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*Original Article*

# The Influence of Various Parameters on the MDOF Structure Response to Harmonic Excitation

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***Abstract*** *- Assessing the dynamic behavior of structures is a highly intricate task, often involving complex and challenging calculations. This complexity is particularly evident when analyzing how Multi-Degree Of Freedom (MDOF) buildings respond to various excitations, such as harmonic forces and seismic forces. These models implicate a coupled set of equations of motion, in which the interaction between various structural components creates more sophisticated dynamic behavior. This work focuses on investigating the impact of several dynamical characteristics on MDOF buildings and examining their performance by systematically varying diverse parameters. A two-step methodology is employed, beginning with analytical solutions and followed by numerical simulations using Newmark-β and Wilson-θ algorithms.*

***Keywords*** *- MDOF, Damping, Dynamical Characteristics, Seismic Force, Newmark-β, and Wilson-θ algorithms.*

## 1. Introduction

Constructing any structure requires the completion of several essential studies [1]. These include a site investigation to evaluate the suitability of the location, geotechnical analysis to assess soil properties and foundation stability, and a comprehensive structural design to optimize the dimensions and arrangement of structural elements, along with an evaluation of potential environmental impacts. Furthermore, it is equally important to account for the various loads that the structure will experience throughout its service life [1-5], including dead loads (self-weight and construction materials such as concrete), live loads (occupants and usage), and external actions such as wind and seismic forces.

Civil engineering is the field dedicated to designing, building, and maintaining structures in the natural environment. Its main priority is the construction of buildings and structures that are safe and capable of withstanding the elements to which they will be exposed, such as wind, waves, traffic, earthquakes, and blasts. Indeed, these different are the main issue on the durability of constructions and buildings, and it is crucial to carry out a series of studies and calculations [3], whether dynamic or static, which ultimately make it possible to make decisions on either the properties of the materials used, the dimensions, etc [6, 7].

It is important to perform structural dynamic analysis to ensure the stability of dwellings. This study aims to assess how a structure responds to dynamic loads. For the complex structures, these differential equations are used for analysing the performance of buildings under different loads and conditions. These equations can be very difficult to solve analytically, especially for large and complex structures. It can indeed be a time- consuming process due to the intricate calculations involved, because this method is based on a series of calculations, and they are sometimes more complex to solve.

Recent reinforcement strategies, particularly supplemental damping devices such as fluid viscous and friction dampers, effectively enhance energy dissipation. However, their inclusion increases the complexity of the governing equations, often leading to strongly nonlinear systems that require advanced numerical methods [8]. This study examines efficient time-integration schemes for linear structural dynamics, especially in high-rise buildings with discretized equilibrium equations. For solving this equation for a 3DOF system using the approaches mentioned above, the model is designed as a system of interconnected masses and springs, with each degree of freedom corresponding to one of the three translation motion modes of the structure. The model is simplified and streamlined to simulate several MDOF scenarios, computing peak displacement, velocity, and acceleration. The model estimates various characteristics of buildings, like mass, stiffness, damping, and the characteristics of the excitation, such as the magnitude and frequency of the harmonic load. These parameters and their variations are crucial in controlling the structural response. They are inherent to every structure, with slabs and beams contributing to the mass, and columns influencing the stiffness of the member. Therefore, variations in these parameters play a substantial role in shaping the structural behaviour. Several works have been reported during the past years ago dealing with the dynamic response of dwellings under external loads, but there exists a very limited number of researches have been estimating the dynamic response of buildings by testing the different parameters affected their stability Rakshita R et al [6] and many other authors also studied the influence of various dynamic parameters especially mass and stiffness in controlling The structural response to ground motion is analysed through numerical approaches.

This study aims to evaluate the influence of several dynamical properties of the structure of three floors subjected to a harmonic force. In this study, we are interested in the following variants: ratio damping, excitation frequency, and beam-to-column stiffness ratio. For each one, we have executed several tests without modifying the other properties, characterized the structure to evaluate the effect of each variant separately, and interpreted the results obtained by using the analytical method, namely, modal analysis and numerical approaches that are Newmark's method and Wilson method. This parametric study led to the conclusion of the influence of each characteristic on the structure.

## 2. Related Work

The identification of dynamic characteristics of structures from different types of vibration is an important task in civil engineering structures. Indeed, dynamic loads are the main issue in the durability of constructions and buildings, and then an understanding of structural dynamics is the first and most interesting step in the design and retrofit of structures. Various works have been devoted to the dynamic analysis of MDOF structures in the literature [1-14]. To investigate the influence of various dynamic characteristics of existing buildings on their dynamic response, Ahmed Tuken et al. have performed in [4] a dynamic analysis of three-story buildings subjected to a harmonic loading by using an analytical approach. Similarly, in [2, 3], a dynamic analysis is conducted on reinforced concrete multistory buildings by two different methods. The same dynamic analysis was performed in [12] by Mohit Sharma et al. using design data.

In the study presented by Ahmed Ibrahim et al. [7], a new element is added to their analysis, which is the Floating Columns (FC). A field study using a shaking table has been presented on a three-story frame by Rafiqul Islam et al in [13]. Instead of using the analytic method, the dynamic analysis presented in [14] by K. J. BATHE et al is based on the following numerical methods: Runge-Kutta, Houblot, Newmark and Wilson Theta, Hiroshi Yamakawa [15], et al [8] A. A. Gholampour et al have developed some approaches, the first researcher suggested the new approach of dynamic behavior known as the "step-by-step transfer matrix" technique, while A. A. Gholampour developed Newmark's method which will be applicable to nonlinear systems, and they study her effectiveness by varying the height, the stiffness of structure. The above-related work shows that there is various research available on the dynamic analysis of MDOF systems under wind, wave, and earthquake...Etc, but harmonic loads are limited. The principal contribution of this work is to examine the great impacts of dynamic properties on the response of buildings excited by this type of load, to evaluate the impact of these characteristics on buildings, and to understand better the dynamic vulnerability of existing reinforced concrete buildings using analytical and numerical methods.

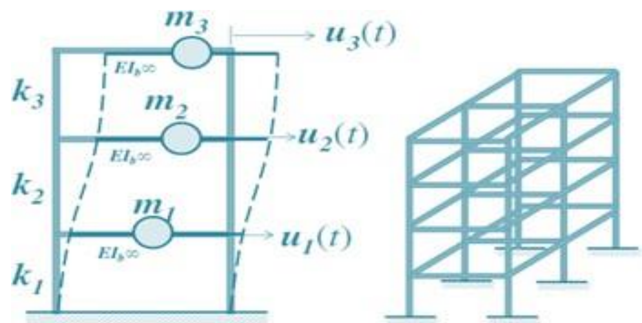



**Fig. 1 Structural configuration of a three-story shear frame**

## 3. Materials and Methods

Any project study for a building aims to guarantee the stability and resilience of the structure, prioritizing user safety. In our investigation, we focus on the dynamic analysis of a G+3 reinforced concrete building, aiming to understand the alterations in the dynamic properties of typical structures from one to 3 floors by two methods. Firstly, we start our study by modelling the variants of dynamical characteristics of buildings to evaluate the effect of these properties on the stability and durability of the model from the different responses of the dynamic analysis obtained by different methods; secondly, we compare and interpret the different results obtained by the approaches chosen according to several criteria.

### *3.1. Analytical Study*

The modal method, acknowledged as a widely used and practical approach in structural analysis and identification, revolves around the examination and characterization of multi-degree-of-freedom systems. This method relies on input and response measurements for a comprehensive understanding and evaluation. It is considered an indispensable tool in understanding the structural dynamics. In the analysis of natural frequencies for dwellings, it is commonly considered that the dwelling modelled as a cantilever beam with uniform properties securely fixed to the foundation. A 3-DoF model of this structure can be easily derived by assuming that each floor can be represented by a lumped mass and the four beams between the stories as four springs in parallel with negligible weight, as shown in Figure 2.

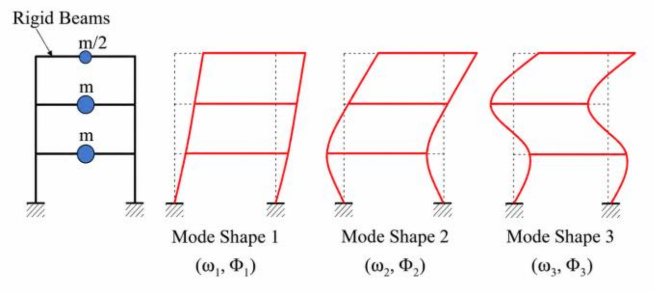



**Fig. 2 Modal shapes of a 3DOF structural model**

Modal analysis of MDOF systems subjected to harmonic loading:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \quad (1)$$

$f(t)$: Defined as the external harmonic load characterized by:

$$\mathrm{f(t)} = \mathrm{F} \sin \omega_{\mathrm{f}} \mathrm{t} \quad (2)$$

In the provided dynamic equation, the parameters M, C, K, and $f(t)$ defines the matrices of inertia, viscous damping, stiffness coefficients, and external force, respectively. The external force, denoted by f(t), undergoes harmonic variation with an amplitude F and frequency. $\omega_f$. To solve the motion equation analytically for an MDOF system, it is important to make certain assumptions to identify the uncoupled equation by assuming that.

$$x(t) = \sum_{i=1}^{3} \varphi_i q_i(t) \quad (3)$$

By Substituting 2 in the equation of motion 1:

$$M \sum_{i=1}^{3} \varphi_i \ddot{q}_i(t) + C \sum_{i=1}^{3} \varphi_i \dot{q}_i(t) + K \sum_{i=1}^{3} \varphi_i q_i(t) = f(t) \quad (4)$$

Applying the transposed modal matrix $\varphi^T$, to the equation and using orthogonality properties, the equation becomes:

$$\varphi^T M \varphi \ddot{q}_i(t) + \varphi^T C \varphi \dot{q}_i(t) + \varphi^T K \varphi_i = \varphi^T F \sin \omega_f t \quad (5)$$

The application of modal decomposition enables the transformation of the coupled system of differential equations into independent modal equations, thereby simplifying the solution process. This is the advantage of using the method of modal superposition to resolve dynamical problems. Solving each equation separately to obtain the final displacements of each floor of the structure, thus the total response is given by:

$$q(t) = e^{-\omega_n \zeta_n}(A \cos(\omega_D t) + B \sin(\omega_D t) + C \sin(\omega_f t) + D \cos(\omega_f t) \quad (6)$$

Where:

$$\omega_d = \omega_n \sqrt{1 - 2\zeta_n} \quad (7)$$

### *3.2. Numerical Study*

In structural dynamics, Dynamic equations of motion describe how a system evolves over time, often taking the form of second-order Ordinary Differential Equations (ODEs). Resolving these problems typically involves employing numerical step-by-step integration algorithms. We divide the period of excitation into small time steps, and then we get the dynamic response through each time step based on calculated properties at the onset and the conclusion of each time step. There are two types of approaches: implicit approaches and explicit approaches. The second family solves the equilibrium equation for each degree of freedom independently. This technique does not require matrix inversion and is quite sensitive to conditions, requiring a very small time step. The implicit approach is more cumbersome to implement, but it makes it possible to considerably increase the time step. For all these reasons, we have chosen to study the numerical structure by the implicit approaches of Newmark and Wilson [9, 10, 14].

#### *3.2.1. Newmark's Method*

The Newmark method, as introduced by Newmark (1959) [9], incorporates various time-step techniques for solving linear or nonlinear equations. The formulation employs a numerical variable denoted as β. In its original version, the method included a second parameter, γ, in addition to β. Different numerical values assigned to these parameters correspond to well-known methods for solving the equation of motion (8), namely the constant acceleration method and the linear acceleration method. This approach is based on a truncated Taylor series expansion of a time-dependent function, approximating successive derivatives using two parameters: β for displacement and γ for velocity.

Numerous studies have established that this approach is unconditionally stable for linear systems when the parameters satisfy $\beta \geq 1/4$ and $\gamma \geq 1/2$, which makes it well-suited for structural dynamic analysis [16-18, 20]. Owing to its robustness and flexibility, it has been widely applied to the performance of buildings under harmonic and seismic excitations.

Nevertheless, it has been reported that an inappropriate choice of parameters or the use of excessively large time steps may lead to numerical oscillations and a loss of accuracy, particularly in stiff systems [16].

#### *3.2.2. Wilson's Method*

The Wilson-θ method, as outlined by Wilson [10], is an implicit numerical time integration approach proposed and employed to resolve the dynamic equilibrium equations of structures. It extends the Newmark formulation, providing improved numerical stability, especially for stiff systems and seismic analysis. The method assumes that a linear variation of acceleration over an extended time interval θΔt where $\theta \geq 1$ is a stability parameter.

Previous studies have widely used this method for seismic analysis and showed that it is unconditionally stable for $\theta \geq 1.37$, making it suitable for large time steps and complex problems. However, it may cause excessive numerical damping and increase the period. To improve accuracy, modified versions with better interpolation and control parameters have been developed. In addition, this technique has demonstrated strong performance in dynamic vibration analysis, providing accurate results with high numerical stability, even under complex loading conditions like an earthquake. Therefore, it remains a reliable and widely used approach in structural dynamics, notably when stability and convergence are critical.

### *3.3. Description of Model*

In this work, a building with three stories, subjected to harmonic loading and constructed using a moment-resisting Reinforced Concrete (RC) frame, is chosen for analysis. The height of the frame is 9m, with each story having a height of 3.0 meters. The study considers variations in structural damping, specifically modified for four scenarios: 0%, 5%, 10%, and 20%. To maintain consistency, the model utilizes girders with a width of b = 0.40 m and square columns with a side dimension of h = 0.40 m. The material properties are defined by a concrete modulus of elasticity of E = 25 GPa.

This study investigates the influence of key parameters on structural behavior through multiple model variations. Each model is then analyzed using three essential methods: analytical approach, Newmark Beta method, and Wilson Theta method.

Mass matrix: $M = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix} (kg)$

And $K = 47407.41 \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} (KN/m)$

And Harmonic excitation: $F = \begin{bmatrix} 10000\sin(10t) \\ 20000\sin(10t) \\ 30000\sin(10t) \end{bmatrix} (N)$

**Table 1. The different parameters characterizing the structure**

| Parameter | Value |
|---|---|
| Height (floor) | 3m |
| Beam size | 0.4m |
| Column size | 0.4m |
| Aspect ratio | 0%, 5%, 10%, 20% |
| Young's modulus of concrete | 25 × 106 KN/m2 |

The building model's three resonance frequencies are determined through eigenvalue-eigenvector analysis and denoted as $\omega_1$, $\omega_2$, and $\omega_3$. Also, the response of each floor is collectively represented by the vector x = [x1, x2, x3].

**Table 2. Dynamic Properties of the Shear Building**

| Mode | $\omega$ (rad/s) | f (Hz) | T (s) |
|---|---|---|---|
| 1 | 11.27 | 1.79 | 0.56 |
| 2 | 30.79 | 4.9 | 0.2 |
| 3 | 42.1 | 6.68 | 0.15 |

The corresponding vibration modes are:

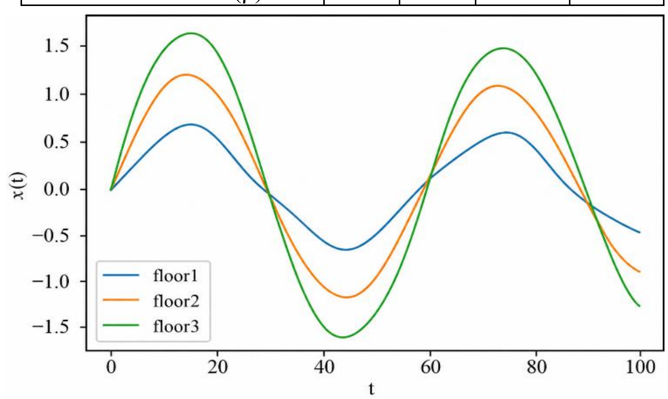
$$\begin{bmatrix} 0.5 & -1 & 0.5 \\ 0.866 & 0 & -0.866 \\ 1 & 1 & 1 \end{bmatrix}$$

## 4. Results and Discussion

The aforementioned computational research on harmonic loads relies primarily on sophisticated numerical software. Consequently, the employment of simpler analytical methods to determine the peak displacement of buildings under harmonic excitation remains restricted to a limited number of studies. Three-story buildings have been selected to explore the influences of dynamic parameters and their implications on the structural response and its stability. The study considers various variations, which are described in the following Table 3:

**Table 3. Three Variants Considered for Four Cases**

| Cases | a | b | c | d |
|---|---|---|---|---|
| Ratio Damping (%) | 0% | 5% | 10% | 20% |
| Excitation frequency (rad/s) | 10 | 20 | 30 | 60 |
| Beam-to-column stiffness ratio (ρ) | 0 | 1/8 | 1/4 | ∞ |

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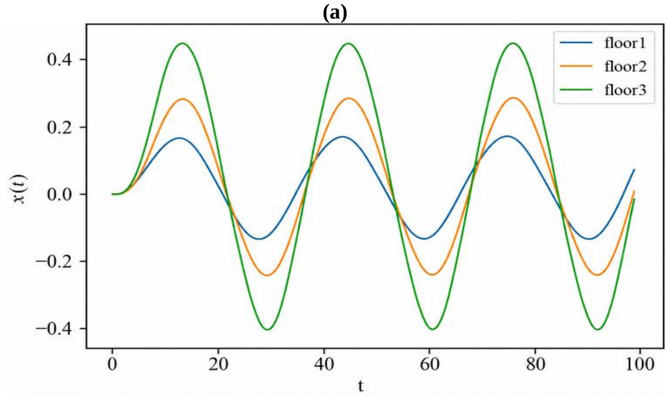


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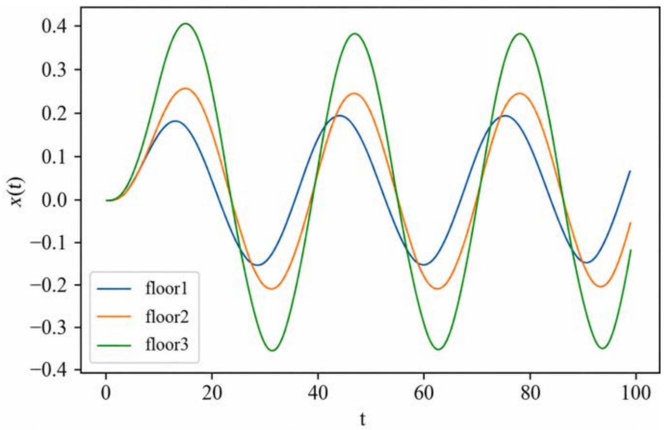


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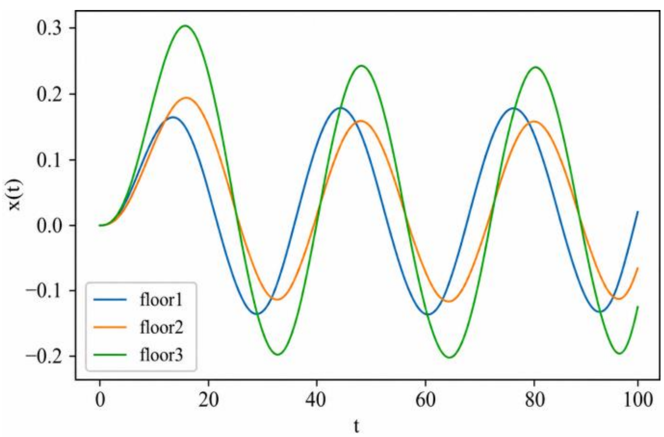
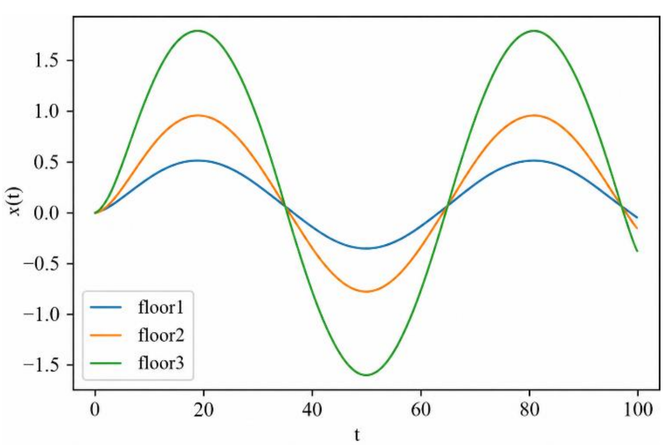



**(d)**

**Fig. 3 The analytical response of the model for different cases of ζ (a: 0%, b: 5%, c: 10%, d: 20%)**

Figures 3(a)–(d) present the variation of top displacement under harmonic loading for damping ratios ranging from 0% to 20%, as obtained from the analytical approach. The results describe that higher values of ζ significantly reduce the amplitude of the structural response, highlighting its effectiveness in dissipating energy and improving the dynamic behavior of the dwelling.

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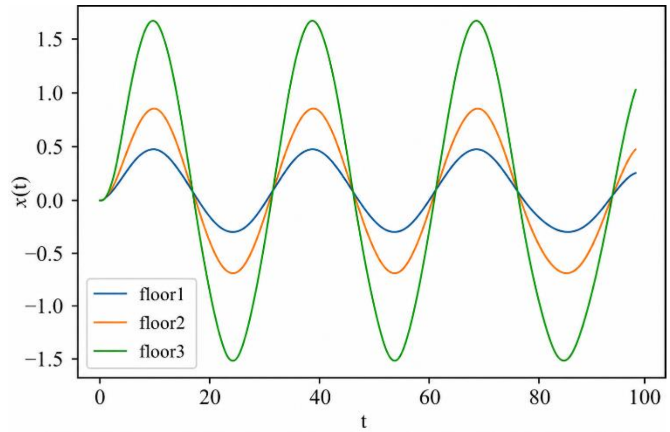


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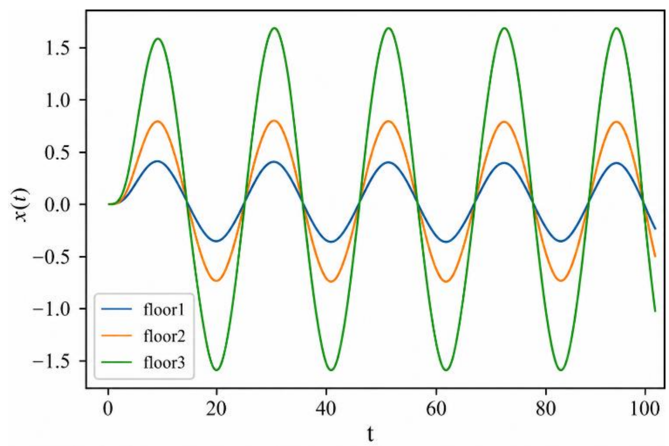


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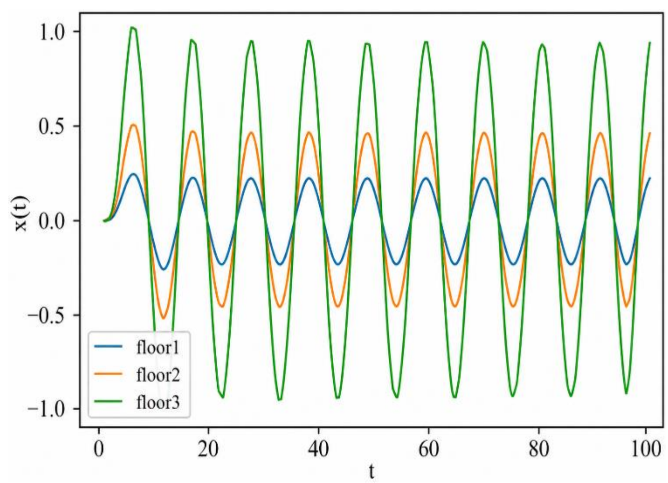
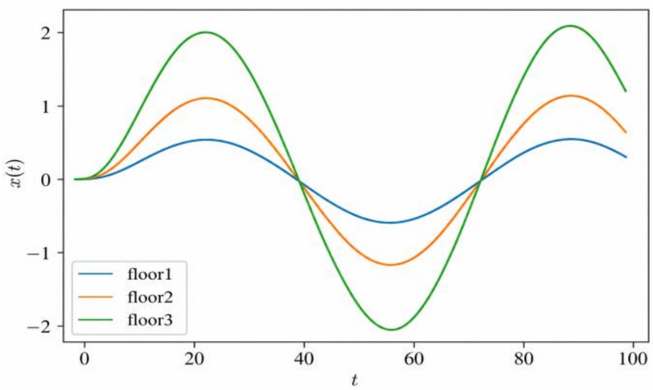



**(d)**

**Fig. 4 The analytical response of the model for different cases of ωf (a: 10rad/s, b:20rad/s, c:30rad/s, d:60rad/s).**

Figures 4(a)–(d) present the impact of excitation frequency on the displacement of the model using the analytical approach. It is observed that increasing the excitation frequency from 10 rad/s to 60 rad/s results in a marked decrease in displacement amplitude, accompanied by a reduction in the apparent structural response period.

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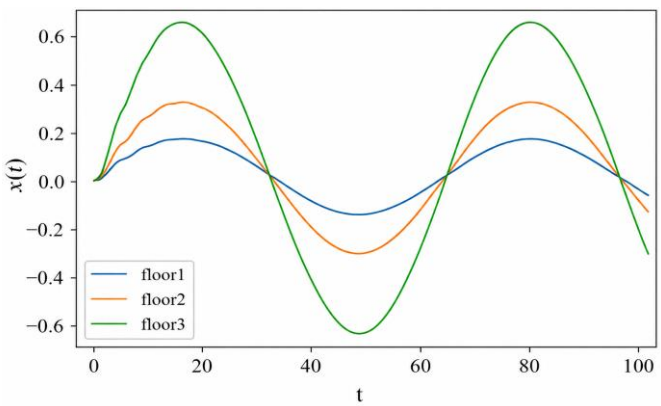


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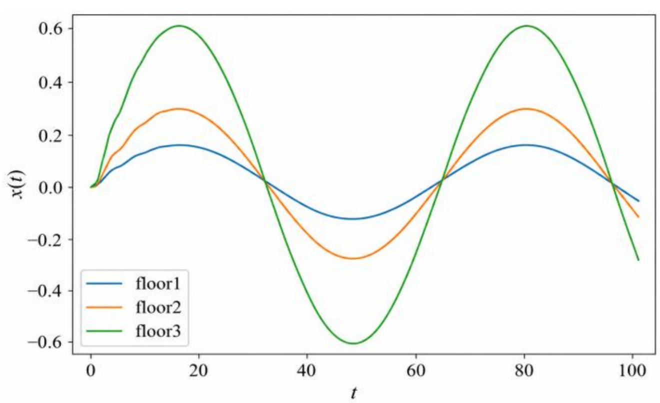


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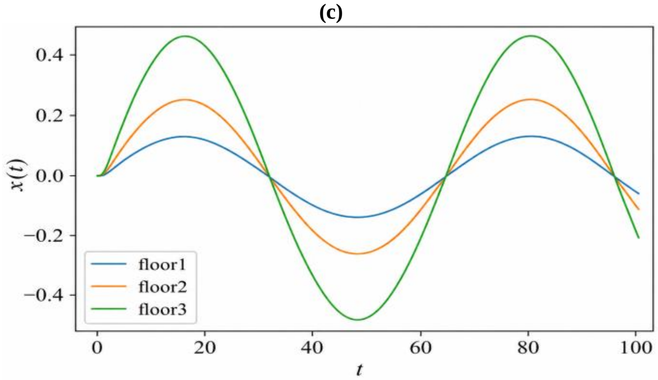
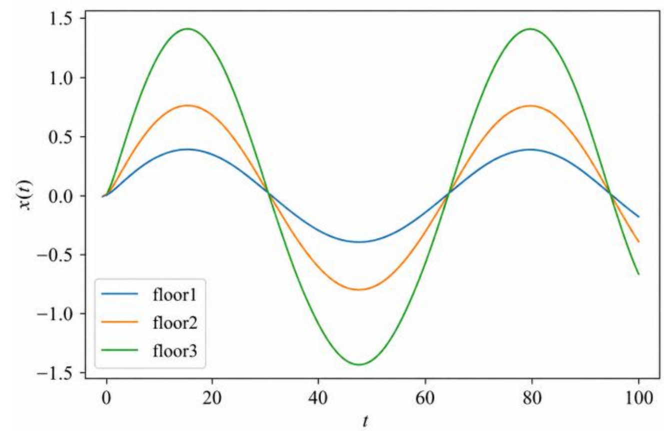



**(d)**

**Fig. 5 The analytical response of the model for different cases of ρ**

Figures 5(a)–(d) indicate that a growth in structural stiffness leads to an elevation of the natural frequencies of the model. Moreover, the maximum vibration amplitudes are not uniform and differ across the various floors of the structure. For the third floor, the maximum vibration range is from 2cm to 0.4cm. On the second floor, the maximum vibrations range from 1cm to 0.23cm, and finally, the first floor maximum vibrations range from 0.4cm to 0.1cm.

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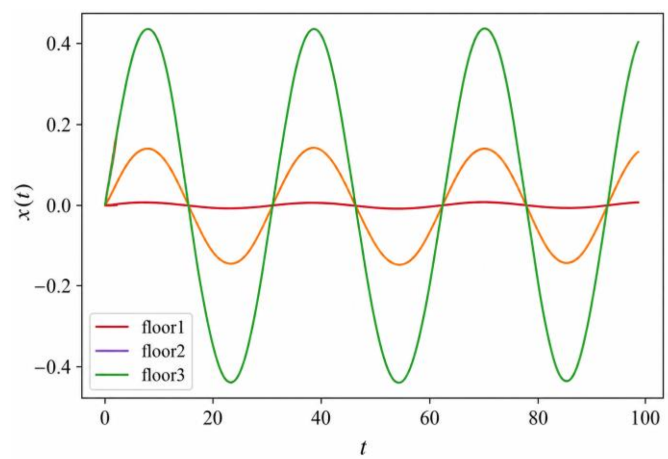


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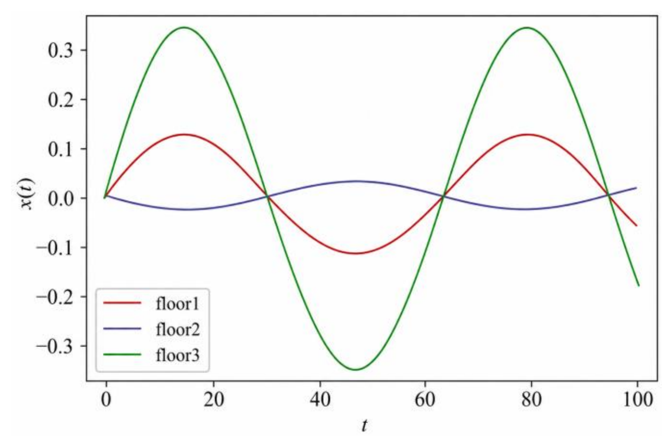


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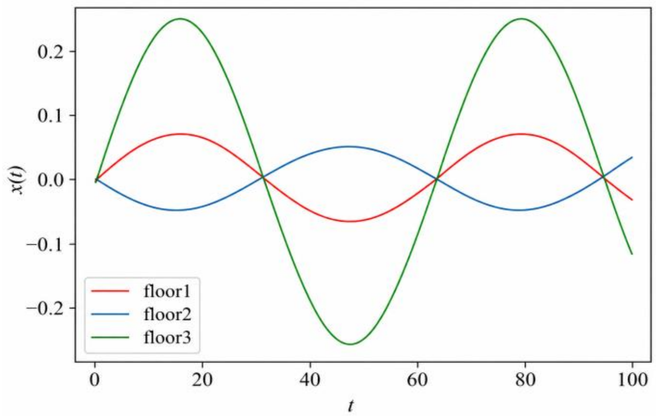
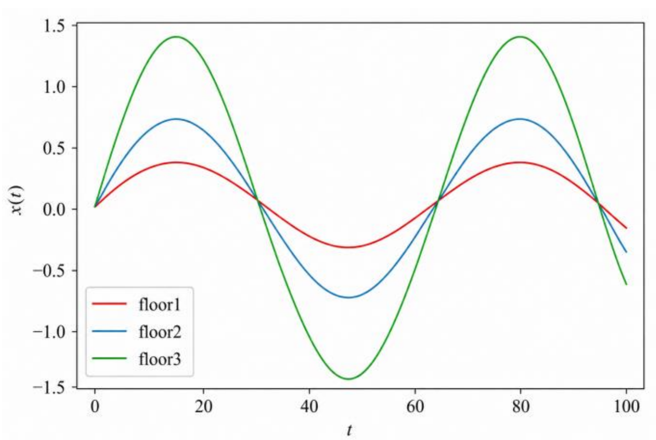



**(d)**

**Fig. 6 The Newmark-β response for different damping ratios ζ(a: 0%, b: 5%, c: 10%, d: 20%)**

The representative examples of the results for Newmark's Figure 6 method, constant time step δ=0.01s, have been employed for solving the equation of motion numerically. Three-story buildings are presented in Figures showing the peak displacement on each floor and the impact of the ratio damping on structures. It can be seen from all figures that an increase in the damping coefficient leads to a substantial reduction in peak displacements.

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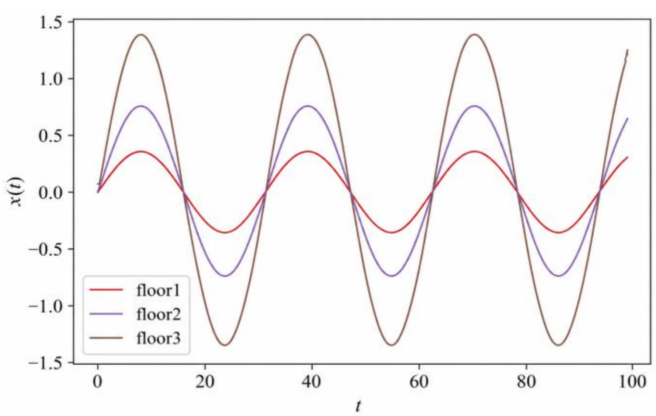


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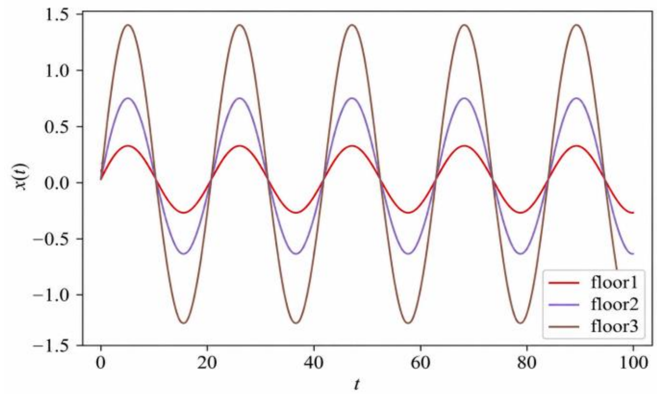


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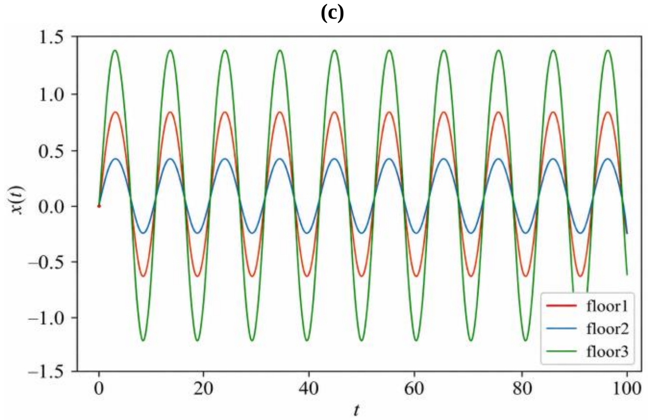
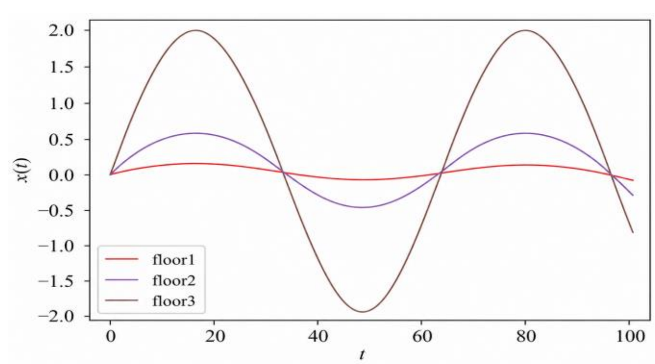



**(d)**

**Fig. 7 The Newmark-β response for different values of ωf (a:10rad/s, b:20rad/s, c:30rad/s, d:60rad/s).**

Figures 7(a)–(d) demonstrate that, based on the Newmark-β method, increasing the excitation frequency results in a noticeable reduction in both the structural response period and vibration amplitude. This trend can be explained by the detuning effect, as the excitation frequency moves away from the structure's natural frequencies, thereby diminishing resonance amplification and leading to reduced displacement responses. Furthermore, higher excitation frequencies generate rapid oscillations, leading to a reduction in the effective response period. Consequently, the structure exhibits reduced sensitivity to external dynamic loading as the excitation frequency increases.

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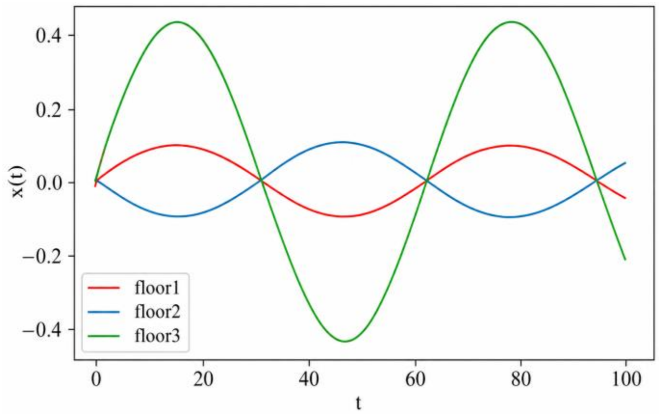


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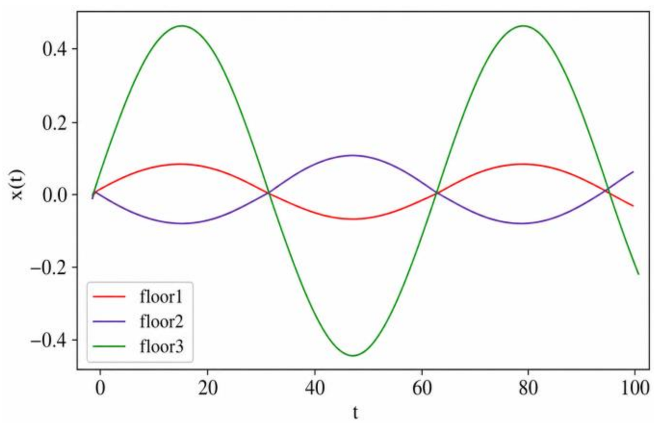


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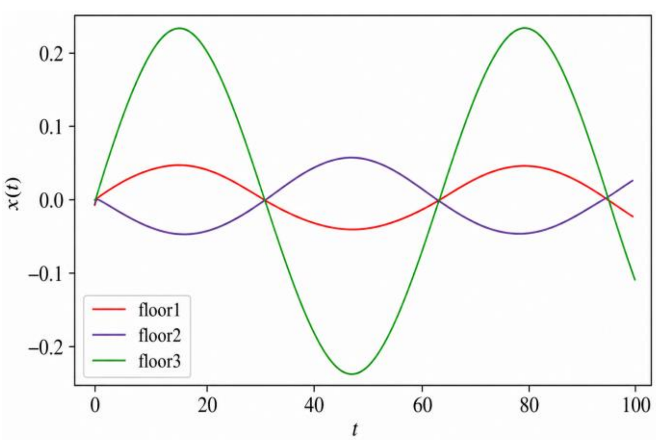
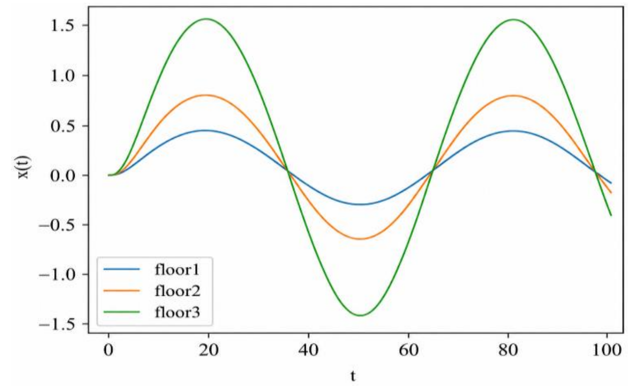



**(d)**

**Fig. 8 The Newmark-β response for different values of ρ**

Figures 8(a)–(c) demonstrate that increasing the structural stiffness, through variation of the beam-to-column stiffness ratio (ρ), results in a substantial reduction in the response of the structure. This behavior can be attributed to the increased resistance to deformation and the enhanced rigidity of the system, which limits lateral displacements under dynamic loading.

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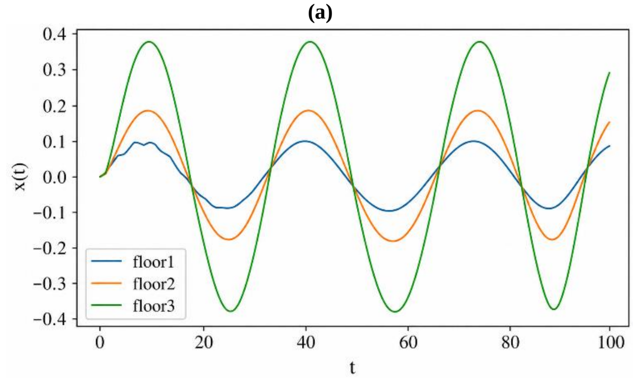


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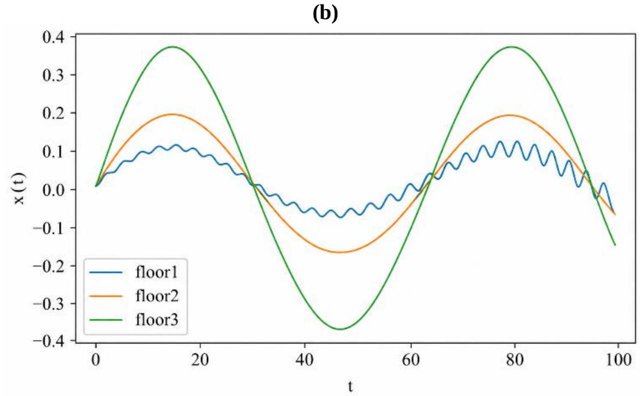


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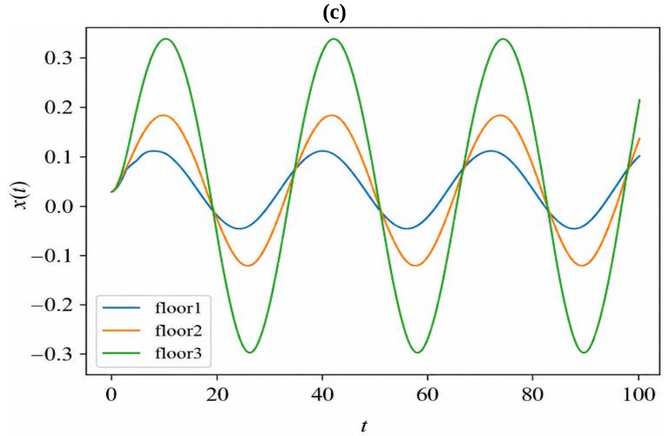
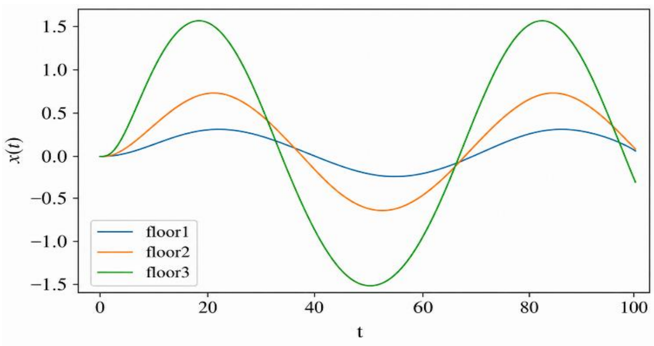


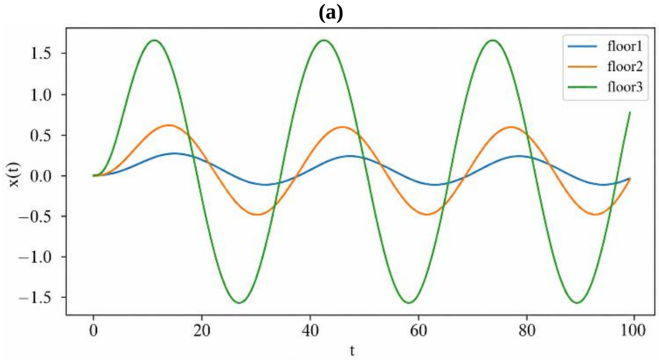

**(d)**

**Fig. 9 The Wilson theta response for several values of ζ (a: 0%, b: 5%, c: 10%, d: 20%)**

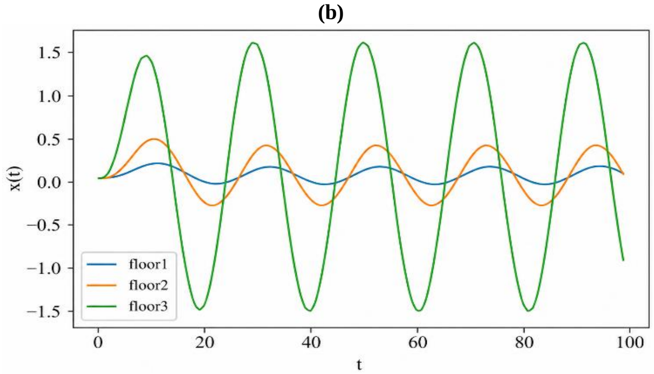
It is observed from Figure 9 (a to d) that according to the graphs obtained, the displacement significantly decreases with an increase in the damping ratio, which means that a low damping ratio can lead to a large amplitude response, while a high damping ratio can lead to a small amplitude response.



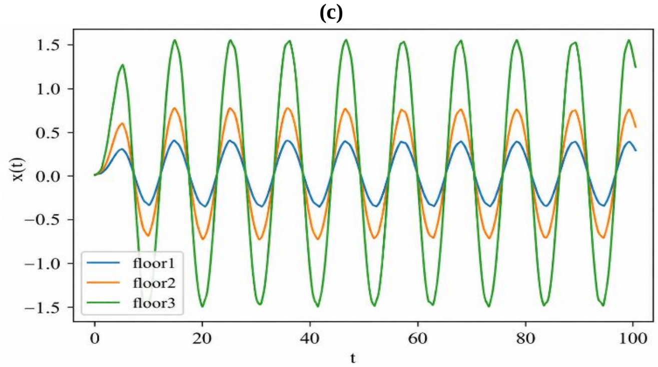

**(a)**




**(b)**

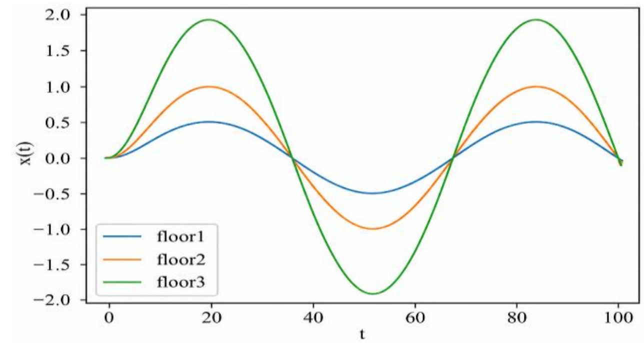



**(c)**

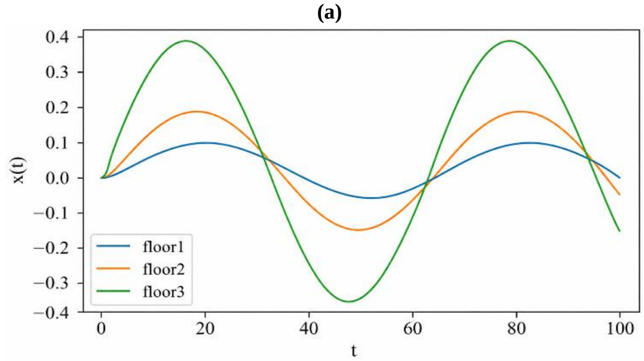



**(d)**

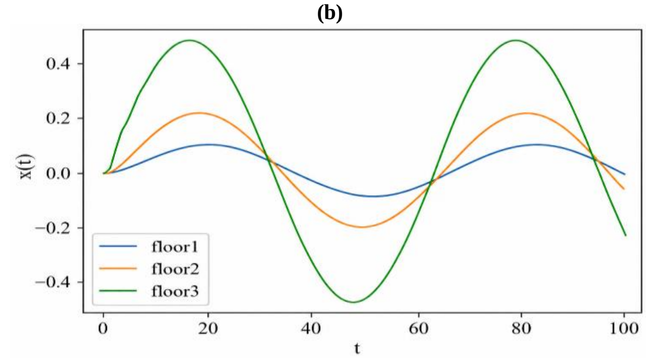
**Fig. 10 The Wilson theta response for different values of ωf (a:10rad/s, b:20rad/s, c:30rad/s, d:60rad/s).**



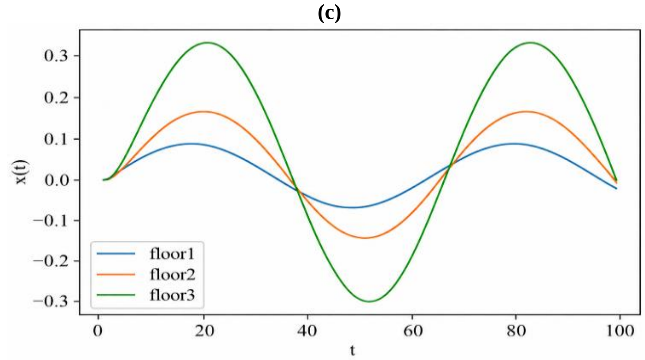

**(a)**




**(b)**




**(c)**




**(d)**

**Fig. 11 The Wilson theta response for different values of ρ**

Also from Figure 10 (a to d), it is observed that when the value of the excited frequencies is increased, the results obtained by the Wilson approach show a reduction in the response at the level of the first two floors.

As depicted in Figures 11(a)–(d), the Wilson-θ results indicate that the beam-to-column stiffness ratio plays a crucial role in governing the structural response. Increasing the structural stiffness results in a pronounced reduction in the top-floor peak displacement, from approximately 2 cm to 0.3 cm, demonstrating the effectiveness of enhanced rigidity in controlling lateral deformations and improving overall structural stability under dynamic loading.

## 5. Conclusion

This work was applied to study the influence of diverse dynamical characteristics in performing and estimating the lateral response of the RC frame building. Here, for the considered project, we have done the analysis procedure as specified in the previous steps and given the comparative results for every story and every method, which includes a graphical representation.

We have evaluated the effect of each parameter separately using three different methods, analytical and numerical (Wilson and Newmark). To do this, we used each model while leaving the other parameters unchanged. The following are the key results of the current investigation:

- Ratio damping severely affects the response of the structure; it is recommended to choose a higher damping ratio to ensure the safety of a structure, to control excessive vibrations, reduce the risk of resonance, and improves the structure's ability to dissipate energy.
- Structural stiffness has a pronounced impact on the response of the model, as evidenced by the results obtained using the three methods. This influence is clearly observed through the variation of the beam-to-column stiffness ratio, ranging from a flexible beam with negligible stiffness to a fully rigid beam.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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