Comparison of Combined Multi-rate Barker Codes and Multi-rate Barker Codes For WOFDM Systems

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ABSTRACT : *In this paper channel estimation is achieved through the use of two novel algorithms involving Barker codes. Comparisons between the delay estimation percentage error of the Least Linear Minimum Mean Square Error (LMMSE) conventional channel estimation technique with the two newly proposed techniques takes place. The first proposed algorithm embeds a Barker code to the transmitted wavelet orthogonal frequency division multiplexing (WOFDM) data block and uses the correlation between that Barker code and the received signal in order to accurately estimate the channel delay at the receiving end. Two new ideas were investigated in an attempt to generate multi-rate versions of the Barker code of length 13. For the first investigated technique combined Barker codes were generated where a barker code. For the second investigated technique each bit of the initial Barker 13 code was repeated a varying number of times in order to generate mutli-rate versions of the code. Those multi-rate Barker codes generated using the second technique were used in creating a training sequence and using the difference in peak positions of the auto correlation of the transmitted and received sequences the delay introduced by the channel was estimated. These comparisons were performed with the four most common wavelet filters used in the WOFDM modulation. The simulation and results show the advantages of the two new proposed techniques.*

Keywords -*Barker codes, channel estimation, correlation, LMMSE, Multi-rate codes*

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has a lot of advantages which makes it perfect for usage in a lot of modern day communication systems. The use of orthogonal subcarriers makes inter symbol interference (ISI) minimal with the use of a mere cyclic prefix as shown in [1].Wavelet based OFDM (WOFDM) has the same advantages as that of the OFDM but the use of a mother wavelet to achieve the orthogonality of the subcarriers eliminates the need for the cyclic prefix and in doing so increases the bandwidth available for actual data transmission and thus improves the bandwidth efficiency as proposed in [2].

The need for channel estimation is essential making it a popular research field and explains the reason for existence of a wide range of channel estimation techniques. The LMMSE channel estimation technique is perhaps one of the most widely used methods for it provides accurate estimation of the channel delay. Perhaps the greatest disadvantage of the LMMSE channel estimation method is that it is an iterative technique making it necessary to continuously update the calculation of the channel covariance matrix which is relatively timely compared to the two new algorithms presented in this paper.

In the next section, the basics of WOFDM are reviewed, the conventional LMMSE algorithm is discussed in section III. Section IV will cover the system model and section V will list and discuss the simulation results in details and finally the last section will present the conclusions.

II. WOFDM PRINCIPLES

OFDM is regarded as a Multi-carrier transmission technique which is particularly useful for frequency selective channels and high data rates explained in [3]. This technique transforms a frequency-selective wide-band channel into a group of non-selective narrowband channels, making it perfect for use by systems that are subjected to channels with large delay spreads. The transmitted

OFDM signal maintains its orthogonality in the frequency domain. The assurance that the orthogonality of each of the used subcarriers hold is done through the use of a cyclic prefix in the beginning of each of the transmitted OFDM blocks. This cyclic prefix uses a part of the bandwidth which could otherwise be used to transmit data and this is considered a waste of useable bandwidth and so decreases the bandwidth efficiency.

WOFDM works the same way as the OFDM in terms of principle but instead of using IFFT in order to generate the orthogonal subcarriers as is the case in OFDM, IDWT is used instead. The advantage of doing so is that the maintaining of the orthogonality of the subcarriers is automatically achieved due to the nature of the wavelet families and does not need the addition of any cyclic prefix. This means that the entire OFDM block can be used for data transmission and so bandwidth is more efficiently employed as illustrated in [4].

While OFDM uses the orthogonal complex exponential carriers generated by the IFFT having frequencies which are multiples of f_o as shown in Equation (1).

$$subc(k) = e^{jk2\pi f_o t} \qquad (1)$$

Where $subc(k)$ denotes the k-th subcarrier used. WOFDM makes use of the fact that the members of a wavelet family will satisfy the same property as shown in Equation (2).

$$\langle \Psi_{j,k}(t), \Psi_{m,n}(t) \rangle = \begin{Bmatrix} 1, j = m, k = n \\ 0, otherwise \end{Bmatrix}$$

(2)

Such a family can be obtained by translating and scaling the wavelet mother function $\Psi(t)$ as in Equation (3):

$$\Psi_{j,k}(t) = s_0^{-j/2} \Psi(s_o^{-j} \cdot t - k\tau_o)$$

(3)

Equation 3 corresponds to a sampled version of a wavelet family, the discrete variables being s_o (the scale) and k (the position within the scale)as shown in[5] and [6]. Since $\Psi_{j,k}(t)$ forms an orthonormal, family any signal s(t) can be written as a weighted sum of wavelet functions (4), the weights being given by the wavelet coefficients ($w_{j,k}$), as stated in Equation(4) :

$$s(t) = \sum_{j \in Z} \sum_{k \in Z} w_{j,k} \Psi_{j,k}(t)$$

(4)

The signal s(t) can be interpreted as a “WOFDM symbol”. Any wavelet from a certain scale j can be composed as a weighted sum of scaling functions from the previous scale j-1. This, a different form of signal synthesis equation will be obtained in Equation (5):

$$s(t) = \sum_{j} \sum_{k} w_{j,k} \Psi_{j,k}(t) + \sum_{k} a_{L,k} \varphi_{L,K}$$

(5)

In equation (5),j=L is the coarsest level used for signal composition, and $a_1,\ldots,a_L$ are called the approximation coefficients. The discrete version of the signal, s[n] is calculated by performing the Inverse Discrete wavelet transform (IDWT) as proposed in [7] and [8]. At each iteration, an up sampling operation is performed with a factor 2 and followed by two quadrature mirror filters. Fig. 1 shows the overall process of the IDWT for four decomposition levels. G and H are the impulse responses of the synthesis low-pass and high-pass filters respectively. Each iteration of the algorithm illustrated in Fig. 1, can be described by Equation(6):

$$x[n] = \sum_{k} (w[k]h[n-2k] + a[k]g[n-2k])$$

(6)

We can consider that the data we have to transmit is a set of approximation and wavelet (detail) coefficients, as follows in Equation (7)

$$data = \{[a_L],[w_L],[w_{L-1}],\ldots,[w1]\}$$

(7)

The input data which is usually a set of modulated data bits is considered as the wavelet coefficients in WOFDM systems as explained in [9]. The data sequence is brought to the input of the IDWT processor, whose output will be the discrete version of the WOFDM symbol.

After each iteration the number of wavelet coefficients halves (because each iteration will rely on a down-sampling and filtering couple). Thus, considering that the data vector from (6) has N samples, then half of them will be stored in [w_1] and they will be transmitted in the channel using the upper half of the dedicated bandwidth. Since WOFDM relies on an IDWT modulator, it follows that the transmission performance can be influenced by the parameters of this modulator. As stated earlier such as the choice of wavelet filter.

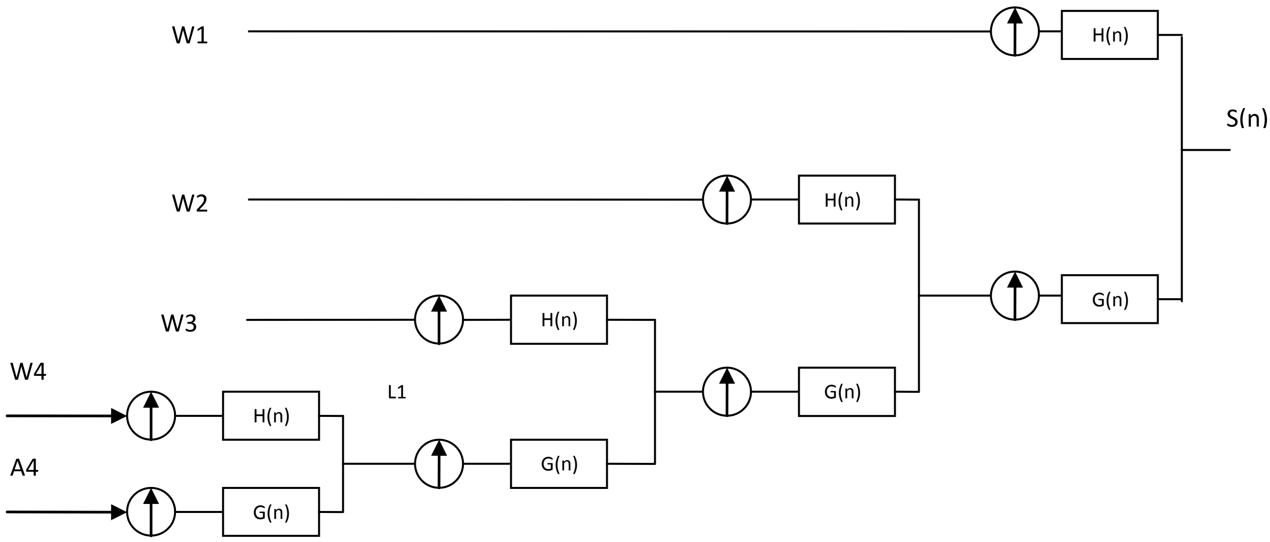

Fig. 1: IDWT implementation using filter bank

III. CHANNEL ESTIMATION

Channel estimation is achieved in most cases through the use of pilot data which is well known to the receiver. Based on the system model, various channel estimation schemes have been proposed in [10].In block-type pilot based channel estimation, the pilot is sent in all sub-carriers with a specific period. Assuming the channel is constant during the block, it is insensitive to frequency selectivity. Since the pilots are sent at all carriers, there is no interpolation error. The estimation can be performed by using LS and LMMSE channel estimator. For frequency domain channel estimates, MSE is usually considered as the performance measure of channel estimates, and it is defined by in Equation (9)

$$MSE = E\left\{\left|H[k] - \hat{H}[k]\right|^2\right\} \qquad (8)$$

Where, $\hat{H}[k]$ is the estimated channel. The simplest channel estimator is to divide the received signal by the input signals, which should be known pilot symbols as shown in [11] and [12]. This is known as the LS Estimator. So the LS estimator is more commonly used due to its ease of implementation and acceptable performance and can simply be expressed as shown in Equations(10) and(11) respectively.

$$\hat{H}_{LS}(k_p) = Y(k_p)/X(k_p) = H(k_p) + N^{'}(k_p) \qquad (9)$$

Where,

$$Y(k_p) = H(k_p)\cdot X(k_p) + N(k_p) \qquad (10)$$

And $N(k_p)$ is AWGN in the frequency domain

This channel estimator is only accurate when there is no noise in the channel. The LMMSE Estimator minimizes the MSE by using the frequency correlation of the slow fading channel. This is achieved through a optimizing linear transformation applied to the LS estimator. From adaptive filter theory, the optimum solution in terms of the MSE is given by the Wiener-Hopf equation illustrated in Equation (12).

$$h = R_{hhls} R_{hlshls}^{-1} h_{ls} \qquad (12)$$

Where, R_{hhls} is the cross correlation between channel attenuation vector h and the LS estimate h_{ls} and R_{hh} is the auto correlation matrix of the LS estimate h_{ls}, given by Equations (13) and (14).

$$R_{hhls} = E\{hh_{ls}^H\} \tag{13}$$

$$R_{hlshls} = E\{h_{ls}h_{ls}^H\} \tag{14}$$

$$\hat{H}_{LMMSE} = [R_{HH} /(R_{HH} + \frac{\sigma_n^2}{XX^H} I)]\hat{H}_{LS} \tag{15}$$

Since white noise is uncorrelated with the channel attenuation, the cross correlation between the channel h and noisy channel h_{ls} is the same as the autocorrelation of the channel h. Thus we can replace R_{hhls} with R_{hh} *and c with* R_{hh} plus noise power σ_n^2 as illustrated in [13]. So the estimator is re-written in Equation (15)

IV. THE PROPOSED ALGORITHMS

In this section transmission will be considered in a flat Rayleigh fading channel where the signal is being corrupted by additive white Gaussian noise. Two channel estimation algorithms are proposed in this paper. They both use Barker codes in order to establish an accurate estimation of the channel delay in WOFDM systems. The first algorithm replaces the first 5 bits of the transmitted data block s(n) shown in Fig. 1 with a five bit Barker code. This data block consisted of 1024 BPSK data bits but after the replacement it consists of a 5 bit Barker code then 999 data bits. The correlation between the Barker code and the received data block gives an accurate estimation of the delay caused by the channel.

Two new concepts were investigated in an attempt to create multi-rate barker 13 codes. For the first attempt combined barker codes were tested. Each bit of a barker code of length 13 was replaced by a barker code of length 4 {1,1,-1,1}.This created a code of length 52 bits and zero padding was later on used in order to extend the length to 64 bits. In order to create the three remaining multi -rate codes, more zeros were added in order to create Barker codes of length 128,256 and 512. These different multi-rate combined barker codes are shown in Fig. 2 (a,b,c,d) respectively. Each of these combined multi-rate barker codes' autocorrelation are shown in Fig. 3(a,b,c,d) respectively

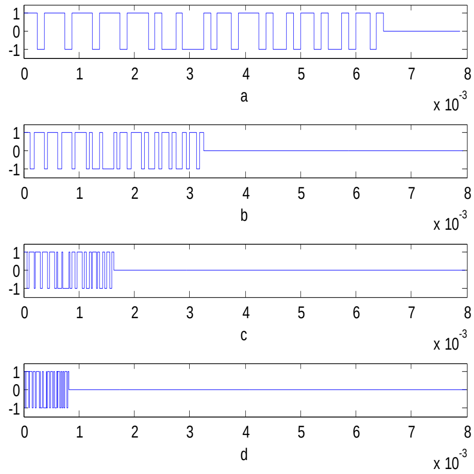

Fig. 2: Combined Multi-rate Barker codes

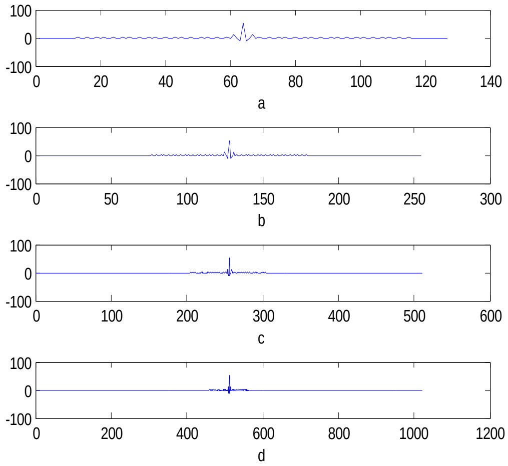

Fig. 3: Autocorrelations of combined multi-rate Barker codes

Fig. 3 shows clearly that all the peaks of the autocorrelations are of the same value and so they don't provide any additional advantage in channel estimation.In the second proposed algorithm instead A4,W4,W3,W2 and W1-shown in Fig. 1 being the inputs to the IDWT , the multi-rate Barker codes shown in Fig. 4 are the inputs to the IDWT. Where, a Barker code of length 64 replaces A4 and W4.A Barker code of length 128 replaces W3, a Barker code of length 256 replaces W2 and a Barker code 0f length 512 replaces W1. This arrangement generates a 1024 bit signal (S(n)) for transmission in the channel.

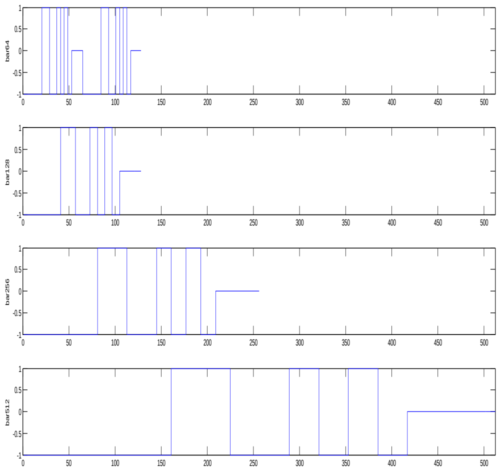

Fig. 4: Multi-rate Barker codes

Fig. 5(a,b,c,d) shows the autocorrelations of the multi-rate barker codes of length 64,128,256 and 512 respectively. and show clearly the varying values of the peaks that change with the length of code and this provides the advantage sought after which allows for better estimation of the channel delay in case of noisy channel for it will be easier to detect the peak.

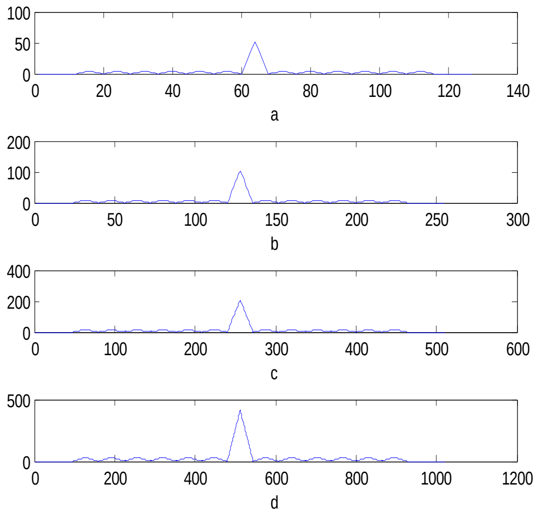

Fig. 5: Autocorrelations of Multi-rate Barker codes

V. SIMULATION RESULTS

In the first algorithm discussed in the previous section. A five bit Barker code is inserted at the beginning of the WOFDM data block. This code is later on used at the receiving end. The received data sequence is correlated with the five bit Barker code at the receiving end. The difference between the location of the maximum value of that process is subtracted from the max of the correlation between the five bit Barker code and the transmitted data at the transmitting end is obtained. This difference is then multiplied by the sampling time (Ts) and so the delay is obtained.

This procedure was repeated several times for each of the four chosen wavelet filters for the Doppler shift (f_d) value of 0.05. The delay was calculated when four different wavelet filters were used as OFDM carriers. The four wavelet filters used were the Haar, Symlet 6 ,Bior thogonal 3 and Daubechies 4.

For the second algorithm a Multi-rate Barker code was the input to the WOFDM system. A Barker code of different length was generated and inputted at different stages in the Mallat wavelet tree shown in Fig. 1. The difference between using multi-rate Barker code training sequence and not normal data is the gained advantages of the Barker code itself such as the lowest side lobe to main lobe ratio level of all other codes. This will provide the most accurate reception regardless of the SNR. The percentage error in the delay estimation for each of the two novel algorithms versus the traditional LMMSE algorithm when each of the four wavelet filters chosen earlier were used are calculated and plotted.

The first filter used was the HAAR filter. The percentage error in the channel delay estimation for each of the three algorithms used are plotted in Fig. 3 versus SNR. The first advantage of the multi-rate Barker code algorithm is the fact that it proves immune to changes in the SNR in the channel and in doing so it proves better than the conventional LMMSE algorithm but at the expense of the bandwidth efficiency. On the other hand the embedding of the Barker code-first proposed algorithm- provides a lower percentage of error than the LMMSE in the delay estimation as well as the lowest percentage error of all three techniques at high values of SNR.

The second filter used was the Daubechies 4 filter. The percentage error in the channel delay estimation for each of the three algorithms versus SNR are plotted in Fig. 4. Once again the multi-rate Barker code algorithm proves immune to changes in the SNR in the channel at the expense of the bandwidth efficiency and again the first algorithm gives the best delay estimation of all three algorithms for high SNR values and overall still performs better than the LMMSE

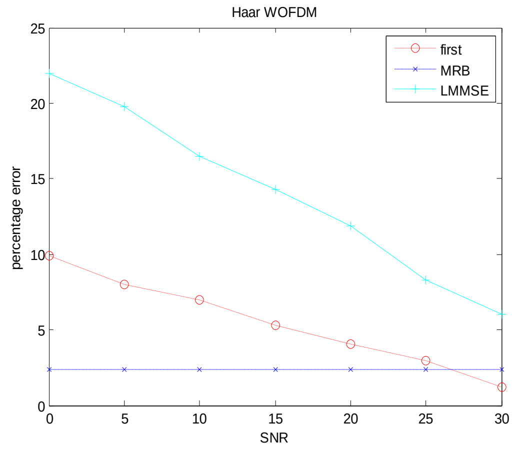

Fig. 3: Haar filter results

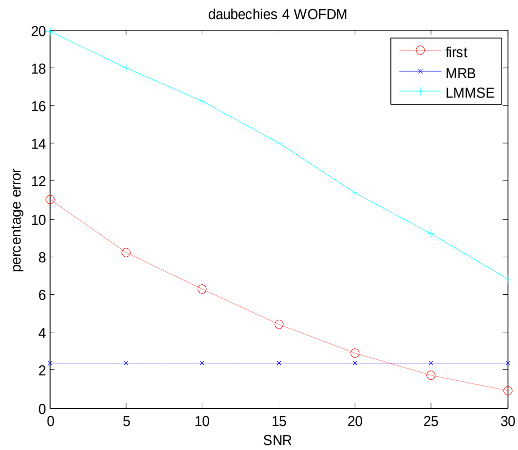

Fig. 4: Daubechies 4 filter results

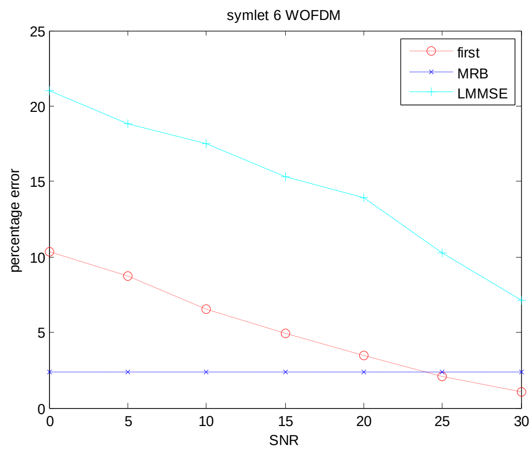

Fig. 5: Symlet 6 filter results

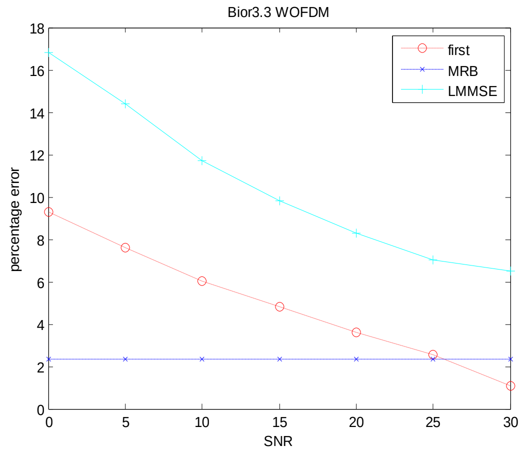

Fig. 6: Biorthogonal 3 filter results

The third filter used was the symlet 6 filter. The percentage error in the channel delay estimation for each of the three algorithms used are plotted in Fig. 5 versus SNR. The multi-rate Barker code algorithm remains immune to changes in the SNR in the channel. Again the first algorithm gives the best delay estimation at high SNR values and the LMMSE.

The last filter used was the biorthogonal three filter .The results were plotted in Fig. 6. The multi-rate Barker code algorithm remains immune to changes in the SNR in the channel at the expense of the bandwidth efficiency. The first algorithm gives the best delay estimation at high SNR values and performs better than the LMMSE .

VI. CONCLUSIONS

In this paper two novel algorithms are introduced. The two algorithms present channel estimation using Barker codes. The first algorithm uses the idea of embedding Barker codes in the WOFDM data block. Based on the correlation between the embedded code and the received signal, the channel delay is estimated with accuracy. The second technique mixes the advantages of the Barker codes with the multi-rate concept of the wavelet based WOFDM modulation. Before the choice was made to use the multi-rate Barker codes in the delay estimation a combined multi-rate barker code was tested first but proved to have smaller autocorrelation peaks. In this algorithm multi-rate Barker codes are used to generate a training sequence that allows accurate estimation of the delay of the channel .The percentage error performance of these two new novel algorithms was tested against the traditional

LMMSE channel estimation technique. The performance was examined for the four commonly used wavelet filters with the WOFDM system. The system was examined for a flat fading channel at different values of SNR. The results show that the multi-rate Barker code channel estimation technique outperforms the embedded Barker code technique and the conventional LMMSE technique for SNR less than 25dB. The results also showed that the multi-rate Barker code technique introduces a constant performance for the different signal to noise ratios. The advantage of using the multi-rate comes at the expense of the bandwidth efficiency. As indicated from the simulation results the Biorthogonal 3 wavelet filter introduces the best performance over the Haar, Daubechies 4 and Symlet 6.

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