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Original Article

Structural Multimodal Analysis of Bridges Using Adaptive Neuro-Fuzzy Inference System and Artificial Intelligence Metamodels

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Abstract *- This study investigates using the Adaptive Neuro-Fuzzy Inference System (ANFIS) and machine learning methods for predicting the natural vibration properties of a bridge modeled as a three-degree-of-freedom system. The ANFIS method, which integrates Fuzzy Logic and Neural Networks, was used to analyze a dataset generated from four input and three output variables representing the first three natural vibration periods. A Monte Carlo simulation created a probability density distribution for these variables. The study aims to enhance the precision of bridge response predictions using artificial intelligence, offering a more adaptable alternative to conventional analytical approaches for structural dynamic analysis. The results demonstrate the effectiveness of ANFIS in predicting natural mode periods, with a comparative analysis revealing that ANFIS models, particularly those using triangular membership functions, provide accurate predictions. The findings highlight the importance of dataset partitioning and membership function selection in optimizing ANFIS performance. A comparative evaluation with Artificial Neural Networks (ANN) and the Response Surface Method (RSM) shows that ANFIS and ANN closely match the reference values from algebraic modal analysis, while RSM exhibits some deviations. The study concludes that ANFIS is a viable method for real-world applications in structural engineering, offering a balance between computational efficiency and interpretability.*

Keywords *- Reinforced Concrete Bridges, Adaptive Neuro-Fuzzy Inference System, Bridge engineering, Structural dynamics, Monte carlo simulation, Artificial Neural Network, Response Surface Methodology.*

1. Introduction

1.1. Literature survey of ANFIS Method in Engineering

The ANFIS model combines numerical and linguistic knowledge, leveraging Artificial Neural Networks' classification and pattern recognition capabilities (ANNs). In contrast to traditional ANNs, the ANFIS model provides better interpretability for the user and is less prone to memorization errors [1]. ANFIS generates an output prediction model by integrating human knowledge (as fuzzy if-then rules) and empirical input-output data sets through least squares estimation techniques [2]. The ANFIS method has been applied to various problems in civil engineering that can be cited; Fallahian [3] applied ANFIS in a two-stage method for identifying structural damage combined with particle swarm optimization—Mittal [4] compared ANFIS and ANN to predict peak ground acceleration in the Indian Himalayan region. Chen [5] applied ANFIS to model and analyze emergency evacuations from metro stations. Kabalan [6] utilized ANFIS in their framework for centralized and dynamic pedestrian management in railway stations. Alawad [7] explored wireless sensor networks with ANFIS to create smarter railway stations. Mamat [8] applied the ANFIS method to predict road embankments' performance on soft soil stabilized with prefabricated vertical drains. In addition, several applications related to bridges have been mentioned. Danilatos [9] used Neuro-Fuzzy architectures, including ANFIS, for bridge structural health. (Nguyen 2024) applied ANFIS, alongside optimized Artificial Neural Networks (ANN), for vibration-based Structural Health Monitoring (SHM) of the Dębica railway steel bridge. Mainly in Civil Engineering, we can cite the four main areas where ANFIS has been applied:

- Application of ANFIS in Civil Engineering System Control: A notable application of the ANFIS method in civil engineering concerns the design of intelligent controllers for vibration mitigation in building structures. A study conducted by Palizvan [11] proposed a type 2 ANFIS controller combined with a robust PID to mitigate vibrations in an uncertain building structure.

This approach demonstrated superior effectiveness to conventional controllers, particularly in the presence of structured and unstructured uncertainties.

- Application of ANFIS in forecasting in civil engineering: A notable application of the ANFIS method is predicting the success of construction projects. A study conducted by Moghimi [12] developed an ANFIS system to predict the success of medium and large construction projects. This system was validated on a real project in Western Australia and showed a forecasting accuracy of 97.46% using linear membership functions. Identified success factors, such as clear objectives, management support, project manager competence, and transparent procurement processes, were incorporated into the ANFIS model to improve forecasting accuracy.
- Application of ANFIS in Pattern Recognition in Civil Engineering: ANFIS is used in civil engineering for pattern recognition, particularly in analysing structures and materials. For example, a study by Jafari [13] developed a hybrid model combining ANFIS and fuzzy clustering to predict groundwater level fluctuations, illustrating the effectiveness of ANFIS in analyzing complex data related to water resources. Similarly, research published in 2024 by Sekfali and Lafifi [14] used ANFIS to analyze the reliability of reinforced earth retaining walls, taking into account geotechnical uncertainties, demonstrating the application of ANFIS in the evaluation of geotechnical structures.
- Application of ANFIS in Civil Engineering Energy Systems: ANFIS is widely used in civil engineering to optimize and predict energy consumption in buildings and infrastructure. ANFIS can effectively model energy consumption profiles by considering multiple complex variables such as climate conditions, room occupancy, and equipment characteristics. This accurate modeling helps improve energy management, leading to better planning and significant cost reduction.

Moreover, the advantages and disadvantages of the ANFIS method are presented in Tables 1 and 2:

Table 1. Advantage of ANFIS Method [15-18]

Advantages of the ANFIS Method	Significance
Ability to model the non-linearity of a system	ANFIS can accurately represent complex relationships between variables, even if they do not follow linear logic.
Automatic adaptation	The system automatically adjusts its parameters to adapt to the input data, making it autonomous.
Fast-Learning	ANFIS learns efficiently from data, reducing the time required for training.
Good generalization ability	It can predict never-before-seen data well after good training, even outside the training sample.
Great flexibility:	Its architecture allows for many variations depending on the specific needs of a problem.
Smooth adaptability of rules	The rules in ANFIS are also adaptable and can be modified depending on the problem to be modeled.
Rapid learning capability	Thanks to its learning mechanisms, ANFIS can quickly adapt to new data, providing increased responsiveness.

Table 2. Disadvantage of ANFIS Method [15, 19, 20]

Disadvantages	Meaning
Dimensionality problem	Increasing the number of inputs can lead to exponential complexity, making the model difficult to manage.
Underfitting, overfitting, and difficult convergence.	Underfitting occurs when the model is too simple to represent the data, overfitting when it is too complex and adapts too much to noise, and hard convergence refers to failing to find a stable solution during training.
Complex choice of membership functions	Determining the right type and number of fuzzy functions can be difficult.
Complexity explodes with the number of inputs.	The more input variables there are, the more complex and difficult it is for the model to manage.
A tradeoff between accuracy and readability:	Improving the model's accuracy may reduce its ability to be easily interpreted by a human.
High computational cost.	The computations required to train the model can be resource and time-intensive.
Challenging positioning of membership functions	Optimal localization of fuzzy functions in the input space is often challenging.

Table 3. Limits of some methods of artificial intelligence and solutions provided by ANFIS

	Limits	Solutions Provided by ANFIS Method	References
ANN	- Black-box behavior, difficult to interpret - Requires a large amount of data - Risk of Overfitting - Poor ability to explain the underlying rules	- Incorporates interpretable fuzzy logic - Better ability to model systems with little data - Less susceptible to overfitting thanks to fuzzy regularization	[1, 21, 22]
SVR	- Rigid statistical approach - Limited to linear/quadratic models - Poorly suited to complex nonlinear systems	- ANFIS efficiently captures nonlinear relationships - Combines empirical data with human logic (via fuzzy rules)	[23, 24]
RSM	- Sensitive to the choice of kernel and hyperparameters - Difficulty integrating a priori knowledge - Poor interpretability	- ANFIS does not require complex kernel functions - Integrates prior knowledge via fuzzy rules - More transparent in terms of decision structure	[21, 24, 25]

Table 3 presents some limitations of Artificial Intelligence methods like neural networks, SVR, and RSM methods and how ANFIS can overcome these limitations.

1.2. Problematic and Novelty of this Research

This research is distinguished by applying multimodal spectral modal analysis to a bridge in Morocco, whose structural configuration presents a particular complexity: a span resting on a central pier composed of three columns framed by two end supports. Unlike the often-used single-modal approaches, multimodal analysis makes it possible to consider all significant vibration modes, thus providing a more realistic and precise assessment of dynamic behavior under seismic stress. To optimize the prediction of natural vibration periods, this study combines this advanced analytical approach with artificial intelligence techniques, particularly the Adaptive Neuro-Fuzzy Inference System (ANFIS). The latter is explored through various membership functions and dataset configurations and compared to two other commonly used metamodels: Artificial Neural Networks (ANN) and the Response Surface Method (RSM). The results highlight the superiority of the ANFIS model—especially with triangular membership functions—in terms of predictive accuracy, thus strengthening the reliability of seismic analyses applied to bridge structures.

1.3. Paper Organization

This work is structured to systematically explore the application of the Adaptive Neuro-Fuzzy Inference System (ANFIS) in computing the natural vibration properties of bridge structures. The methods section introduces the ANFIS method, outlining its main principles while detailing its methodology, including its five-layer architecture and mathematical formulation. The results section presents the structural configuration of the bridge, the dynamic multimodal analysis, and the dataset used for training and testing the ANFIS models, along with the results and performance metrics. The discussion section discusses the comparative analysis of ANFIS with other machine learning techniques, such as Artificial Neural Networks (ANN) and the Response Surface Method (RSM). The Conclusion section presents the key findings and future research directions. The paper aims to demonstrate the effectiveness of ANFIS in structural engineering applications, emphasizing its balance between computational efficiency and interpretability.

2. Methods

The Adaptive Neuro-Fuzzy Inference System (ANFIS) is a hybrid intelligent system. It combines the strengths of artificial neural networks and fuzzy logic to model complex systems effectively. ANFIS employs a Sugeno-type fuzzy inference system, which is particularly suited for generating precise outputs through rule-based systems. This hybrid model optimizes the fuzzy rules using neural network learning techniques, balancing interpretability and performance. Figure 1 represents the general structure of ANFIS. The ANFIS model consists of a structured five-layer architecture, with each layer playing a distinct role in the fuzzy inference process. The first layer (Input Layer) is responsible for fuzzifying the input variables. Each node in this layer corresponds to an input variable and computes the degree to which the input belongs to a fuzzy set, as defined by its membership function.

In the second layer (Rule Layer), the firing strength of each fuzzy rule is calculated based on the membership values obtained in the first layer. This layer combines the membership degrees of multiple input variables to determine the activation strength of each rule. The third layer (Normalization Layer) ensures that the firing strengths of all rules are normalized, such that their total contribution adds up to one. This step ensures the proportional influence of each rule in the subsequent processing. The fourth layer (Defuzzification Layer) computes the output of each rule by applying specific parameters associated with that rule. This step translates the fuzzy rule evaluations into numerical values based on the inputs and their weights. Finally, the fifth layer (Output Layer) aggregates the contributions of all rules to produce the final output. This layer combines the weighted outputs of the rules into a single crisp value, representing the system's overall prediction or response. This layered structure ensures that the input data is systematically transformed into an accurate and interpretable output through fuzzy logic and adaptive learning. Table 1 explains each layer in the structure of the ANFIS Method with the corresponding analytical formulation.

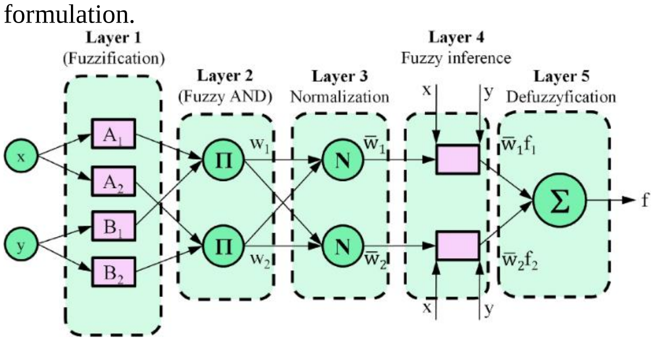

Fig. 1 Structure of ANFIS [26-28]

Table 4. Explanation of the layers in the structure of ANFIS Method[26-28]

Layers	Explication of layers
Layer 1 (Input Layer):	The input variables are fuzzified using membership functions. Each node represents an input variable and calculates the degree of membership. $O_i^{(1)} = u_A(x)$
Layer 2 (Rule Layer):	The firing strength of each rule is calculated based on the degrees of membership of the input variables. $O_i^{(2)} = w_i = u_A(x).u_B(y)$
Layer 3 (Normalization Layer):	The firing strengths are normalized so that their sum equals 1. $O_i^{(3)} = \frac{w_i}{\sum_{i=1}^{N} w_i}$
Layer 4 (Defuzzification Layer):	The outputs of each rule are calculated by applying the rule parameters. $O_i^{(4)} = w_i(p_1x + p_2y + p_3)$
Layer 5 (Output Layer):	The final output is the weighted sum of the rule outputs. $O_i^{(5)} = \sum_{i=1}^{N} O_i^{(4)} = \sum_{i=1}^{N} w_i(p_1x + p_2y + p_3)$

In ANFIS, the choice of membership functions plays a crucial role in the system's performance. Several membership functions are commonly used to represent fuzzy sets, and their selection depends on the specific characteristics of the input variables and the application. The most widely used membership functions include:

2.1. Gaussian Membership Function

The Gaussian membership function is popular due to its smooth nature and ability to model uncertainty effectively. It is represented as follows in (1), where $u_A(x)$ is the membership degree of x in fuzzy set A, c_A is the center of the Gaussian function, and σ_A is the width (standard deviation), controlling how spread out the function is.

$$u_A(x) = exp\left(-\frac{(x-c_A)^2}{2{\sigma_A}^2}\right) \quad (1)$$

2.2. Triangular Membership Function

The Triangular membership function is defined by three parameters: the left edge a, the peak b, and the right edge c. The Triangular membership function is defined as follows in (2):

$$u_A(x) = \begin{cases} \frac{x-a}{b-a} & for\ a \leq x \leq b \\ \frac{c-x}{c-b} & for\ b \leq x \leq c \\ 0 & Otherwise \end{cases} \quad (2)$$

2.3. Trapezoidal Membership Function

The Trapezoidal membership function is represented by four parameters a, b, c and d The Triangular membership function is defined as follows in (3):

$$u_A(x) = \begin{cases} 0 & if\ x < a\ or\ x > d \\ \frac{x-a}{b-a} & for\ a \leq x \leq b \\ 1 & for\ b \leq x \leq c \\ \frac{d-x}{d-c} & for\ c \leq x \leq d \end{cases} \quad (3)$$

2.4. Bell Shaped Membership Function

The Bell Shaped Membership Function is represented by three parameters a, b and c. It is defined as follows in (4):

$$u_A(x) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \quad (4)$$

These membership functions provide flexibility in representing the fuzziness of real-world problems and contribute to the effectiveness of the ANFIS model in predicting and analyzing complex systems.

3. Results

3.1. State of the art: Artificial Intelligence Methods for Dynamic Analysis of Bridges

The study of bridge dynamics aims to understand and predict the response of structures under various stresses, including those due to moving loads and natural phenomena such as earthquakes. Artificial intelligence (AI) has thus been widely adopted to overcome these limitations by proposing models capable of learning directly from experimental or simulation data. Response Surface Methodology (RSM) has also been used to simplify the prediction of dynamic responses, although its effectiveness decreases when faced with highly nonlinear phenomena typical of seismic events.

In addition, more recent methods, such as deep learning neural networks and hybrid algorithms combining fuzzy logic and machine learning, such as ANFIS, have improved the accuracy and understanding of seismic responses of bridges. These advances allow for better estimating dynamic displacements, velocities, and stresses and improve real-time monitoring systems and seismic risk management.

3.2. Structural Configuration

The study uses an application of ANFIS metamodeling to estimate the natural properties of vibration for a bridge structure. Therefore, the structure considered in our study is a two-span concrete bridge composed of five prestressed girders with a length of 39 m for each span and a width of 12 m. The pier is described as having a pier cap and three columns with a diameter of 1.4 m for each. Figure 2 displays the longitudinal profile of the bridge structure.

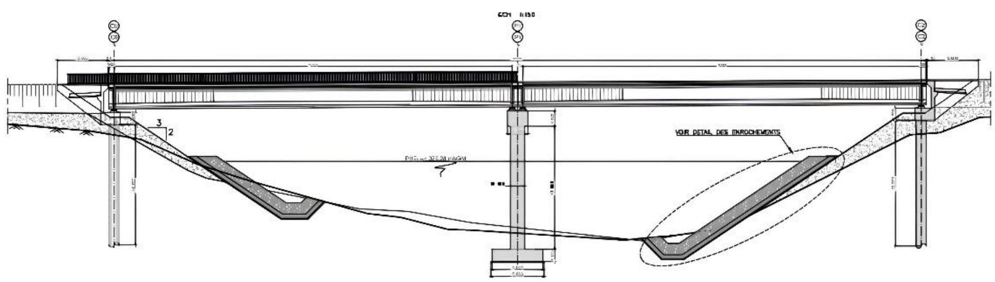

Fig. 1 Longitudinal profile of the bridge

3.3. Dynamic Multimodal Analysis: Mathematical Problem Formulation

Multimodal spectral analysis is more sophisticated than the monomodal method, and it is very effective in analyzing the response of complex linear elastic structures to earthquake excitation. For a seismic analysis, this analysis considers all the vibration modes that contribute to the structure's response to seismic excitation. It is based on a dynamic calculation spectral calculation and takes a static account of differential displacements. Statistical combinations of the maximum modal contributions obtain the overall response. The deck is considered an infinitely rigid diaphragm. It can, therefore, be represented as a mass concentrated and applied at its center of gravity, with half the mass of the piers taken into account. The system is represented by three degrees of freedom governing the longitudinal translation movement, as illustrated in Figure 3. The system's kinetic energy is formulated as follows in (5).

$$T = \frac{1}{2}M_t\dot{x_1}^2 + \frac{1}{2}M_p\dot{x_2}^2 + \frac{1}{2}M_t\dot{x_3}^2 \qquad (5)$$

The system's potential energy is also written in (6).

$$V = \frac{1}{2}K_a{x_1}^2 + \frac{1}{2}K_a(x_2 - x_1)^2 + \frac{1}{2}K_p{x_2}^2 + \frac{1}{2}K_a(x_3 - x_2)^2 \qquad (6)$$

The equations of motion using the energy approach of Lagrange theorem are given as follows in (7), and for an undamped free system, they are written as given in (8).

$$L = T - V \qquad (7)$$

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \qquad (8)$$

By applying Lagrange's equation for each degree of freedom, the following system of equations is obtained (9-11):

$$M_t\ddot{x}_1 + K_a x_1 - K_a(x_2 - x_1) = 0 \qquad (9)$$

$$M_p\ddot{x}_2 + K_p x_2 + K_a(x_2 - x_1) - K_a(x_3 - x_2) = 0 \qquad (10)$$

$$M_t\ddot{x}_3 + K_a(x_3 - x_2) = 0 \qquad (11)$$

By expressing the above equations in matrix form, the resulting system can be written as shown in Equations (12–13), where M denotes the mass matrix, and K represents the stiffness matrix.

$$[\mathrm{M}]\{\ddot{x}\} + [\mathrm{K}]\{x\} = \{0\} \qquad (12)$$

$$[\mathrm{M}] = \begin{pmatrix} M_t & 0 & 0 \\ 0 & M_p & 0 \\ 0 & 0 & M_t \end{pmatrix} ;\ [\mathrm{K}] = \begin{pmatrix} 2K_a & -K_a & 0 \\ -K_a & 2K_a + K_p & -K_a \\ 0 & -K_a & K_a \end{pmatrix} \qquad (13)$$

The computation of natural periods consists of resolving the determinant of the following system (14).

$$|[\mathrm{K}] - [\mathrm{M}]\{\omega^2\}| = 0 \qquad (14)$$

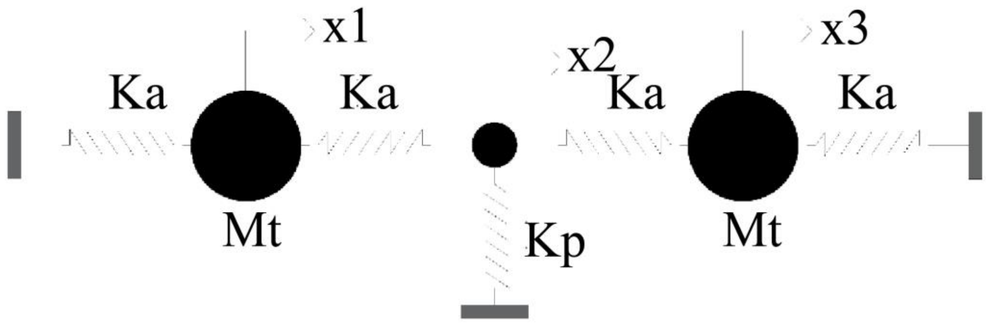

Fig. 3 Dynamic model of the bridge

The masses and stiffness values required for the multimodal analysis of the corresponding bridge structure are provided in Table 5 below.

Table 1. Parameters of the case study

Description	Unit	Value
Mass of the deck M_t	T	1200
Mass of the pier M_p	T	16.5
Bearing stiffness K_a	kN/m	18500
Pier stiffness K_p	MN/m	170

3.4. Data Set

The model was created utilizing experimental data from various reinforced concrete bridges with the same type of case study and the simulation of Monte Carlo. These data are obtained from technical studies offices based on analyzing technical design notes of 150 bridges with the same configuration as the case study. The parameters and their intervals of variation are summarized in Table 6. The probability density functions of all data are given in Figures 4, 5 and 6.

Table 2. Data parameters and variation ranges

Type	Description	Unit	Variation interval
INPUTS	Mass of the deck M_t	T	$727 \leq M_t \leq 1250$
	Mass of the pier M_p	T	$1.785 \leq M_p \leq 20.135$
	Bearing stiffness K_a	kN/m	$18404 \leq K_a \leq 20000$
	Pier stiffness K_p	MN/m	$1.28 \leq K_p \leq 1287.9$
OUTPUT	Period of 1st mode	S	$1.23 \leq T_1 \leq 2.15$
	Period of 2nd mode	S	$0.86 \leq T_2 \leq 1.19$
	Period of 3rd mode	S	$0.075 \leq T_3 \leq 0.12$

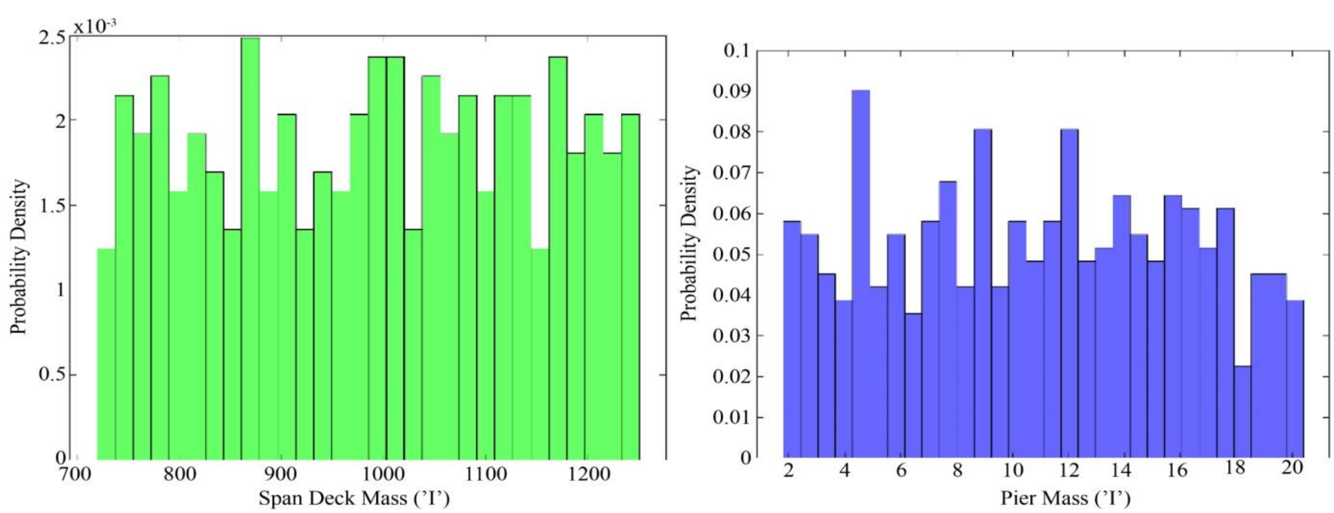

Fig. 4 Probability Density Function of the span deck and pier mass

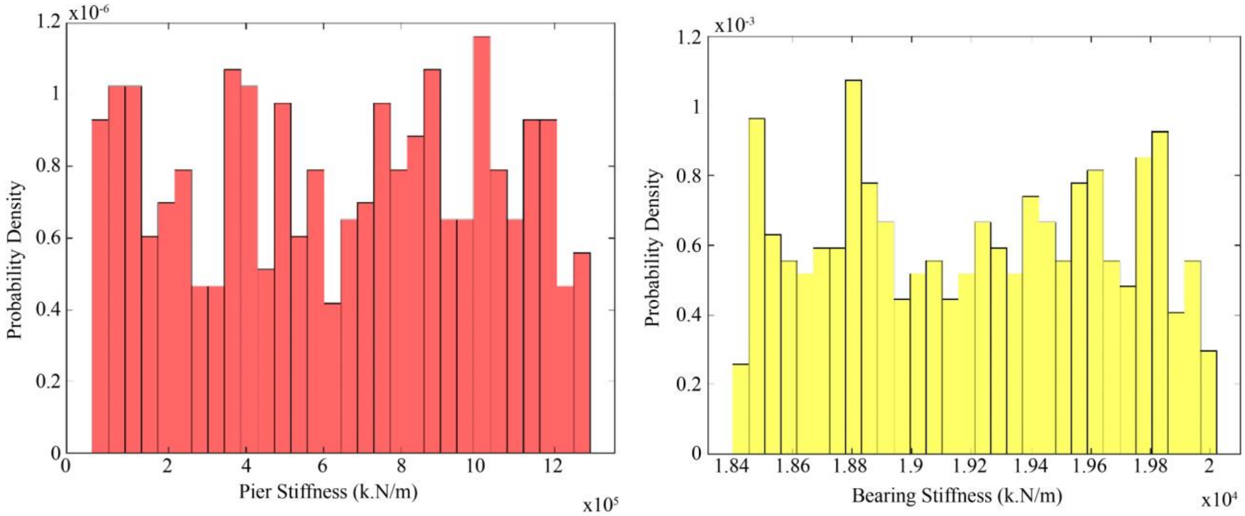

Fig. 5 Probability density function of the pier and bearing stiffness

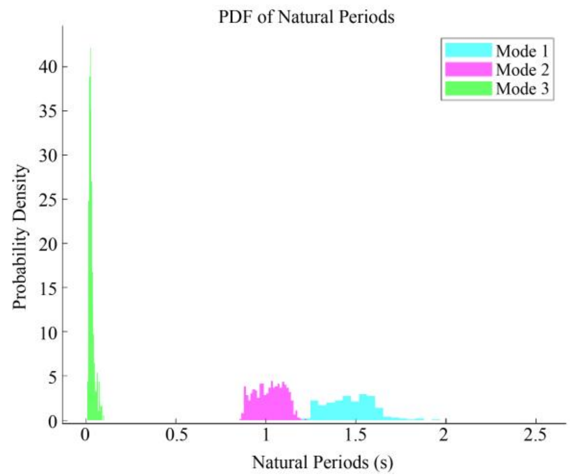

Fig. 6 Probability density function of the longitudinal natural periods of vibration modes

Table 7. Performance metrics of ANFIS for model 1

Membership function	RMSE	
Triangular	$RMSE_{T1}$	0.1068
	$RMSE_{T2}$	0.0147
	$RMSE_{T3}$	0.0072
Trapezoidal	$RMSE_{T1}$	0.1269
	$RMSE_{T2}$	0.0774
	$RMSE_{T3}$	0.0103
Generalized Bell-Shaped	$RMSE_{T1}$	0.1181
	$RMSE_{T2}$	0.0269
	$RMSE_{T3}$	0.0116
Gaussian	$RMSE_{T1}$	0.1312
	$RMSE_{T2}$	0.0463
	$RMSE_{T3}$	0.0105

3.5. Application of ANFIS Method

An Adaptive Neuro-Fuzzy Inference System (ANFIS) is developed using three models trained on a dataset comprising 150 samples. The dataset is divided into training and testing subsets, with varying proportions across the models. Model 1 utilizes 60% of the data for training and 40% for testing, Model 2 employs a 70%-30% split, and Model 3 adopts an 80%-20% division. The membership functions implemented in this study include triangular, trapezoidal, generalized bell-shaped and Gaussian functions. The ANFIS approach was developed using MATLAB software, and the training process was conducted over 100 epochs. Since the prediction problem under investigation is a regression task, the performance of the models is evaluated using the root mean squared error (RMSE). Tables 7, 8, and 9 present the performance metrics related to the implementation of the ANFIS method for Model 1, Model 2, and Model 3 across three natural mode periods. The plots in Figures 7 to 9 illustrate the predicted mode periods for each membership function corresponding to the training ratio of Model 1. Similarly, Figures 10 to 12 show the results for Model 2, and Figures 13 to 15 present the results for Model 3.

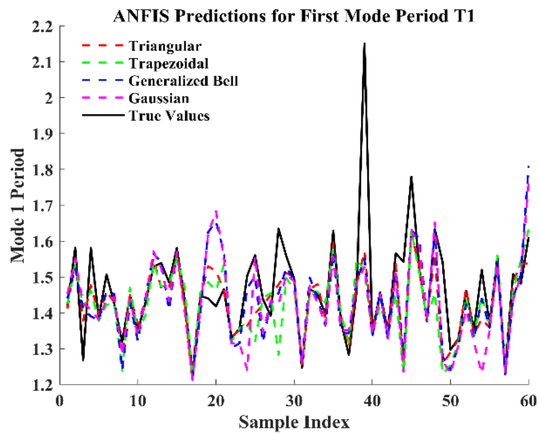

Fig. 7 Model 1 - ANFIS prediction of T1 for all membership functions

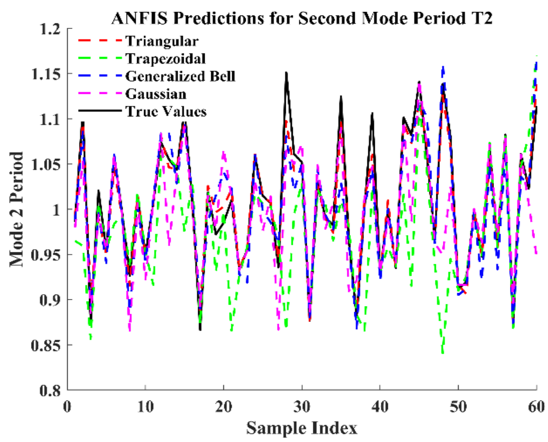

Fig. 8 Model 1 - ANFIS prediction of T2 for all membership functions

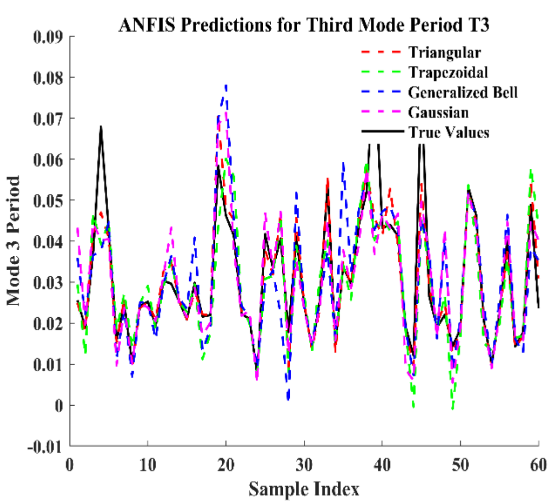

Fig. 9 Model 1 - ANFIS prediction of T3 for all membership functions

Table 3. Performance metrics of ANFIS for model 2

Membership function	RMSE	
Triangular	$RMSE_{T1}$	0.041
	$RMSE_{T2}$	0.0182
	$RMSE_{T3}$	0.0076
Trapezoidal	$RMSE_{T1}$	0.0779
	$RMSE_{T2}$	0.0635
	$RMSE_{T3}$	0.007
Generalized Bell-Shaped	$RMSE_{T1}$	0.0871
	$RMSE_{T2}$	0.0399
	$RMSE_{T3}$	0.007
Gaussian	$RMSE_{T1}$	0.1172
	$RMSE_{T2}$	0.037
	$RMSE_{T3}$	0.0082

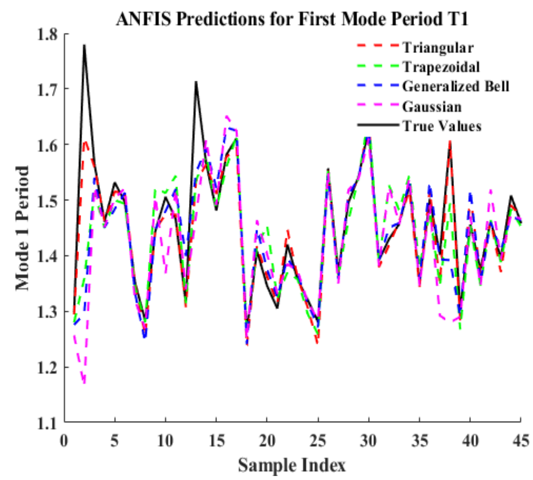

Fig. 10 Model 2 - ANFIS prediction of T1 for all membership functions

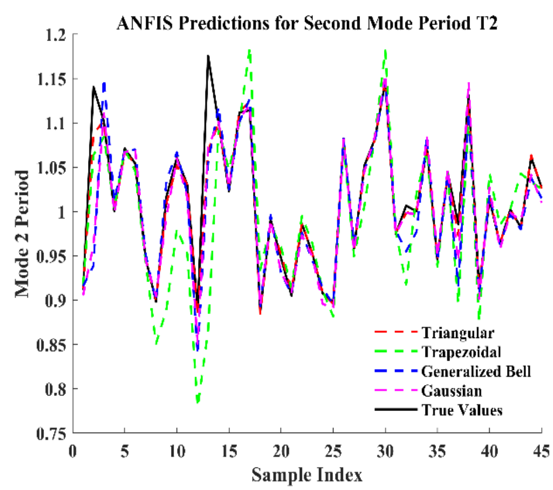

Fig. 11 Model 2 - ANFIS prediction of T2 for all membership functions

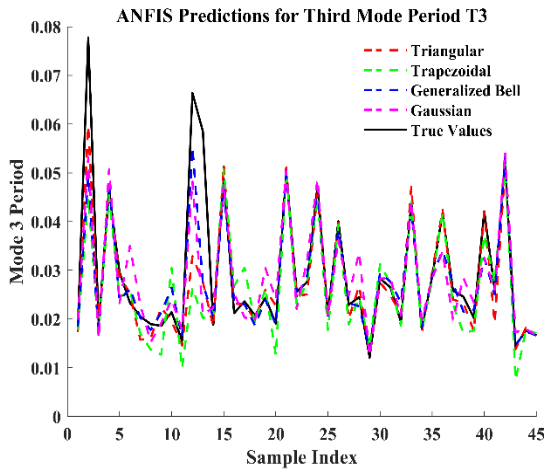

Fig. 12 Model 2 - ANFIS prediction of T3 for all membership functions

Table 4. Performance metrics of ANFIS for model 3

Membership function	RMSE	
Triangular	$RMSE_{T1}$	0.042
	$RMSE_{T2}$	0.0128
	$RMSE_{T3}$	0.0037
Trapezoidal	$RMSE_{T1}$	0.0771
	$RMSE_{T2}$	0.0158
	$RMSE_{T3}$	0.0111
Generalized Bell-Shaped	$RMSE_{T1}$	0.0655
	$RMSE_{T2}$	0.0106
	$RMSE_{T3}$	0.0094
Gaussian	$RMSE_{T1}$	0.1106
	$RMSE_{T2}$	0.0223
	$RMSE_{T3}$	0.0123

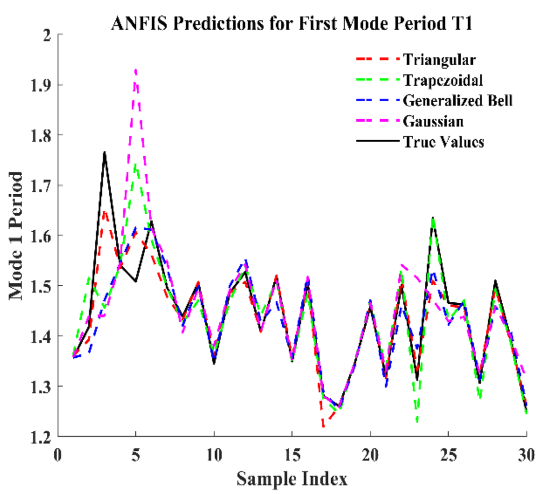

Fig. 13 Model 3 - ANFIS prediction of T1 for all membership functions

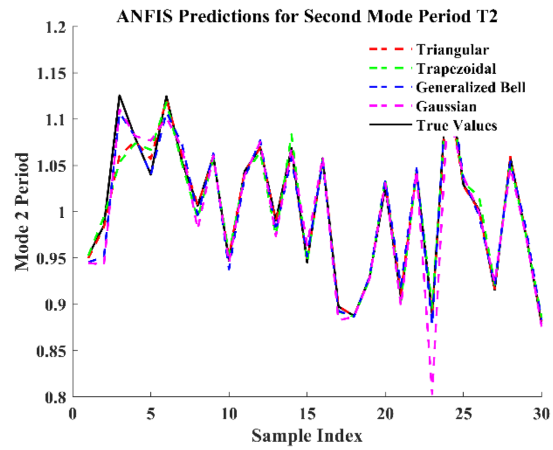

Fig. 14 Model 3 - ANFIS prediction of T2 for all membership functions

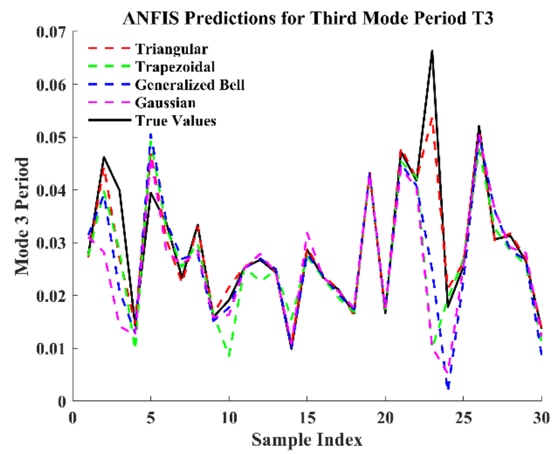

Fig. 15 Model 3 - ANFIS prediction of T3 for all membership functions

3.6. Comparison Analysis: Artificial Neural Networks and Response Surface Method

The learning process was subdivided into 80% for training, 10% for validation, and 10% for testing. It was carried out using a Neural Network topology with dimensions of 4-10-15-10-3. The architecture is illustrated in Figure 18. The training utilized the Levenberg-Marquardt algorithm with feed-forward backpropagation, employing Sigmoid and Rectified Linear Unit (ReLU) activation functions. Figure 16 presents the training performance, characterized by the descending evolution of the Mean Squared Error (MSE).

Meanwhile, Figure 17 illustrates the data regression, showing the approximation functions between the target and the trained values based on the accuracy of the data fit. Additionally, the Response Surface Method (RSM) is recognized as one of the most commonly used data regression and prediction techniques. The fundamental approach involves determining a quadratic polynomial equation to approximate the analytical formula based on the regression of the collected input and output data. The final quadratic regression equations for the three natural periods corresponding to the natural vibration modes are provided in Equations (15), (16) and (17).

$$T_1 = 0.67 + 3.08\text{x}10^{-4}\mathrm{M}_t - 0.0118\mathrm{M}_P - 10^{-6}K_P + 8.8x10^{-5}K_a \quad (15)$$

$$T_2 = 0.98 + 9.51\text{x}10^{-4}\mathrm{M}_t - 0.00101\mathrm{M}_P - 4.6x10^{-5}K_a \quad (16)$$

$$T_3 = 0.054 + 1.12\text{x}10^{-5}\mathrm{M}_t + 0.00305\mathrm{M}_P - 4x10^{-6}K_a + 3.8x10^{-5}{\mathrm{M}_P}^2 \quad (17)$$

To supply an enhanced insight into practical considerations of metamodeling, the obtained values considering the input parameters of the studied bridge configuration are compared to the results generated by the proposed models of the Adaptive Neuro-Fuzzy Inference System (ANFIS) method. Table 10 presents the findings derived from the elaborated methods.

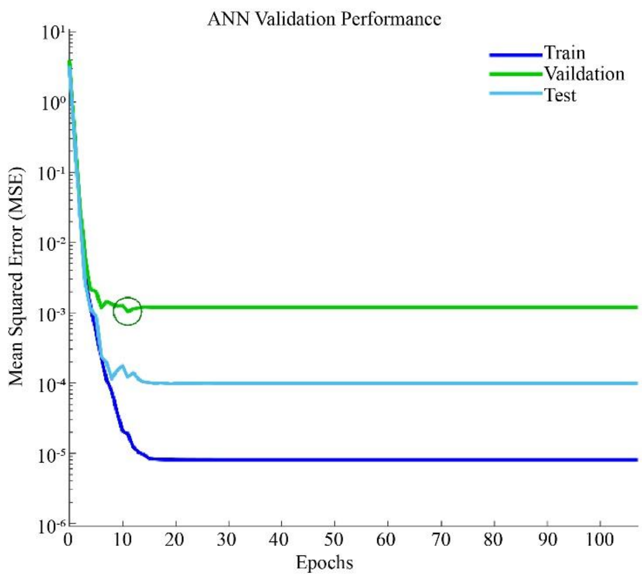

Fig. 16 Neural networks training performance

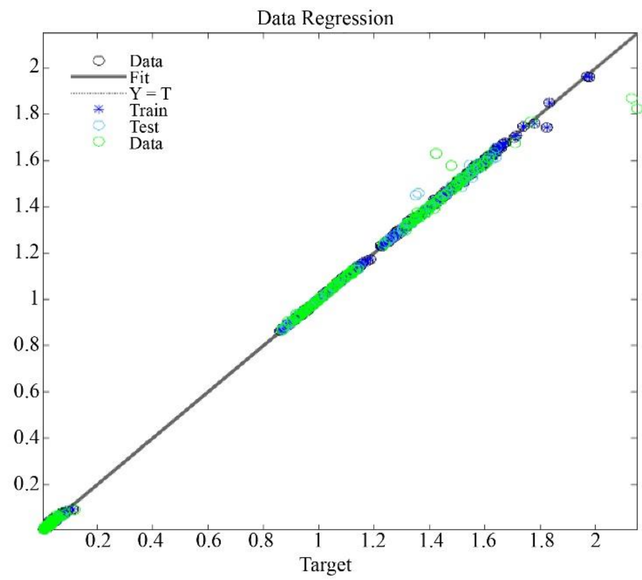

Fig. 17 Neural networks training regressions

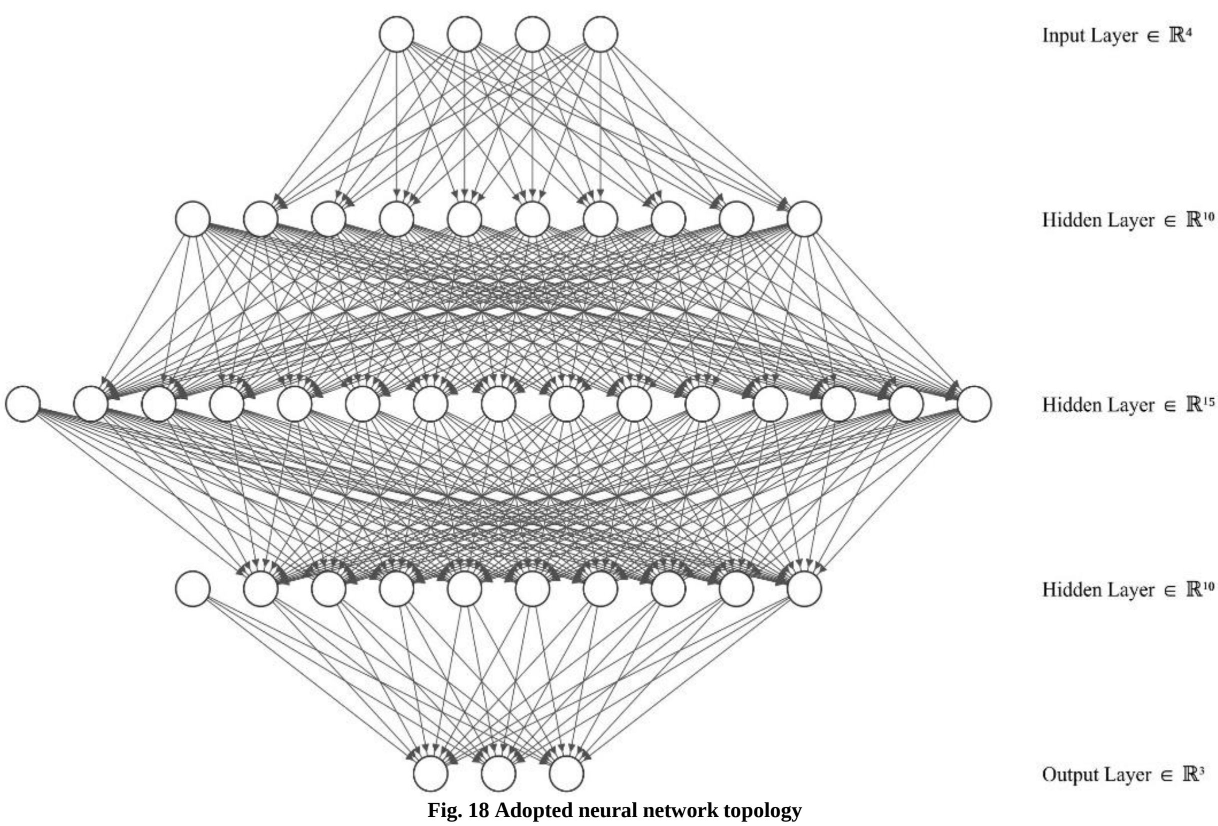

Fig. 18 Adopted neural network topology

Table 5. Values of longitudinal mode periods predictions

Machine Learning Method		First mode Period (s)	Second mode Period (s)	Third mode Period (s)
ALGEBRAICAL MODAL ANALYSIS		1.68	1.15	0.056
ARTIFICIAL NEURAL NETWORKS		1.69	1.16	0.05
RESPONSE SURFACE METHOD		1.45	1.25	0.052
ANFIS MODEL 1	Triangular	**1.68**	**1.16**	**0.06**
	Trapezoidal	1.68	1.16	0.055
	Generalized Bell-Shaped	1.68	1.15	0.55
	Gaussian	1.68	1.16	0.056
ANFIS MODEL 2	Triangular	1.45	1.04	0.044
	Trapezoidal	1.23	0.91	0.008
	Generalized Bell-Shaped	1.26	0.92	0.021
	Gaussian	1.22	0.09	0.012
ANFIS MODEL 3	Triangular	1.86	1.18	0.08
	Trapezoidal	1.70	1.15	0.067
	Generalized Bell-Shaped	1.77	1.17	0.073
	Gaussian	1.75	1.16	0.074

4. Discussions

4.1. Interpretation of Results

The results obtained from the application of the Adaptive Neuro-Fuzzy Inference System (ANFIS) method demonstrate its effectiveness in predicting the natural mode periods of the studied bridge configuration. The comparative analysis between the different ANFIS models, Artificial Neural Networks (ANN), and the Response Surface Method (RSM) provides insightful conclusions regarding these machine-learning approaches' predictive performance and efficiency. The findings of this study highlight the impact of dataset partitioning and membership function selection on the performance of ANFIS. Three models were trained using a dataset of 150 samples with different training-to-testing ratios: Model 1 (60%-40%), Model 2 (70%-30%), and Model 3 (80%-20%). The implemented membership functions include Triangular, Trapezoidal, Generalized Bell-Shaped and Gaussian functions. The Root Mean Square Error (RMSE) metric was employed to evaluate model performance. These results underscore the significant influence of dataset partitioning on model performance across the three modes.

Additionally, the choice of membership function plays a role in optimizing predictive accuracy. The observed variations suggest that an appropriate balance between training and testing data and a well-suited membership function is essential for enhancing ANFIS efficiency in similar applications. The RMSE values in Tables 7, 8, and 9 indicate that the choice of membership function plays a crucial role in the accuracy of the ANFIS predictions. Across all models, the Triangular membership function consistently yields approximately the lowest RMSE values, particularly for the first and second natural mode periods. While still effective, the Gaussian and generalized bell-shaped functions show slightly higher error margins, suggesting their flexibility in capturing input-output relationships may introduce additional complexity. In addition, Table 7 compares the predicted natural mode periods of different machine learning models with Algebraic Modal Analysis, revealing key performance variations. Artificial Neural Networks (ANNs) closely match the reference values, with minimal deviation in all three modes, confirming their reliability. The Response Surface Method (RSM) shows noticeable deviations in its predictions. At the same time, it underestimates the first mode period (1.45 s vs. 1.68 s) while overestimating the second (1.25 s vs. 1.15 s), suggesting a different error pattern than ANN and ANFIS models. Among the ANFIS models, Model 1 demonstrates the best agreement with the reference, particularly for Triangular, Trapezoidal, and Gaussian functions.

In contrast, the Generalized Bell-Shaped function significantly overestimates the third mode period (0.55 s), indicating instability. Model 2 exhibits a systematic underestimation across all modes, with the Gaussian function performing the worst (1.22 s for the first mode vs. 1.68 s reference and a second mode period of only 0.09 s), suggesting that its training ratio or membership function selection may not be optimal.

Conversely, Model 3 tends to overestimate the first and third mode periods, particularly with the Triangular function (1.86 s for the first mode) and Gaussian function (0.074 s for the third mode), hinting at potential overfitting due to a higher training ratio. These findings confirm that ANN and ANFIS Model 1 (excluding the Generalized Bell-Shaped function) provide the most accurate results, whereas ANFIS Models 2 and 3 require further optimization to enhance prediction accuracy. From a practical standpoint, the ANFIS-based predictions exhibit a high level of agreement with the ANN results, making it a viable method for real-world applications where computational efficiency and interpretability are critical. Additionally, the comparative analysis suggests that using a sufficient training dataset is crucial for improving model accuracy. This consideration should be considered when deploying machine-learning techniques in structural engineering.

4.2. Comparison with Other Studies

Table 11 below compares our study with other studies related to the application of ANFIS in dynamic bridges.

Table 11. Comparison with other studies

Study	Application Domain	Context	Findings	Scientific Contribution / Added Value	Future Work / Perspectives
Our Study	Structural Dynamic Multimodal Analysis of Bridges Using The Adaptive Neuro-Fuzzy Inference System and Artificial Intelligence Metamodels	Application to a complex Moroccan bridge with a three-column central pier span; a combination of multimodal analysis with ANFIS, RNA and RSM for the prediction of natural periods	The ANFIS model with triangular membership functions shows superior predictive accuracy compared to RNA and RSM.	Innovative integration of multimodal modal analysis and AI for better seismic dynamic modeling of complex bridges	Extension to various structural configurations; optimization of ANFIS membership functions and exploration of hybrid models
Ding and Li [29]	Structural health monitoring of long-span suspension bridges	Use of wavelet analysis to detect structural damage	Effective damage detection from ambient vibration responses	Application of wavelet analysis for the structural health monitoring of suspension bridges	Integration with other monitoring techniques and improvement of damage detection accuracy
Muzzammil [30]	Bridge Engineering – Scour Prediction	Use of ANFIS to predict local scour depth around bridge abutments; comparison with traditional regression models	ANFIS outperformed regression models, providing more accurate scour depth predictions	Application of AI (ANFIS) for precise hydraulic risk modeling around bridge structures	Extension to other structural types and integration with real-time monitoring systems for dynamic risk assessment

4.3. Contributions

- The first application of multimodal spectral modal analysis to a real bridge in Morocco, with a complex structural configuration.
- Improving the accuracy of seismic analysis by considering multiple vibration modes (multimodal approach).
- Innovative integration of artificial intelligence, particularly the ANFIS model, to predict natural vibration periods.
- Rigorous comparison with other metamodels (ANN and RSM), highlighting the superiority of ANFIS, especially with triangular functions.
- Model optimization through experimentation with different membership functions and data distributions.

4.4. Perspectives

- Extension to bridges with more varied configurations - Apply the developed method to other bridges (arch, cable-stayed, multiple-girder, etc.) to validate its robustness on different geometries.
- Consideration of nonlinear behavior-Integrate nonlinear effects (plasticization, local damage, nonlinear supports) for more realistic modeling of seismic behavior.
- Analysis under multi-hazard stresses - Extend the analysis to cases of combined stresses (earthquake + wind, earthquake + scour, etc.) to better assess the overall vulnerability of the structure.
- Improving the ANFIS model using hybrid techniques ANFIS with other optimization algorithms (e.g., genetic algorithms, PSO, or deep learning) to increase prediction accuracy
- Development of an intelligent database--Build a structured database from various seismic simulations on different bridges to train more generalizable AI models.
- Implementation in a real-time environment: Adapt the model for integration into real-time seismic monitoring or warning systems for critical infrastructure.
- Comparison with other advanced AI approaches: Extend the comparison to more recent models such as deep neural networks (DNNs), transformer-based models, or random forests.

5. Conclusion

The study successfully applied the Adaptive Neuro-Fuzzy Inference System (ANFIS) to predict the natural vibration modes of a bridge structure, demonstrating its effectiveness in comparison to Artificial Neural Networks (ANN) and the Response Surface Method (RSM). The results indicate that ANFIS models, particularly those using triangular membership functions, provide accurate predictions of the natural mode periods.

The comparative analysis revealed that ANFIS Model 1, with a 60%-40% training-to-testing ratio, performed the best, closely matching the reference values from algebraic modal analysis. The study underscores the importance of dataset partitioning and membership function selection in optimizing ANFIS performance.

Future research could explore using optimization algorithms such as Particle Swarm Optimization (PSO) or Ant Colony Optimization (ACO) to enhance the ANFIS model's learning process. Additionally, cross-validation methods could be employed to determine the optimal number of rules, preventing overfitting and improving the model's predictive accuracy. The findings suggest that ANFIS is a reliable and efficient method for predicting bridge vibration modes, making it a valuable tool for structural engineering applications.

References

[1] J.S.R. Jang, “ANFIS: Adaptive-Network-Based Fuzzy Inference System,” *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 23, no. 3, pp. 665-685, 1993. [CrossRef] [Google Scholar] [Publisher Link]

[2] M. Masher, “*Flexure and Shear Behavior of Ferrocement Members: Experimental and Theoretical Study*,” PhD Thesis, Civil Engineering, Basrah University, Iraq, 2010. [Google Scholar]

[3] S. Fallahian, and S.M. Seyedpoor, “A Two Stage Method for Structural Damage Identification Using an Adaptive Neuro-Fuzzy Inference System and Particle Swarm Optimization,” *Asian Journal of Civil Engineering (Building and Housing)*, vol. 11, no. 6, pp. 795-808, 2010. [Google Scholar]

[4] Abha Mittal, Shaifaly Sharma, and D.P. Kanungo, “A Comparison of ANFIS and ANN for the Prediction of Peak Ground Acceleration in Indian Himalayan Region,” *Proceedings of the International Conference on Soft Computing for Problem Solving (SocProS 2011)*, vol. 2, pp. 485-495, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[5] Shaokuan Chen et al., “Modelling and Analysis on Emergency Evacuation from Metro Stations,” *Mathematical Problems in Engineering*, vol. 2017, no. 1, pp. 1-11, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[6] Bachar Kabalan et al., “Framework for Centralized and Dynamic Pedestrian Management in Railway Stations,” *Transportation Research Procedia*, vol. 27, pp. 712-719, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[7] Hamad Alawad, and Sakdirat Kaewunruen, “Wireless Sensor Networks: Toward Smarter Railway Stations,” *Infrastructures*, vol. 3, no. 3, pp. 1-17, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[8] Rufaizal Che Mamat, Anuar Kasa, and Siti Fatin Mohd Razali, "Performance Prediction of Road Embankment on Soft Soil Stabilized with Prefabricated Vertical Drains Using Numerical Method," *International Journal of Advanced Trends in Computer Science and Engineering*, vol. 8, no. 4, pp. 1601-1608, 2019. [Google Scholar] [Publisher Link]

[9] D. Danilatos, "*Bridge Health Monitoring through Neuro-Fuzzy Architectures*," Master of Science Thesis, Missouri University of Science and Technology, 2008. [Google Scholar]

[10] Duc C. Nguyen et al., "Vibration-Based SHM of Dębica Railway Steel Bridge with Optimized ANN and ANFIS," *Journal of Constructional Steel Research*, vol. 215, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[11] Javad Palizvan Zand, Javad Katebi, and Saman Yaghmaei-Sabegh, "A Generalized ANFIS Controller for Vibration Mitigation of Uncertain Building Structure," *Structural Engineering and Mechanics*, vol. 87, no. 3, pp. 231-242, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[12] Neda Kiani Mavi et al., "Forecasting Project Success in the Construction Industry Using Adaptive Neuro-Fuzzy Inference System," *International Journal of Construction Management*, vol. 24, no. 14, pp. 1550-1568, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[13] Mohammad Mahdi Jafari et al., "Application of a Novel Hybrid Wavelet-ANFIS/Fuzzy C-Means Clustering Model to Predict Groundwater Fluctuations," *Atmosphere*, vol. 12, no. 1, pp. 1-15, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[14] Nasser Sekfali, and Brahim Lafifi, "Mechanically Stabilized Earth Wall Reliability Analysis Using Response Surface Methodology, ANN, ANFIS and Multi-Objective Genetic Algorithm," *Periodica Polytechnica Civil Engineering*, vol. 69, no. 1, pp. 159-174, 2025. [CrossRef] [Google Scholar] [Publisher Link]

[15] Shivali Chopra et al., "Taxonomy of Adaptive Neuro-Fuzzy Inference System in Modern Engineering Sciences," *Computational Intelligence and Neuroscience*, vol. 2021, no. 1, pp. 1-15, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[16] Ahmed F. Bendary et al., "Proposed ANFIS Based Approach for Fault Tracking, Detection, Clearing and Rearrangement for Photovoltaic System," *Sensors*, vol. 21, no. 7, pp. 1-16, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[17] S.O. Sada, and S.C. Ikpeseni, "Evaluation of ANN and ANFIS Modeling Ability in the Prediction of AISI 1050 Steel Machining Performance," *Heliyon*, vol. 7, no. 2, pp. 1-9, 2021. [Google Scholar] [Publisher Link]

[18] Adaptive Neuro-Fuzzy Inference System (ANFIS), Schneppat AI. [Online]. Available : https://schneppat.com/adaptive-neuro-fuzzy-inference-system-anfis.html

[19] Mohd Najib Mohd Salleh, Noureen Talpur, and Kashif Hussain, "Adaptive Neuro-Fuzzy Inference System: Overview, Strengths, Limitations, and Solutions," *Data Mining and Big Data, Lecture Notes in Computer Science*, vol. 10387, pp. 527-535, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[20] Sri Sumarlinda, "The Improvement Prediction Model Using Anfis for Medical Dataset," *Journal of Theoretical and Applied Information Technology*, vol. 102, no. 5, pp. 1663-1672, 2024. [Google Scholar] [Publisher Link]

[21] Simon S. Haykin, *Neural Networks and Learning Machines*, 3rd ed., Pearson, pp. 1-934, 2009. [Google Scholar] [Publisher Link]

[22] L.A. Zadeh, "Fuzzy Logic = Computing with Words," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 2, pp. 103-111, 1996. [CrossRef] [Google Scholar] [Publisher Link]

[23] Raymond H. Myers, Douglas C. Montgomery, and Christine M. Anderson-Cook, *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, 3rd ed., Wiley, 2009. [Google Scholar] [Publisher Link]

[24] Alex J. Smola, and Bernhard Schölkopf, "A Tutorial on Support Vector Regression," *Statistics and Computing*, vol. 14, pp. 199-222, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[25] Vladimir N. Vapnik, *The Nature of Statistical Learning Theory*, 2nd ed., Springer New York, NY, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[26] Ajith Abraham, "Neuro Fuzzy Systems: State-of-the-Art Modeling Techniques," *Connectionist Models of Neurons, Learning Processes, and Artificial Intelligence, Lecture Notes in Computer Science*, vol. 2084, pp. 269-276, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[27] Mohammad Fazle Azeem, *Fuzzy Inference System - Theory and Applications*, InTechOpen, pp. 1-518, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[28] Robert Fuller, *Neural Fuzzy Systems*, Abo Akademi University, pp. 1-348, 1995. [Google Scholar]

[29] Ding Youliang, and Li Aiqun, "Structural Health Monitoring of Long-Span Suspension Bridges Using Wavelet Packet Analysis," *Earthquake Engineering and Engineering Vibration*, vol. 6, pp. 289-294, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[30] Mohammad Muzzammil, "ANFIS Approach to the Scour Depth Prediction at a Bridge Abutment," *Journal of Hydroinformatics*, vol. 12, no. 4, pp. 474-485, 2010. [CrossRef] [Google Scholar] [Publisher Link]