifies the fixed cost associated with establishing the recycling plant. Following that, the second term accounts for transportation costs from source i to collection centre j for recycling plant k. The third term encompasses sorting costs incurred during the waste plastic recycling process, while the fourth term represents the processing costs. Lastly, the fifth term in the Equation denotes the disposal costs incurred during the transfer from the recycling plant to landfills.

The initial constraint (2) guarantees that the cumulative quantity of waste plastic allocated to a collection centre (j) from all sources (i) remains within the capacity limits of that particular collection centre (j). Moving to the second constraint (3), it ensures that the overall amount of waste plastic transported from all collection centres (j) does not surpass the capacity of the designated recycling plant (k).

The third constraint (4) safeguards that the total quantity of waste plastic allocated from all recycling plants (k) to a landfill (l) adheres to capacity restrictions. Incorporating the fourth constraint (5) ensures that the collective amount of waste plastic assigned from all collection centres (j) to a recycling plant (k) meets or exceeds the demand (D_k) of that specific recycling plant. This guarantees that recycling plants receive an ample supply of waste plastic to fulfill their processing capacities.

Meanwhile, the fifth constraint (6) establishes a balance, ensuring that the total waste plastic allocated from all sources (i) to a collection centre (j) equals the sum of waste plastic assigned from all recycling plants (k) to the same collection centre and the waste plastic designated from all recycling plants (k) to the landfill (l).

The sixth constraint (7) sets an upper limit on sorting costs at collection centres, considering fuzzy sorting cost parameters. Constraint seven (8) ensures that processing costs at recycling facilities do not exceed the respective recycling plant capacities when fuzzy processing cost parameters are considered. Lastly, the eighth constraint (9) guarantees that disposal costs at landfills do not surpass the landfill’s capacity when incorporating fuzzy disposal cost parameters.

4. Fuzzy Programming

In the realm of network design, incorporating uncertainty is imperative. However, the proposed mathematical model encounters limitations due to its reliance on deterministic parameters. A critical point of consideration is the mismatch between the amount of waste plastic supplied to the recycling plant and the demand for the recycling plant to operate at full capacity, making the precise estimation of waste generation a challenging endeavour.

Various methodologies, including fuzzy sets theory, stochastic approaches, and probabilistic optimization, can be employed to tackle this inherent uncertainty. It is essential to note that each of these methods uniquely addresses uncertainty. Significantly, stochastic and probabilistic methods operate under the assumption that probability distributions of uncertain parameters are known. In this study, generating accurate and true random distributions becomes problematic due to a lack of precise historical data, making the application of stochastic techniques unfeasible.

In these circumstances, fuzzy sets theory provides a framework for dealing with a wide range of uncertainty issues. The transformation of fuzzy numbers into their crisp equivalents within the context of the fuzzy model is accomplished by the process of defuzzification, which requires the satisfaction of membership functions at specified degrees. Researchers have proposed various methods to convert fuzzy models into crisp models [23].

This study places particular emphasis on the defuzzification process, specifically highlighting the concepts of the 'expected value' and 'expected interval' of a fuzzy number. This approach stands out by maintaining the model's complexity without introducing additional variables or constraints, as seen in other methods like robust programming [23].

Adopting this specific approach provides decision-makers with valuable insights into potential risk factors associated with constraint violations at each step of the solution process. Moreover, the model accommodates the use of nonlinear membership functions and offers decision-makers the flexibility to set their aspiration levels. Ultimately, it strikes a balance between achieving these aspiration levels and effectively managing the risk of constraint violations [23]. Jiménez et al. [23] described the membership function as follows:

$$\mu_{\tilde{c}}(x) = \begin{cases} f_c(x) = \frac{x - c^p}{c^m - c^p} & if\ c^p \le x \le c^m \\ 1 & if\ x = c^m \\ g_c(x) = \frac{x - c^o}{c^m - c^o} & if\ c^m \le x \le c^o \\ 0 & if\ x\ (c^p or\ x\ )c^o \end{cases} \quad (10)$$

Where, $\tilde{c}$ is a triangular fuzzy number (c^p,c^m,c^o).

Jiménez [24] defined the expected interval (EI) and expected value (EV) of the triangular fuzzy number c¯ as shown using Equations 11 and 12:

$$EI\,(\check{c}) = [E_1^C . E_2^C] = [{}_0^1 f_c^{-1}(x)\,dx.\ {}_0^1 g_c^{-1}(x)dx] \quad (11)$$

$$= \left[\frac{1}{2}\,(c^p + c^m).\ \frac{1}{2}\,(c^m + c^o)\right]$$

$$EV\,(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2\,c^m + c^0}{4} \quad (12)$$

Xanthopoulos and Iakovou [12] provided the degree to which $\bar{a}$ is greater than $\bar{b}$, for any set of fuzzy numbers. $\bar{a}$ and $\bar{b}$.

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)}, & 0 \in [\,E_1^a - E_2^b,\ E_2^a - E_1^b] \\ 1, & E_1^a - E_2^b > 0 \end{cases} \quad (13)$$

If $\mu_M(\bar{a},\bar{b}) \ge \alpha$, $\bar{a}$ is greater than or equal to $\bar{b}$ at least in degree α and denoted by $\bar{a} \ge\ \alpha^{\bar{b}}$.

When $\frac{\alpha}{2} \le \mu_M\,(\bar{a}, \bar{b}) \le 1 - \frac{\alpha}{2}$, $\bar{a}$ is equal to $\bar{b}$ in degree α. Hence, it presented a fuzzy mathematical programming of the kind.

Min z = $\bar{c}^T$x, subject to,

$$\begin{aligned} &\tilde{a}_i x \ge \tilde{b}_i, && i = 1,\ldots\ldots,l \\ &\tilde{a}_i x = \tilde{b}_i, && i = l+1,\ldots\ldots M \\ & && x \ge 0, \end{aligned} \quad (14)$$

The constraints $\tilde{a}_i$x $\ge \tilde{b}_i$ and $\tilde{a}_i$x = $\tilde{b}_i$ can be presented as equivalent forms respectively:-

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \ge \alpha, \quad i = 1,\ldots\ldots,l, \quad (15)$$

and

$$\frac{\alpha}{2} \le \frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \le 1 - \frac{\alpha}{2},\ i = l+1..M \quad (16)$$

Xanthopoulos and Iakovou [12] asserted that a solution x_o is an α-acceptable optimal solution of the model among all feasible decision vectors x if and only if the following Equation is satisfied:

$$c^{-t}x \ge \frac{1}{2}\,c^{-t}x_o \quad (17)$$

Applying Equation 15, this Equation can be written as:

$$\frac{E_2^{c^t x} + E_1^{c^t x}}{2} \ge \frac{E_2^{c^t x_o} + E_1^{c^t x_o}}{2} \quad (18)$$

In conclusion, the transformation of the model given by Equation 14 into an equivalent crisp α-parametric model is outlined as follows:

Min $EV\,(\bar{c})^T$x, subject to

$$[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]\,x \ge \alpha E_2^{b_i} + (1-\alpha)\,E_1^{b_i}, \quad i = 1,\ldots\ldots, l$$

$$\left[\left(1 - \frac{\alpha}{2}\right)E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}\right]x \ge \frac{\alpha}{2}E_2^{b_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{b_i}, \quad i = l+1\ldots M$$

$$\left[\frac{\alpha}{2}E_2^{a_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{a_i}\right]x \le \left(1 - \frac{\alpha}{2}\right)E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \quad i = l+1,\ldots..,M$$

$$X \ge 0 \quad (19)$$

Jiménez et al. [23] introduced an interactive technique aimed at discovering optimal solutions that strike a balance between reducing the objective function value and enhancing the satisfaction of constraints. Assume $x^0(\alpha_k)$ is the α_k-acceptable optimal solution obtained by solving Equation 19, where α =α_k.

By Equation 19, the corresponding fuzzy number representing the objective function is computed as $\tilde{z}^0(\alpha_k)$ $=\tilde{c}^T x^0(\alpha_k)$. The set Q consists of discrete values for solving $\tilde{z}^0(\alpha_k)$ is determined by

$$Q = \left\{\alpha_{k=} \ \alpha_0 + 0.1k | k = 0,1, \ldots \ldots, \frac{1-\alpha_0}{0.1}\right\} \quad (20)$$

Where α_0 is a random value chosen by the decision maker (DM), $0 \leq \alpha_0 \leq 1$. After obtaining all the values of $\tilde{z}^0(\alpha_k)$, the next step is to decide on a value goal $\underline{G}$ and its tolerance threshold $\bar{G}$ as decided by the DM.

This is then used to construct a fuzzy set $\tilde{G}$ to calculate the degree of satisfaction of the DM of the objective value. The membership function of $\tilde{G}$ and degree of satisfaction of the fuzzy goal $\tilde{G}$ by each $\tilde{z}^0(\alpha_k)$ is given as follows:-

$$\mu_{\tilde{G}}(z) = \begin{cases} 1 & z \leq \underline{G}, \\ \frac{z-\bar{G}}{\underline{G}-\bar{G}} & \underline{G} \leq z \leq \bar{G}, \\ 0 & z \geq \bar{G}, \end{cases} \quad (21)$$

$$K_{\tilde{G}}\left(\tilde{z}^0(\alpha)\right) = \frac{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z). \ \mu_{\tilde{G}}(z) dz}{\int_{-\infty}^{+\infty} \mu_{\tilde{z}^0(\alpha)}(z) dz} \quad (22)$$

The degree of balance of each solution is equivalent to α_k is calculated by,

$$\mu_{\tilde{R}}\left(x^0(\alpha_k)\right) = \alpha_k * K_{\tilde{G}}\left(\bar{z}^0(\alpha_k)\right) \quad (23)$$

Where * denotes a t-norm such as the minimum or algebraic product, among others. As indicated, the ideal solution x^* has the highest degree of balance, as represented by,

$$\mu_{\tilde{R}}(x^*) = \max_{\alpha_k \in Q}\{\alpha_k * K_{\tilde{G}}\left(\bar{z}^0(\alpha_k)\right)\} \quad (24)$$

Above Equations 10-24, the proposed fuzzy reverse logistics network model can be completely translated into a corresponding crisp α-parametric model. Subsequently, this transformed model can be efficiently addressed as a mixed-integer linear programming problem.

5. Model Applications and Results

The proposed model is applied to numerical tests to determine the optimal number of recycling plants in Patna, India, with the objective of minimizing the total reverse logistics cost associated with waste plastic recycling. Due to a dearth of information regarding potential sources of waste plastics and demand points for recycled plastics in the Patna region, the identification process involves 15 major sources of waste plastics, 20 major collection centres, 30 major fixed-type recycling plants, and 5 landfills.

The information was gathered through collaboration with officials from the Municipal Corporation of Patna, and on-site reconnaissance was conducted to verify the relevant details. The potential locations for recycling plants are determined based on the availability of land and its current land use, focusing primarily on waste or barren lands and existing waste dumping areas. These identified locations are illustrated in Figure 2.

The overarching goal of the model is to ascertain optimal locations and establish a reverse logistics network at the strategic level, given the insufficiency of data to delve into minute details of the network.

As recycling of waste plastic is not much practiced in the city of Patna, the relevant data has been taken from waste plastic recycling plants located in Indore, the cleanest city in India, by doing reconnaissance and questionnaire survey. Also, some of the relevant data was taken from the Centre for Science and Environment [3] and the Central Pollution Control Board [2].

The fixed cost associated with setting up the recycling plants is US$ 60979, including initial land and machinery investment. The collection cost, including the cost of purchasing unsegregated waste plastic, is US$ 43830 every year. The sorting cost done manually to segregate different types of waste plastic is US$ 46740.

The cumulative processing cost of electricity and the cost of processing the sorted and shredded waste plastic is US$ 28630. The transportation cost is US$ 36.59 per ton, which is included in the collection cost. The disposal cost or tipping charge is the disposal cost of waste plastics or materials after processing, which is US$ 12.20 per ton.

The cost related to the environment has been incorporated into disposal costs. Data that are used in the implementation of the model are shown in Table 1. The possibility distributions of objective values are calculated for every discrete value in the set Q = {0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}.

After evaluating the possibility distributions of the objective values, DM is needed to evaluate $\underline{G}$ and $\bar{G}$ as 1.2×10^6 and 2.1×10^6 respectively. The compatibility index and the degree of balance of each solution can be easily computed using the t-norm minimum and referring Equations 23 and 24.

The results obtained are shown in Table 2, and the solution of the corresponding crisp α-parametric model with α = 0.7 has the highest degree of balance. If the DM is dissatisfied with this solution, one can change the values of $\underline{G}$ and $\bar{G}$. Continuing with this modification allows for the utilization of results from the equivalent crisp α-parametric model.

Table 2. Data used to implement the model

Description	Value
Total Supply of Waste Plastic in Patna Region	32,850 tons/year
Total Demand for Waste Plastic	7673 tons/year
Processing Capacity of Recycling Plants	2800 tons/year
Handling Capacity of Recycling Plants	2800 tons/year
Storage Capacity of Recycling Plants	600 tons/year
Fixed Cost of Opening a Recycling Plant	US$ 60979
Transportation Cost	US$ 36.59 per ton per km
Sorting Cost	US$ 116.85 per ton/year
Processing Cost	US$ 71.57 per ton/year

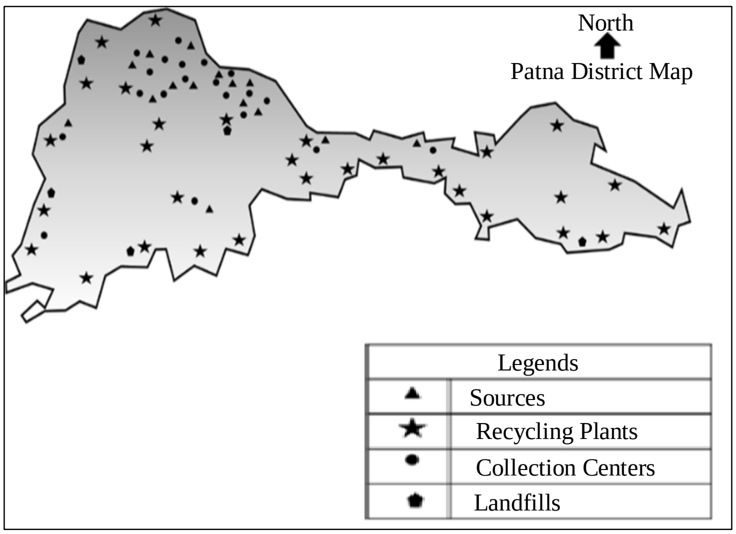

Fig. 2 Potential reverse logistics network for the fuzzy model

Consequently, this approach tends to maintain the overall complexity of the problem without introducing additional complexities. The FMILP solution procedure was implemented using MATLAB 2020b, supported by a configuration of 4GB RAM, 500GB memory, and an i7 core processor.

Examining Figure 3 in detail unveils that approximately 49% of the total cost of the waste plastic recycling network is attributed to processing, sorting, and transportation expenses. Upon further breakdown of the total cost, it becomes evident that the fixed cost for establishing a recycling facility constitutes roughly 50% of the overall expenditure.

This is followed by sorting costs at 17.01%, transportation costs at 16.96%, processing costs at 14.71%, and the least contribution coming from disposal costs at 2.5%. The following assumptions are also made for the application of the developed reverse logistics network of waste plastic.

- Mechanical efficiency remains consistent across similar types of facilities, such as collection centres and recycling plants.
- Municipalities are the city's primary suppliers of waste plastics. All the waste is collected from different households and collection points located in that particular municipality from where it is transported to the collection centres where all the different plastics and non-plastic materials are separated and shredded based on the type of plastics. In the final stage, the waste products reach the recycling plants, and subsequently, the residual waste from these recycling plants is disposed of in landfills.
- The waste belonging to the non-plastic categories like dirt, moisture, aluminium, and iron are not sorted to maintain mechanical efficiency, and this disposal happens only in collection centres.
- The network considers eight types of products, encompassing non-plastic, PET, HDPE, V, LDPE, PP, PS, and other resins, along with layered multi-material.

Table 3. α-acceptable optimum solutions for the fuzzy MILP model

Feasibility Degree (α)	Possibility Distribution of Objective Value	Compatibility Index	The Degree of Balance of Each Solution	No. of Recycling Plants
0.4	87242,120638,167560	0.8407	0.4000	16
0.5	90145,120750,178798	0.8077	0.5000	16
0.6	91454,120978,183550	0.7724	0.6000	16
0.7	91853,121413,187770	0.7335	0.7000	16
0.8	92670,121856,194879	0.6912	0.6912	17
0.9	93568,122322,204565	0.6445	0.6445	17
1	94665,122965,208957	0.5937	0.5937	17

6. Sensitivity Analysis

Sensitivity analysis is an important part of any optimization study since it determines how changes in a model's input parameters impact its output. In the context of the research paper, a fuzzy MILP model for waste plastic recycling sensitivity analysis can provide insight into the model's robustness and reliability under varying conditions and different feasibility degrees (α).

Table 4. Values of different cost items associated with varying feasibility degree (α)

Feasibility Degree (α)	Fixed Cost (US$)	Transport Cost (US$/tons)	Sorting Cost (US$/tons)	Processing Cost (US$/tons)	Disposal Cost (US$/tons)	Total Cost (US$)
0	60778	20594	19650	16679	1054	118755
0.1	60778	20594	19887	16890	1093	119242
0.2	60778	20594	20255	17106	1133	119866
0.3	60778	20594	20538	17255	1155	120320
0.4	60778	20594	20691	17404	1171	120638
0.5	60778	20594	20778	17258	1342	120750
0.6	60778	20594	20816	17350	1440	120978
0.7	60778	20594	20650	17864	1527	121413
0.8	60778	20594	21223	17616	1645	121856
0.9	60778	20594	21721	17543	1686	122322
1	60778	20594	21782	17950	1861	122965
Deterministic Model	60778	20594	21850	18263	1889	123374

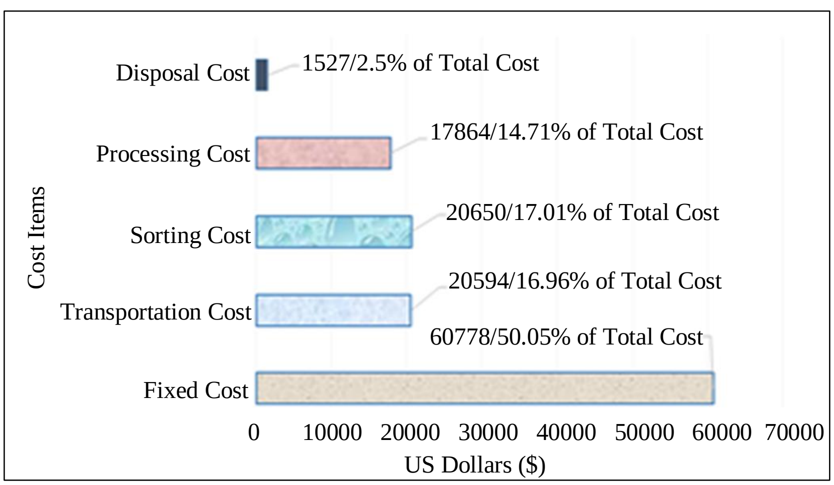

Fig. 3 Distribution of different costs in the objective function

Table 5. Results of optimization with varying transportation costs

Condition	Transportation Cost (US$/tons)	Total Cost (US$)	No. of Plants Required
Base	20594	121413	16
10% Reduction	18535	118726	16
20% Reduction	16475	113475	16
10% Increase	22653	125,297	17
20% Increase	24713	130,158	17

Table 6. Results of optimization with varying sorting costs

Condition	Sorting Cost (US$/tons)	Total Cost (US$)	No. of Plants Required
Base	20650	121413	16
10% Reduction	18585	120832	16
20% Reduction	16520	118080	16
10% Increase	22715	122,377	16
20% Increase	24713	124,269	16

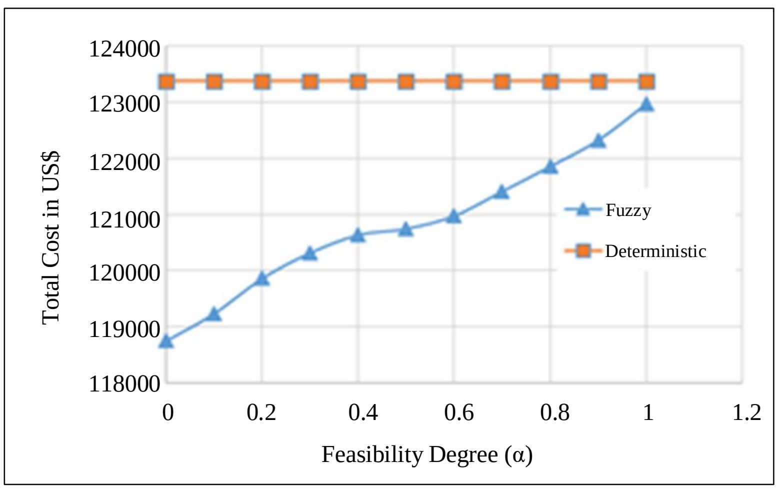

Fig. 4 Variation of total costs of fuzzy and deterministic models

Table 7. Results of optimization with varying processing costs

Condition	Processing Cost (US$/tons)	Total Cost (US$)	No. of Plants Required
Base	17864	121413	16
10% Reduction	16078	120572	16
20% Reduction	14291	119726	16
10% Increase	19650	122248	16
20% Increase	21437	123085	16

Table 8. Results of optimization with varying disposal costs

Condition	Disposal Cost (US$/tons)	Total Cost (US$)	No. of Plants Required
Base	1527	121413	16
10% Reduction	1374	121025	16
20% Reduction	1222	120637	16
10% Increase	1680	121800	16
20% Increase	1832	122188	16

7. Conclusion and Future Study

Disposing waste plastic in developing countries, which generate large amounts of plastic waste and have limited landfill space, is challenging, especially in urban areas where recycling infrastructure is lacking. The study aims to adopt sustainable practices in plastic recycling to mitigate the environmental impact of plastic pollution.

By analyzing the entire lifecycle of waste plastic materials, from collection to recovery, the research identifies key tailbacks and inefficiencies in existing reverse logistics systems. The strategic plan proposed for facility allocation optimization and integration of reverse logistics strategies offers a way to enhance efficiency and promote a more circular approach to plastic consumption. This research contributes novel insights into the often-unnoticed aspects of reverse logistics in waste plastic recycling, providing valuable knowledge for policymakers, industry stakeholders, and practitioners working towards a more sustainable and eco-friendly environment.

This work provides a robust reverse logistics model that utilizes fuzzy logic and Mixed-integer Linear Programming (MILP) to optimize the reverse logistics flow of waste plastic from various sources to collection centres, recycling companies, and landfill sites. Stakeholders in the waste plastic recycling industries can use this model to make data-driven decisions that improve sustainability and efficiency and reduce overall reverse logistics costs, including fixed, transportation, sorting, processing, and disposal costs. The model's adaptability in managing multiple sources, collection centres, recycling plants, and landfills enables it to be scaled to fulfil various waste management requirements.

Furthermore, a sensitivity analysis was carried out to determine the practicality of the proposed fuzzy MILP model by adjusting the proportions of various expenditures involved with waste plastic recycling. Examining the findings reveals the efficacy of the fuzzy MILP approach for reverse logistics of waste plastic recycling under uncertainty. The suggested fuzzy MILP model outperforms deterministic methods in real-world issues where accurate data on supply and demand are unavailable.

Although this study has produced promising results, it has some limitations that need to be addressed. Firstly, the model's effectiveness is highly dependent on the accuracy of the input data, which covers transportation costs, processing costs, and demand estimates. The use of inaccurate or outdated data may lead to suboptimal solutions.

Additionally, the model's complexity may require extensive computational resources, especially when dealing with a large number of sources, collection centres, recycling plants, and landfills. This could pose difficulties for real-time decision-making. Although the fuzzy MILP model approach addresses uncertainty and ambiguity, it still makes several assumptions that may not fully capture the dynamic nature of waste plastic recycling supply chains.

Possibilistic mixed-integer programming problems present a significant challenge, notably regarding solution stability. Analyzing these problems equivalently is considerably more intricate compared to traditional linear programming, where shadow pricing and sensitivity analysis are straightforward. Additional factors like environmental impact and regulatory constraints are recommended to enhance the model's practical applicability.

Exploring future research directions could yield more stable solutions. For instance, simulation-based optimization approaches can be employed, and evolutionary computational techniques may offer solutions for the corresponding fuzzy MILP model. Furthermore, expanding the model to accommodate multiple competing objectives such as cost reduction, resource efficiency, and minimized environmental impact can provide decision-makers with a more comprehensive set of trade-offs for evaluation.

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