Innovative filter analysis in Image Processing

T.Saravanan
Assistant Professor,
Department of Computer Science
Roever College of Engineering, Perambalur.

***Abstract*— The wavelet transform has become the most interesting technology for still images. Yet there are many parameters within a wavelet analysis and synthesis which govern the quality of a reconstructed image. An evaluation of the visual quality of images for different wavelet filter leads to recommendations on the wavelet filter to be used in image coding. The discrete wavelet decomposition and reconstruction remains one of the main issues of current signal and image processing. The reconstruction performance of the various wavelet filter family approaches that of discrete time domain filter coefficients used specifically for reconstruction for better visualization. The wavelet lifting scheme divides the wavelet transform into a set of steps. One of the elegant qualities of wavelet algorithms expressed via the lifting scheme is the fact that the inverse transform is a mirror of the forward transform. The comparative analysis of various wavelet filters using lifting wavelet transforms has been performed for image quality evaluation.**

***key wordss*—**Discrete wavelet transform, Filter analysis, Frequency Response Characteristics.

1. INTRODUCTION

Most Traditional statistical approaches to filter analysis are restricted to the analysis of spatial interaction over relatively small neighborhoods. One way to overcome limitation of a single Scale analysis is to use the discrete wavelet Transform (DWT), which provides a precise and unifying frame work for the characterization of a signal at multiple resolution[1], there have now been several studies on filter analysis with particular attention to the use of LWT [3], which constitute the pyramid structure wavelet transform.

The present paper work is to explore the orthogonal and biorthogonal wavelet filters for 2D image analysis, which constitutes a many image processing application such as image [2] compression, feature extraction, noise removal and Texture analysis.

The basic idea behind wavelet transform is to analyze different frequencies of a signal using different scales. High frequencies are analyzed using low scales while low frequencies are analyzed in high scales. To be more specific, in wavelet transform, all of the basis functions, which are called wavelets, are derived from scaling and translation of a single function, called mother wavelet. Many types of mother wavelets and associated wavelets exist [9]. Depending on their properties, the wavelet transform can be divided into two categories, i.e., the biorthogonal [1] [2] wavelet transform and the orthogonal wavelet [9] transform. In this study, analyzed both transform through non symmetric [10] method. Orthogonal wavelet transform allows an input image to be decomposed into a set of independent coefficients, corresponds to each orthogonal basis. In other words, orthogonality implies that there is no redundancy in the information represented by the wavelet coefficients, which results in efficient representation and other desirable properties.

Each wavelet, ψ_{ab} , is defined by the scaling and translation of the mother wavelet ψ as follows:

$$\Psi_{a,b}\ (t) = 1/\sqrt{a}\ \Psi\ (t\text{-}b/a) \qquad (1)$$

where a and b are integers representing scale and translation, respectively. For certain other functions, the set of wavelets $\psi_{ab,}$ forms a smooth, compactly supported (finite length), and orthogonal basis. In order to define the wavelet transform we use the wavelets defined in (1) as basis functions, namely,

$$w_{a,b} = 1/\sqrt{a} \int f(t)\ \Psi\ (t\text{-}b/a)\ dt \qquad (2)$$

where, w_{ab} are the wavelet coefficients. The process utilizing this equation is called wavelet decomposition of the profile f(t) by means of the set of wavelets $\Psi_{a,b}$. In (2), each wavelet coefficient represents how well the profile f(t) matches with the wavelet with scale a and translation b. If the profile f(t) is similar to the wavelet at a particular scale and translation, then the coefficient has a large value. A wavelet coefficient, therefore, represents the degree of correlation or similarity between the profile and the mother wavelet at the particular scale and translation. Thus, the set of all wavelet coefficients w_{ab} gives the wavelet domain representation of the profile f(t).

The 1-D DWT can be extended to 2-D transform using separable wavelet filters. With separable filters, applying a 1-D transform to all the rows of the input and then repeating on all of the columns can compute the 2-D transform. When one-level 2-D DWT is applied to an image, four transform coefficient sets are created. As depicted in Figure 1 (c), the four sets are LL, HL, LH and HH where the first letter corresponds to applying either a low pass or high pass filter to the rows, and the second letter refers to the filter applied to the columns.

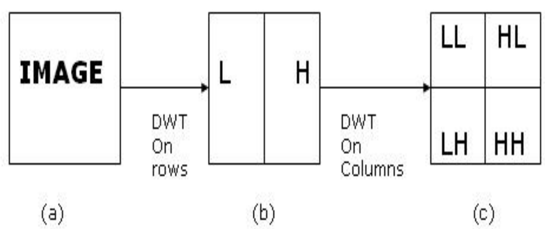

Figure 1 Block Diagram of DWT (a) Original Image (b) Output Image after the 1-D applied on Row input (c) Output Image after the second 1-D applied on Column input

2. LIFTING WAVELET TRANSFORM

Sweldens first introduced [3] [4] the concept of second generation wavelet, which is so called lifting scheme. Lifting wavelet transform is a multiresolution analysis used for the construction of the second generation wavelets. It is an efficient implementation of the wavelet transform algorithm. It provides perfect reconstruction of original signal which is not possible by standard implementation even though they are lossless in principle. It also provides flexibility as compared to classical wavelets.

(a) Decomposition

The image of size 2^J x 2^J is passed through the subband coding structure as described in the Fig.2.

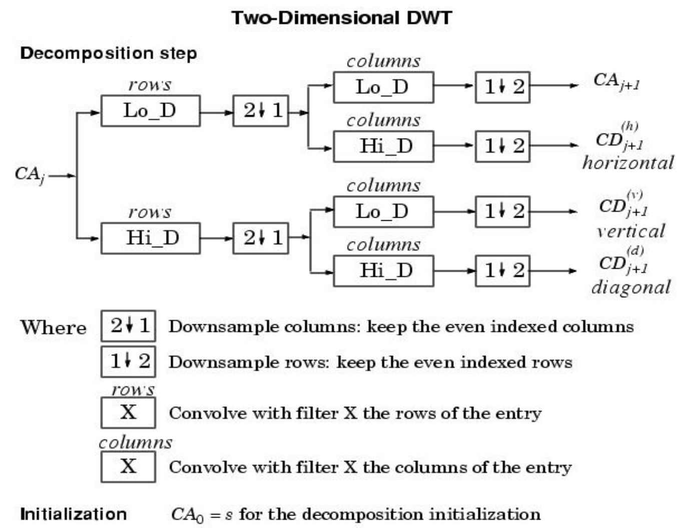

Figure.2 Subband Coding Method - Decomposition

As a first step, this image is convolved with the filters Lo_D and Hi_D and down sampled along the columns to yield a pair of 2^J x $2^{J/2}$ array. Now this array is fed to the pair of decomposition filters Lo_D and Hi_D and they are convolved and down sampled along the rows. The Analysis and Synthesis filter coefficients are shown in Table I and Table II.

Table I Analysis filter coefficient (db3)

Filter	Filter coefficients					
Lo_D	0.0352	-0.0854	-0.1350	0.4599	0.8069	0.3327
Hi_D	-0.3327	0.8069	-0.4599	-0.1350	0.0854	0.0352

Table II Synthesis filter coefficient (db3)

Filter	Filter coefficients
Lo_R	0.3327 0.8069 0.4599 -0.1350 -0.0854 0.0352
Hi_R	0.0352 0.0854 -0.1350 -0.4599 0.8069 -0.3327

Repeating this procedure will yield a set of four $2^{J/2}$ x $2^{J/2}$ arrays called LL1, HL1, HH1 and LH1. Inside these arrays the maximum energy is packed in the LL1 array called the approximate of the image. The remaining three arrays contain main features of the image in Vertical (LH1), horizontal (HL1) and diagonal (HH1) aspects. These four arrays are arranged into a single 2^J x 2^J matrix as shown in Fig 1. This is called the first level decomposition in which the given image is broken down into a set of approximate, horizontal, vertical and diagonal coefficients. Each of them has reduced resolution than the original image. To perform the second level decomposition the approximate array (LL1) is fed back to the same subband coding structure which results in four $2^{J/4}$ x $2^{J/4}$ arrays called as LL2, HL2, HH2 and LH2. Repeating this process, we can obtain higher level of decompositions.

(b) Reconstruction

The reconstruction of the image can be carried out by the following procedure as shown in figure.3. First, all four subbands at coarsest scale are up sampled by a factor of two, and the subbands are filtered using low-pass Lo_R and high-pass Hi_R synthesis filters in each dimension. Then four filtered the subbands are summed to reach the low subbed at the next finer scale. The process is repeated until the image is fully reconstructed. Finaly will get the image like original image size as 2^Jx2^J

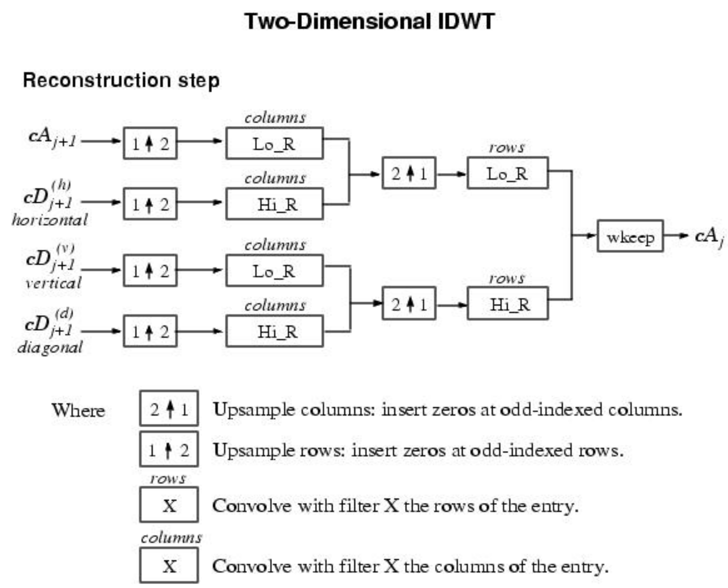

Figure.3 Subband Coding Method - Reconstruction

To perform the three level reconstruction, first the 3rd level approximate array (LL3) and detailed array (LH3) up sampled by row vector then this passes through the synthesis low pass and high pass filter section. These output is combined together for the next stage, where the same output again up sampled by column vector then this passes through the synthesis low pass filter section. The same process is obtained for detailed array (HL3) and (HH3). At the end of the stage (LL3+LH3) and (HL3+HH3) these two outputs are combined together to produces an approximate array (CA2) coefficient. For the next level the same process is repeated, finally can get the reconstructed image as like as original image.

3. FILTER ANALYSIS

BASIC CONDITIONS FOR WAVELET FILTER COEFFICIENTS

Wavelet must satisfy necessary conditions like orthogonality and certain desirable properties for specific kind of applications. These conditions, in turn, put restrictions on scaling and wavelet function coefficients [8].

(a) Normalization

Vector h(k) is normalized when the sum of all the coefficients should be √2
and square of sum of all the coefficients should be 1.

ie, $$\sum_{k=0}^{N-1} h(k) = \sqrt{2} \qquad (3)$$

$$\sum_{k=0}^{N-1} h^2(k) = 1 \qquad (4)$$

(b) Orthogonality

Orthoganality as the attribute that the inner product of the bases equals zero.

$$\sum_{k=0}^{N-1} h(k)h(k-2m) = 0 \quad \text{for all } m \neq 0 \qquad (5)$$

if m = 0 will get the square normalization result.

(c) P^{th} Moment function

Since Φ(t) and Ψ(t) are orthogonal, this means, if Φ(t) is capable of expressing monomial of order up to p then corresponding wavelet function must have its moments of order up to p as zero.

$$\sum_{k=0}^{N-1} (-1)^k k^p h(k) = 0 \qquad (6)$$

(d) Generation of HPF,ILPF AND IHPF from LPF

Let h_0 be the lowpass decomposition filter coefficients

Consider 4 tap filter

h_0 = { h(0),h(1),h(2),h(3) } (LPF)

Reverse of lowpass decomposition filter coefficient is called as inverse lowpass filter h_1

h_1 = { h(3),h(2),h(1),h(0) } (ILPF)

highpass decomposition filter f_0 is a copy of h_1, where the signs of the odd numbered elements (1^{st}, 3rd,……..etc) are reversed.

f_0 = { -h(3),h(2),-h(1),h(0) } (HPF)

Inverse highpass reconstruction filter f_1 is a copy of h_0, where the signs of the even numbered elements ((2^{nd}, 4th,……..etc) are reversed.

f_1= {h(0),-h(1),h(2),-h(3) } (IHPF)

these conditions are applicable for all types of wavelet filter except vaidyanathan filter, the vaidyanathan [5] filter does not satisfy any conditions on the moments but produces exact reconstruction.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this paper, the comparative analysis of image decomposition using Lifting Wavelet Transform[3] [4] for various wavelet filters analyzed. The test results and discussions for one level and 3 level decomposition has been presented in the upcoming sections.

4.1 SYMMETRIC METHOD IMAGE ANALYSIS

The analysis is performed on 20 sets of standard images each image is of various square size (512X512 and 1024X1024). The one level symmetric[1] method image decomposition and reconstruction performance is compared for three filters such as 'db2', 'LeGall 5/3' and 'CDF 9/7' wavelet filters and the test results obtained is presented graphically in figure 4.

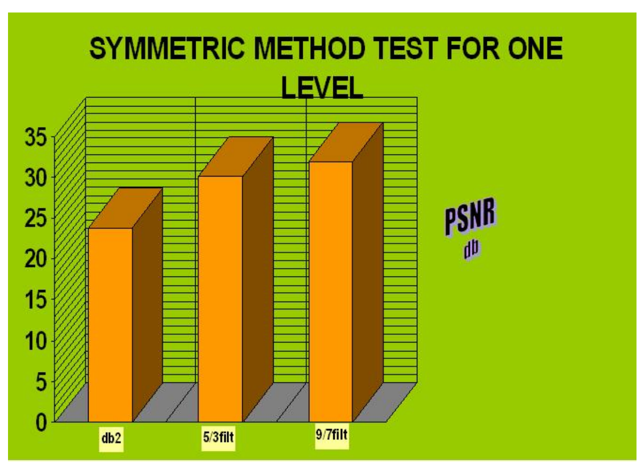

Figure.4 Symmetric method test for one level

Similarly one level Nonsymmetric method image decomposition and reconstruction performance is compared for the same three filters such as 'db2', 'LeGall 5/3' and 'CDF 9/7' wavelet filters and the test results obtained is presented graphically in figure.5.

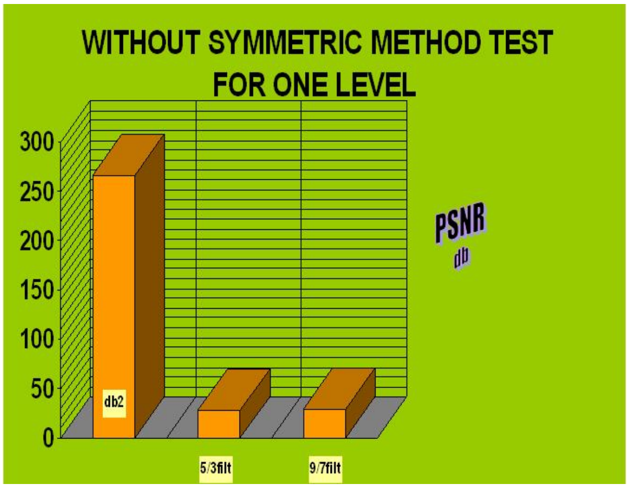

Figure.5 NonSymmetric method test for one level

Grater the PSNR value implies, better the quality of the reconstructed image. From the PSNR values obtained, it is found that Symmetric [1] method, CDF 9/7 wavelet filter provides better performance than the other two filters, but in Nonsymmetric method result is opposite to that of previous one. Where 'db2' wavelet filter provides more and better than the Symmetric approach. So further 3 level image decomposition and reconstructin process chosen NonSymmetric method using lifting wavelet approach. Such process described in the next section.

4.2 NONSYMMETRIC METHOD IMAGE ANALYSIS

For the NonSymmetric method three level, image analysis performance is compared for different daubechies family wavelet filters namely db1, db2, db3, db4, db5, db6,db25 and db35 using LWT and the test results obtained is presented graphically in figure.6. The PSNR value clearly displayed in the chart says that, among these filters 'db1' filter provides better image quality than the other daubechies family. Average RMSE and PSNR value of 20 images for each filters presented in the table III

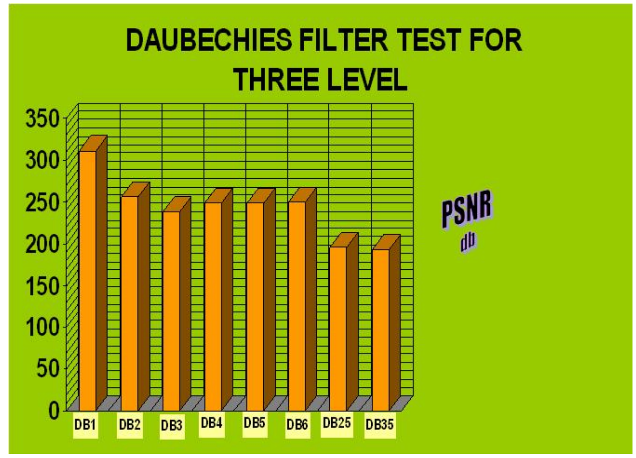

Figure.6 NonSymmetric method test for three level – Daubechies family

Table III Daubechies wavelet family filter comparison Test Results

Filter Bank	(Average for 20 images) PSNR	RMSE
Daubechies-1	310.3505	7.89E-14
Daubechies-2	255.9933	4.60E-11
Daubechies-3	237.3411	3.95E-10
Daubechies-4	248.1647	1.11E-10
Daubechies-5	248.3622	1.09E-10
Daubechies-6	249.103	9.79E-11
Daubechies-25	196.1042	5.98E-08
Daubechies-35	193.281	5.99E-08

* Note E-14 = 10^{-14}

Similarly the test has been conducted for coiflet filters, image analysis performance is compared for different Coiflet family wavelet filters namely coif1, coif 2, coif 3, coif 4 and coif 5 using LWT and the test results obtained is presented graphically in figure 7. The PSNR value clearly displayed in the chart says that, among these filters 'coif3' filter provides better image quality than the other coiflet family used in this project. Average RMSE and PSNR value of 20 images for each filters presented in the table IV.

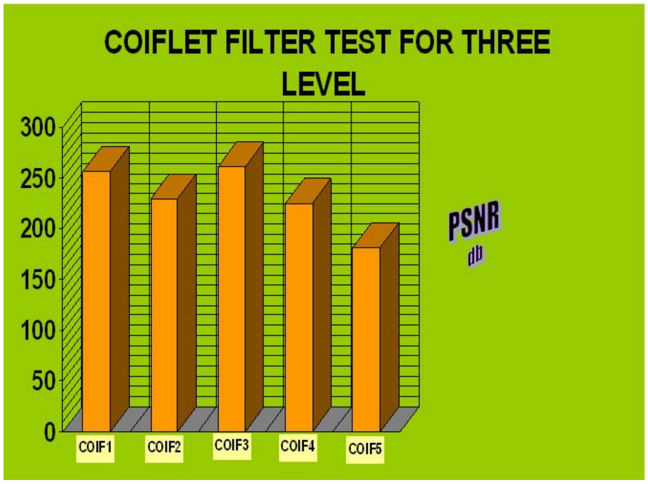

Figure.7 NonSymmetric method test results for three level – coiflet family

Table IV Coiflet wavelet family filter comparison Test Results

Filter Bank	(Average for 20 images) PSNR	RMSE
Coif-1	256.3036	4.45E-11
Coif -2	230.1728	8.91E-10
Coif -3	260.7423	2.62E-11
Coif -4	224.7662	1.64E-09
Coif -5	181.2996	2.46E-07

* Note E-14 = 10^{-14}

4.3 FILTER RESPONSE

In this section frequency versus magnitude response curve plot for all filter analysis made by using DFT[11]. This converts the time domain signal into frequency domain, which can be obtained by

$$H(k) = \sum_{n=0}^{N-1} h(n)\, e^{-i(2\pi kn/N)} \qquad (7)$$

where k = 0, 1,……….N-1
N – Number of samples

In this section described frequency response characteristics curve for all daubechies family, coiflet family,9/7 and 5/3 filter coefficients used in this project, daubechies family alone represented graphically in the figure 8, in that produced dark curve as a db1 filter frequency response says exact reconstruction.

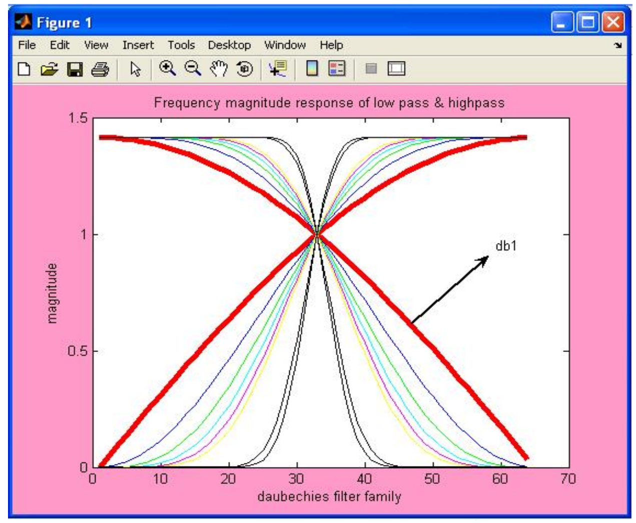

Figure.8 Frequency Vs Magnitude plot for daubechies family

Similary graphical representation of figure.9 produced dark curve says coiflet 3 filter satisfies exact reconstruction among coiflet family and CDF family.

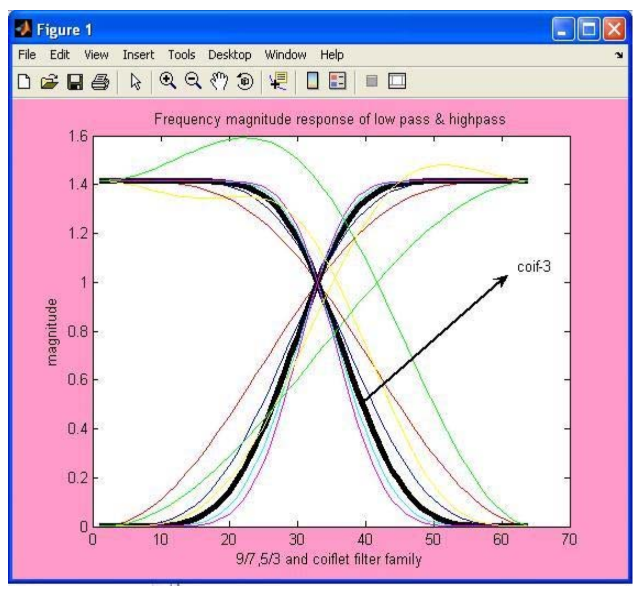

Figure.9 Frequency Vs Magnitude plot for coiflet and CDF family

So the comparison of filter response characteristic curve for db1 and coif3 separately displayed in the figure 10, in that minimum filter coefficient and linearly changing large transition band curve (db1) produces the best image quality result while moves to the higher level decomposition.

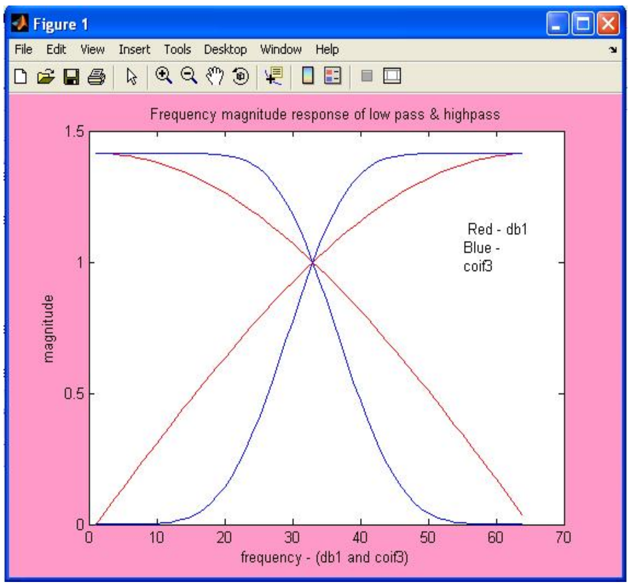

Figure.10 Frequency Vs Magnitude plot fordb1 and coif3 filter

5. CONCLUSION

Image analysis for various wavelet filters using LWT approach has been studied. In the LWT approach the given image is decomposed for 3 levels in each level the wavelet coefficients such as LL, LH, HL and HH are extracted. The inverse process is applied for the extracted coefficients to get the original image.

The process is done by using orthogonal and biorthogonal filters through 3 families. The families are CDF, Daubechies and Coiflet, these are verified through various parameters and the filter which belongs to that family is identified.

The comparative analysis of an image decomposition using LWT for various wavelet filter is analyzed and characterized. The Symmetric and Nonsymmetric method analysis is performed on 20 set of standard images with various square size. The comparative studies are shown in figure 11.

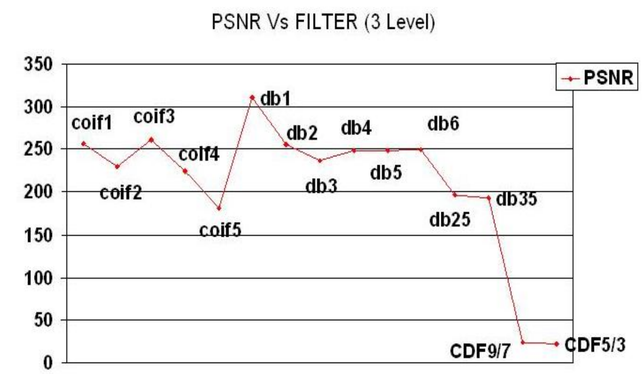

Figure.11 PSNR Vs Various wavelet filter

This graph shows that the PSNR value is high for db1 filter in doubechies family. The nearest value of db1 filter is obtained through coif3 filter of coiflet family, from the metrics analysis which shows results in db1 filter has better performance over the remaining filter for the exact reconstruction of given 2D images.

The characteristics of the above filters are represented through the magnitude Vs Frequency plot. Here it shows linearly changing large transition band curve of db1 filter produces better result even though coif3 filter having the sharp cutoff frequency(small transition band).

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REFERENCES

[1] Jianyu Lin and Mark J. T. Smith "New Perspectives and Improvements on the Symmetric Extension Filter Bank for Subband/Wavelet Image Compression-IEEE Transactions on signal processing, volume. 17, number. 2, February 2008.

[2] Balasingham.I and Ramstad T.A Are the Wavelet Transforms the Best Filter Banks for Image Compression?, research article, Hindawi Publishing Corporation EURASIP Journal on Image and Video Processing, pp 1-7, Article ID 287197 2008.

[3] Sweldens.W, "The lifting scheme: A custom-design construction of biorthogonal wavelets," Appl. Comput. Harmon. Anal., vol. 3, pp. 186–200, 1996.

[4] Sweldens.W, "The lifting scheme: a construction of second generation wavelets," SIAM J. Math. Anal, pp. 511–546, 1997.

[5] P. P. Vaidyanathan and A. Kirac, "Theory of cyclic filter banks," in Proc. IEEE International Conference. Acoust., Speech, Signal Process., Munich, ermany, pp. 2449–2452. May1997.

[6] Torsten Palfner and Erika Muller, Effects of Symmetric Periodic Extension for Multiwavelet Filter Banks on Image Coding. IEEE Institute of Telecommunications and Information Technology University of Rostock, Germany, 1999.

[7] Bryan E. Usevitch, A Tutorial on Modern Lossy Wavelet Image Compression.Foundations of JPEG 2000. IEEE signal processing magazine, September 2001.

[8] Soman K.P and Ramachandran K.I, Insight into wavelets from theory to practice, 2006 Edition. Prentice Hall of India private limited.

[9] Jensen.A, la cour-harbor.A "Ripples in mathematics" The Discrete Wavelet Transfrom. 2003 Edition. Springer(India) private Ltd.

[10] Adiga A, Ramakrishnan K. R, and Adiga B. S "A Design and implementation of orthonoramal symmetric wavelet transform using PRCC Filter Banks"-in proceeding. IEEE International Conference. Acoust., Speech, Signal Processing, Apr. 2003, vol. 6, pp. VI–513.

[11] Michael Weeks, Digital Signal Processing Using MATLAB and Wavelets, 2007 Edition. Infinity Science Press LLC. (An imprint of Laxmi Publications Pvt. Ltd.)

[12] http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html