Phase Effects at Intracavity Cascade Parametric Amplification with Low Frequency Pump

Rena J. Kasumova
[1](Physics Department/Baku State University, acad. Z. Khalilov str. 23, AZ1148, Baku, Azerbaijan)

ABSTRACT: *Theory of intracavity parametric amplification at a low frequency pump has been developed in multilayer domain structure at the constant-intensity approximation. Analytical expression has been obtained for the optimum phase correlation between interacting waves. The numerous estimation of the expected conversion efficiency is presented for conditions of experiment. The author conducts the comparison between conversion efficiency at parametric amplification with a low frequency pump and the general case of frequency up-conversion. It is shown that in the first case, by changing the entrance parameters of the signal wave and the wave at sum frequency, it is possible to affect considerably the dynamics of the nonlinear process and to achieve considerable transfer of pump energy to the wave of sum frequency, which heavily distinguishes this process from a traditional case of frequency up-conversion.*

Keywords *- intracavity frequency conversion, constant-intensity approximation, the regular domain structure.*

I. INTRODUCTION

In recent years nonlinear optical effects, occurring in regular domain structures (RDS), have been investigated broadly. The advantage of such structures is a possibility of realizing phase agreement between interacting waves even, when the conditions of general phase matching are not followed. In similar structures on a boundary between domains, there takes place phase correction of undesirable shift of phases between interacting waves due to the opposite direction of polar axis of the neighboring layers [1-10].

Apparently, there are some ways for the effective conversion of laser frequency which include conversion in multilayer domain structures, cascade parametric amplification with low frequency pump and intracavity geometry of nonlinear medium. Namely in this succession we carried out investigations of nonlinear frequency conversion features in the constant-intensity approximation, that is, depleted pump intensity approximation [11], simultaneously taking into account changes of phases and losses of interacting waves. The following stage deals with the consideration of the case in which all three enumerated (i.e. parametric interaction in active-nonlinear crystals with a regular domain structure) versions have been combined.

Refs. [5, 12-13] report a successful use of semi-conductor structures as periodical domain structures. Similar semi-conductors are transparent in 1-12 mcm range, possess large nonlinearity (d_{14}=90 pm/V, for comparison: the highest nonlinearity in $LiNbO_3$ d_{33}=34 pm/V) and high threshold of optical shooting. For the realization of parametric conversion, in particular, organic polymers possessing high quadratic nonlinearity are interesting [14].

It is known that the cascade parametric process at low frequency pump is a two-step process. At first, there takes place the process initiated by a wave of the pump, i.e. parametric amplification in the medium. This three-wave process is accompanied by the transfer of a part of energy from pump field to two low frequency fields. Then, as a result of nonlinear interaction of pump radiations and one of generated low frequency waves (the signal wave) generation of sum frequency takes place. In this case, pump photons with higher degree of efficacy are transformed into photons with higher frequency than the case of general up-conversion of frequency. A number of works [4, 8] are dedicated to the theme of highly effective parametric conversion at low frequency pump in polydomain consisting of n periods of "grating" of nonlinear susceptibility modulation. In work [4] a similar succession of nonlinear interactions was considered for the case of simultaneously occurring two linked processes without the account of phase effects. In [15] we investigate similar quasi-phase-matched interaction with the account of reversed influence of excited waves on exciting ones.

And finally, as to quasi-phase-matched intracavity interactions, so far such a type of interaction in active-nonlinear elements has been experimentally obtained and investigated, for example, for the processes of self-doubling, self-division and summing up frequencies [9, 16-18]. Here, either a theoretical analysis of frequency conversion has been made, in constant-field approximation (that is, undepleted pump approximation) or numerical methods of analysis are used. At intracavity way of frequency conversion of fundamental radiation, generation and nonlinear frequency conversion take place within the same cavity. Hence, any phase changes of interacting waves have an influence on conversion efficiency. As a consequence, it is expedient to make an analysis of intracavity process of conversion in constant-intensity approximation, where it is possible to take into regard both phases changes and losses of interacting waves at the same time.

The purpose of the present work is the consideration of the peculiarities of nonlinear energy exchange between optical waves at constant-intensity approximation in case of quasi-phase-matched intracavity cascade parametric amplification at low frequency pump, occurring in the consecutive layers arranged one after another. An analysis of intracavity three-frequency wave interactions is made for the five-layer structure, in which stepped frequency conversion occurs. Firstly, in the first layer there takes place an exponential parametric amplification of the signal wave. Then the process of nonlinear conversion of frequency occurs in the following four layers, forming regular domain structure. With this, the values of complex amplitude of fundamental radiation and the wave of sum frequency at an outlet of each domain are entrance values of respective complex amplitudes for the following domain.

II. THEORY

Theoretical investigation was carried out following the optical scheme, suggested in [10]. The considered five-layer structure Nd:Mg:$LiNbO_3$ depicted in details in [15] is arranged inside a laser cavity. It is assumed that the side surfaces of the structure play the reflective surfaces of a cavity on the left (mirror 1) and on the right (mirror 2).

Under the influence of external electromagnetic radiation of the semi-conductor laser pump (functioning at wavelength of 0.81 mcm) falling from the left on the active-nonlinear crystal between working levels (on a rare-earth element Nd) inversion of population is generated. As a result, forced passages accompanied by radiation occur, i.e. laser generation takes place at frequency ω_p (corresponding wavelength equals to 1.084 mcm). Further, a part of energy of the pump wave which is not absorbed by the crystal interacts nonlinearly with the generated laser wave in the crystal, and as a consequence, there takes place the generation of radiation at sum frequency ω_3 (corresponding wavelength of 0.728 mcm is in red range). Laser cavity is supposed to be closed for the wave of pump ($R_1(\omega_p) = R_2(\omega_p) = 1$, and $R_{1,2}$ are the coefficients of reflection from left and right mirrors, respectively). The second right mirror reflects completely the signal wave and also the wave at sum frequency. After reflection from the second mirror a sum frequency wave alongside with two other waves propagate in RDS -structure (now in reverse direction already), again nonlinearly interacting in domain layers, and then leave the structure. Passing waves with frequencies ω_p, ω_3 and ω_1 through a layer of thickness d are not accompanied by energy exchange between extending waves. It is entailed by ignoring the conditions of phase-matched for these waves in this first layer. The exit from the active-nonlinear crystal for the sum frequency wave takes place through left side of the crystal (the first mirror), playing the role of an outlet mirror ($R_1(\omega_3) < 1$).

Theoretical investigation is carried out for two passages of a cavity. Let's consider the first parametric amplification for two running waves in a field of a strong pump wave in the first layer of thickness d. With this, parametric interaction is depicted by the following reduced equations [15]

$$\begin{aligned}
&\frac{dA_1^{\pm}}{dz} \pm \delta_1^{(1)} A_1^{\pm} = \mp i\gamma_1^{(1)} A_p^{\pm} A_2^{\pm *} \exp(\pm i\Delta_1 z),\\
&\frac{dA_2^{\pm}}{dz} \pm \delta_2^{(1)} A_2 = \mp i\gamma_2^{(1)} A_p^{\pm} A_1^{\pm *} \exp(\pm i\Delta_1 z), \qquad (1)\\
&\frac{dA_p^{\pm}}{dz} \pm \delta_p^{(1)} A_p^{\pm} = \mp i\gamma_p^{(1)} A_1^{\pm} A_2^{\pm} \exp(\mp i\Delta_1 z),
\end{aligned}$$

where $A_{1,2,p}^{\pm}$ are complex amplitudes of signal, idler waves and waves at frequencies $\omega_{1,2,p}$ ($\omega_p = \omega_1 + \omega_2$), respectively, in direction of axis z (sign plus) and that opposite to axis z (sign minus). Parameters $\gamma_n^{(1)}$ and $\delta_n^{(1)}$ are nonlinear coupling coefficients and linear absorption of n-th wave (n=1, 2, p), and $\Delta_1 = k_p - k_1 - k_2$ signifies the wave mismatch in the first layer.

We decide the task in a general case when at an entrance to the first layer all three waves are present, i.e.

$$A_n(z=0) = A_{no}\exp(i\varphi_{no}), \quad (n=1,2,p). \quad (2)$$

Here φ_{no} is an initial phase of all three waves at an entrance to the first layer, z=0 corresponds to the entry of the first layer.

For conversion efficiency of the signal wave defined as $\eta_1(d) = I_1 / I_{p0}$, at outlet of the first layer we obtain ($\delta_1^{(1)} = \delta_2^{(1)} + \delta_p^{(1)}$, $\varphi_{p0} = \varphi_{10} + \varphi_{20}$)

$$\eta_1(d) = I_{10}\exp(-2\delta_1^{(1)}d)$$
$$\times\left[\cosh^2 q_3 d + \left(\frac{\Delta_1}{2} + \frac{\gamma_1^{(1)} A_{20}^* A_{po}}{A_{10}}\right)^2 \frac{\sinh^2 q_3 d}{(q_3 d)^2}\right], \quad (3)$$

where

$$q_3^2 = \Gamma_2^2 - (\Gamma_p^{(1)})^2 - \frac{\Delta_1^2}{4},$$
$$\delta^{(1)} = \delta_1^{(1)} + \delta_2^{(1)} + \delta_p^{(1)}, \quad p = \delta^{(1)} - i\Delta_1,$$
$$q_1^2 = p^2/4 - q, \quad q = \Gamma_2^2 - (\Gamma_p^{(1)})^2 + \delta_1^{(1)}\left(\delta_2^{(1)} + \delta_p^{(1)} - i\Delta_1\right),$$
$$\Gamma_2^2 = \gamma_1\gamma_p I_{20}, \quad (\Gamma_p^{(1)})^2 = \gamma_1\gamma_2 I_{p0}, \quad q_2^2 = q - p^2/4,$$
$$P = \left(\delta_1^{(1)} - \delta_2^{(1)} - \delta_p^{(1)} + i\Delta_1\right)/2.$$

The following step is the consideration of sum frequency generation at ω_3 ($\omega_3 = \omega_1 + \omega_p$) in a regular domain structure, consisting of four layer with identical phase mismatches in each of domains (Δ) and optimum lengths of domains, where $l_{j,opt}$ is determined from the condition of quasi-phase matching $\lambda_j l_{j,opt} = \pi/2,\ j = 1 \div 4$.

Generation of sum frequency occurs in a field of the same pump wave as parametric amplification in the first layer. The process is depicted by the following system of reduced equations [15]:

$$\frac{dA_1^{\pm}}{dz} \pm \delta_1^{(2)} A_1^{\pm} = \mp i\gamma_1^{(2)} A_3^{\pm} A_p^{\pm *}\exp(\pm i\Delta z),$$
$$\frac{dA_p^{\pm}}{dz} \pm \delta_p^{(2)} A_p^{\pm} = \mp i\gamma_p^{(2)} A_3^{\pm} A_1^{\pm *}\exp(\pm i\Delta z), \quad (4)$$
$$\frac{dA_3^{\pm}}{dz} \pm \delta_3^{(2)} A_3^{\pm} = \mp i\gamma_3^{(2)} A_p^{\pm} A_1^{\pm}\exp(\mp i\Delta z),$$

Here $A_3^{\pm}$ is complex amplitude of wave at sum frequency ω_3. $\delta_{1,p,3}^{(2)}, \gamma_{1,p,3}^{(2)}$ are coefficients of absorption and nonlinear interaction of waves, while $\Delta = k_3 - k_p - k_1$ stands for phase mismatch between interacting waves in the second layer, i.e. the first domain.

The boundary conditions at the entrance to the first domain will be as follows:

$$A_1(z=0) = A_1(d)\exp\left[i\varphi_1(d)\right],$$
$$A_p(z=0) = A_p(d)\exp\left[i\varphi_p(d)\right], \quad (5)$$
$$A_3(z=0) = A_{30}\exp(i\varphi_{30}),$$

where $\varphi_{1,p}(d)$, φ_{30} are phase changes on all three waves on a boundary between the first and second layers, z=0 corresponds to the entrance to the first domain.

The process of cascade parametric conversion at low frequency pump with respective boundary conditions has been profoundly studied in the work [15] for RDS-crystal in the constant-intensity approximation of fundamental radiation. We'll make consideration on the case of a regular domain structure, comprising two periods of lattice of modulation of nonlinear quadratic susceptibility, i.e. for four domains. Not dwelling on the intermediate calculations, we'll cite the expression for complex amplitude of a sum frequency wave at the outlet of a structure:

$$A_3(\ell_4) = A_3(\ell_3)\exp\left(i\varphi_3(l_3) - \frac{2\delta_3^{(2)} + i\Delta}{2}\ell_4\right)$$
$$\times\left\{\cos\lambda\ell_4 + i\left[\gamma_3^{(2)}\frac{A_p(\ell_3)A_1(\ell_3)}{A_3(\ell_3)}\right.\right. \quad (6)$$
$$\left.\left.\times e^{i\left[\varphi_p(l_3) + \varphi_1(l_3) - \varphi_3(l_3)\right]} + \frac{\Delta}{2}\right]\frac{\sin\lambda\ell_4}{\lambda}\right\},$$

where

$$\frac{A_p(\ell_3)A_1(\ell_3)}{A_3(\ell_3)} = -\frac{\exp(i\Delta l_3)}{\gamma_3^{(2)}}$$
$$\times\left(\frac{\lambda^2}{\frac{\Delta}{2} - \gamma_3^{(2)}\frac{A_p(\ell_2)A_1(\ell_2)}{A_3(\ell_2)}e^{i\left[\varphi_p(l_2) + \varphi_1(l_2) - \varphi_3(l_2)\right]}} - \frac{\Delta}{2}\right),$$

$$\lambda_{1,2,3,4} = \lambda = \sqrt{(\Gamma_p^{(2)})^2 + \frac{\Delta^2}{4}}, \quad (\Gamma_p^{(2)})^2 = \gamma_1^{(2)}\gamma_3^{(2)} I_{po}$$

Two facts should be remembered here. First, the values of complex amplitudes of the sum frequency wave at the outlet of the first, second, third and fourth domains depend on outlet values of complex amplitudes of interacting waves for the previous domains (see, boundary conditions for each domain

[15]). Secondly, a value of complex amplitude of a signal wave at an entrance to the first domain depends on parametric amplification in the first layer of thickness d (see boundary conditions (5)).

At the outlet from the structure the waves reflecting from the mirror 2 (from right lateral surface of crystal), now propagate in reverse direction. With this, there again takes place nonlinear interaction between waves. For waves running in the negative direction of axis z, boundary conditions (after reflection) at the entrance to a structure look as follows:

$$\begin{aligned} A_1(z=0) &= A_1(l_4)\exp\left(i\varphi_{r,1}\right), \\ A_2(z=0) &= A_2(l_4)\exp\left(i\varphi_{r,2}\right), \\ A_3(z=0) &= A_3(l_4)\exp\left(i\varphi_{r,3}\right), \end{aligned} \qquad (7)$$

where $\varphi_{r,1}, \varphi_{r,2}, \varphi_{r,3}$ are changes of wave phases at reflection from the mirror 2 and $z=0$ corresponds again to the entrance to the structure, $A_1(l_4), A_2(l_4), A_3(l_4)$ are complex amplitudes at an outlet from the fourth layer of a domain structure (when moving in positive direction of z).

For clarity of consideration we'll designate the layers-domains passing now through in reverse direction by the fifth (former fourth), sixths (former third) etc. as if a structure consisted of no four, but eight domains. Yet, we take only into consideration that between the fourth and the fifth domains an interchange of signs of nonlinear susceptibility is broken as it is one and the same domain. These two domains, one can say, form one domain of length equal to doubled length of the fourth domain. At the same time it is necessary to take into regard the contribution of intracavity conversion, that is, on a boundary of the fourth and fifth domains there emerges phase shift owing to the reflection of laser mirror waves.

Following the standard procedure of the decision of the system (4), and with boundary conditions (7), it is possible to obtain ($I_{20} \ll I_{10}$, $\delta_3^{(2)} = \delta_1^{(2)} + \delta_2^{(2)}$)

$$\eta_3(l_5) = I_3(\ell_5)/I_{30} = \eta_3(l_4)\exp(-2\delta_3^{(2)} l_5)$$
$$\times\left\{\left(\cos\lambda_5 l_5 + b\sin\psi\frac{\sin\lambda_5 l_5}{\lambda_5}\right)^2 + \left(\frac{\Delta}{2} - b\cos\psi\right)^2\frac{\sin^2\lambda_5 l_5}{\lambda^2{}_5}\right\} \qquad (8)$$

where

$$\operatorname{sinc}x = \sin x/x, \quad I_j = A_j A_j^*,$$
$$\psi = \Delta l_4 + \varphi_{r,1} + \varphi_{r,2} - \varphi_{r,3},$$
$$b = \left(\frac{\Delta}{2} - \frac{\lambda_4^2}{\Delta - \dfrac{\lambda_3^2}{\Delta - \dfrac{\lambda_2^2}{\Delta}}}\right),$$

$$\eta_3(\ell_4) = I_3(\ell_4)/I_{p0} = \eta_3(\ell_3)\exp\left(-2\delta_3^{(2)}\ell_4\right)$$
$$\times\left[\cos^2\lambda_4\ell_4 + \left(\Delta - \frac{\lambda_3^2}{\Delta - \dfrac{\lambda_2^2}{\Delta}}\right)^2\frac{\sin^2\lambda_4\ell_4}{\lambda_4^2}\right],$$

$$\eta_3(\ell_3) = I_3(\ell_3)/I_{p0} = \eta_3(\ell_2)\exp\left(-2\delta_3^{(2)}\ell_3\right)$$
$$\times\left[\cos^2\lambda_3\ell_3 + \left(\Delta - \frac{\lambda_2^2}{\Delta}\right)^2\frac{\sin^2\lambda_3\ell_3}{\lambda_3^2}\right],$$

$$\eta_3(\ell_2) = I_3(\ell_2)/I_{p0} = \eta_3(\ell_1)\exp\left(-2\delta_3^{(2)}\ell_2\right)$$
$$\times\left[\cos^2\lambda_2\ell_2 + \left(\frac{\Delta}{\lambda_2}\right)^2\sin^2\lambda_2\ell_2\right], \quad ,$$

$$\eta_3(l_1) = I_3(\ell_1)/I_{p0} = \eta_1(d)\cdot\gamma_3^{(2)2}$$
$$\times\left(I_{p0}/I_{10}\right)\cdot l_1^2\cdot\operatorname{sinc}^2\lambda_1 l_1\exp(-2\delta_3^{(2)} l_1).$$

Let's define from (8) the optimum phase ratio for efficient intracavity parametric conversion

$$\psi + \operatorname{atan}\left(\frac{2\lambda_5/\Delta}{\tan(\lambda_5 l_5)}\right) = \pi m, \qquad m = 1, 2, \ldots, \quad (9)$$

From (8) and (9) it follows that the conversion efficacy from ψ is being changed by harmonic law, and optimum value of ψ depends on pump intensity according to the analysis made in the constant-intensity approximation.

In analogy with it, calculating conversion efficiency in the subsequent sixth domain (i.e. in the third domain, but at propagating waves in reverse direction) it is possible to obtain the following

$$\eta_3(l_6) = I_3(\ell_6)/I_{p0} =$$
$$= \eta_3(l_5)\exp(-2\delta_3^{(2)} l_6)$$

$$\times\left\{\cos^2\lambda_6 l_6+\left(\frac{\Delta}{\lambda_6}-\frac{\lambda_5/\lambda_6}{\dfrac{\Delta}{2\lambda_5}+\dfrac{b}{\lambda_5}}\right)^2\sin^2\lambda_6 l_6\right\} \quad . (10)$$

We may determine, by analogy, conversion efficacy after the seventh and eight domains. As the analysis has shown, further growth $\eta_3(l_j)$ becomes slower, the process of frequency conversion achieves saturation and slight growth of conversion efficiency is observed.

III. RESULTS AND DISCUSSION

The numerical calculation of analytical expressions, obtained in constant intensity approximation is presented in Figs 1-3 for the efficiency of intracavity parametric conversion of the pump wave radiation energy. Curves are plotted for the case of optimum lengths of domains, where values $l_{j,opt}$ are determined from condition $\lambda_j l_{j,opt}=\pi/2$ ($j=1\div 4$).

The dynamic process of parametrical conversion is shown in Fig.1. Conversion efficiency dependencies $\eta_3(l_j)$ on the given domain lengths $\Gamma_p^{(1)}l_j$ are shown for a value of the given phase mismatch $\Delta/2\Gamma_p^{(1)}$=1. Just here, the dependence of reduced efficiency of the signal wave $\eta_1(d)=I_1(d)/I_{po}$ after parametric gain in the first layer of thickness d (curves 7) is presented. Owing to keeping the quasi-phase-matched condition for the first four domains, conversion efficiency smoothly increases rep to an outlet of waves from the structure (curves 1-4). However, at reverse propagation of waves in the fourth domain (i.e. at propagating in the fifth domain) there is observed a decrease on dependence (dotted curve 5), what is entailed by not following the quasi-phase-matched condition on a boundary between the fourth and fifth domains. However, acquired phase shift may be compensated by a shift of interacting waves phases at their reflection from mirror 2. It is possible to carry out by the choice of the reflection coefficients of mirror 2. As a result further growth of conversion efficacy (compare solid and dotted curves 5) may be achieved. This increase of efficacy is being continued as far as propagation in 5, 6 etc. domains (compare solid curves 5 and 6).

In Fig. 2 there is offered conversion efficiency dependence to the wave of sum frequency in the fifth domain on phase ratio Ψ between interacting waves for two values of losses ($\delta_3^{(2)}=\delta_1^{(2)}+\delta_p^{(2)}$). As it was expected, efficiency $\eta_3(l_5)$ oscillates with change of phase ratio. The dependence maximum is observed at optimum value of phase correlation, which is possible to calculate from expression (9) analytically derived analytically at constant-intensity approximation. With an increase in reduced losses $\delta_3^{(2)}/\Gamma_p^{(1)}$ amplitude of oscillations falls down (compare curves 1 and 2).

It is known that frequency conversion with increasing frequency may be realized by both cascade way of parametric interaction in case of low frequency pump and at a general single conversion during mixed frequencies. If the first case initially supposes only the existence of the pump wave, then in the second case of conversion there is a need for existence of two waves: the pump wave and the signal wave. The investigations in depleted pump intensity approximation show that owing to the exponential growth of the signal wave amplitude and its subsequent interaction with the pump wave at cascade way of conversion, it is possible to achieve considerable increase of conversion efficiency.

In Fig.3 there has been made a comparison of frequency conversion after the first domain length l_1 at cascade parametric interaction ($\omega_p=\omega_1+\omega_2$, $\omega_p+\omega_1=\omega_3$) (curves 1-3) and usually conversion of frequency goes upward ($\omega_p+\omega_1=\omega_3$) (curves 4 and 5). The calculations have been carried out for a value of reduced phase mismatch $\Delta/2\Gamma_1^{(2)}$=3.2. From the comparison of the curves it is seen that at frequency parametric conversion in comparison with a general way of conversion, there is achieved about one order more efficiency of conversion (compare curves 1-3 and 4-5). At cascade conversion a two times increase of entrance intensity from $I_{10}=10^{-4}\cdot I_{po}$ up to value $2\cdot 10^{-4}\cdot I_{po}$ of signal waves leads to increasing in efficiency by 9.6 % (compare curves 2 and 3), i.e. to value $\eta_3(l_1)$ =0,1371. For comparison, in case of general way conversion with two times increase in the signal with intensity at entrance to the structure (it reaches from I_{10}= $0.1 \cdot I_{po}$ to value $0.2 \cdot I_{po}$) an increase in conversion efficiency takes place twice (compare curves 4 and 5), i.e. only to value $\eta_3(l_1)$ =0.0172.

At cascade way of conversion with five times increasing of entrance intensity of sum frequency wave I_{30} reaches from $0.0002 \cdot I_{po}$ to the level $0.001 \cdot I_{po}$ conversion efficacy, i.e. increases by 10% (compare curves 1-3). Thus, even by a slight change of initial parameters of the signal wave and the sum frequency wave at entrance to the structure, it is possible to influence essentially the course of the nonlinear optical process of parametric amplification at a low frequency pump, dynamics of energy exchange. This fact was noted in work [10] as well.

Let's estimate practically the efficiency of intracavity conversion in conditions of experiment for specific samples of domain Nd:Mg:LiNbO$_3$ the most intensive laser generation has wavelength of 1.084 mcm. In the process of nonlinear interaction takes place between the pump wave on the wavelength in 0.81 mcm and laser radiation generation of radiation of red colour (wavelength is 0.728 mcm). Let's estimate conversion efficiency to the wave of this length for experimentally realized value of laser generation intensity, are equal to $2 \cdot 10^7$ W/cm^2. In the constant-intensity approximation the numerical calculation of analytical expressions for conversion efficiency (at phase mismatch $\Delta / 2\Gamma_p^{(1)}$= 3.2) at reduced coherent lengths of domains equal to $\Gamma_p^{(1)} l_1 = \Gamma_p^{(1)} l_2 = \Gamma_p^{(1)} l_3 = \Gamma_p^{(1)} l_4$ =1.2, and gives expected values of efficiencies: $\eta_3(l_1)$ =0.0049; $\eta_3(l_2)$ =0.01467; $\eta_3(l_3)$ =0.0257; $\eta_3(l_4)$ =0.0451; $\eta_3(l_5)$ =0.0544; $\eta_3(l_6)$ =0.06152.

Hence, for the experimental realization of parametric conversion the optimum size of one layer of the investigated domain structure from active-nonlinear crystal Nd:Mg:LiNbO$_3$ should be taken equal to 170 mcm at structure period Λ =340 mcm. In work [10] by a calculation of a sample from nonlinear crystal, there was received a domain structure period equal to Λ =20.7 mcm. In case of using the semi-conductor structure as a regular domain structure with the coefficient of nonlinearity d_{14} = 90 pm/V [5] estimations give a value of coherent length of domains equal to 66 mcm under period in 132 mcm. Indeed, in semi-conductor structures owing to high nonlinearity it is possible to receive an efficient conversion on coherent lengths approximately three times less than in crystal Nd:Mg:LiNbO$_3$. As it is known, an account of losses in layers of crystal Nd:Mg:LiNbO$_3$ leads to decrease in conversion efficiency. It is elucidated in Fig.2 that at losses $\delta_3^{(2)} / \Gamma_p^{(1)}$ =0.1, fall of efficiency makes up 38.3 %.

IV. FIGURES

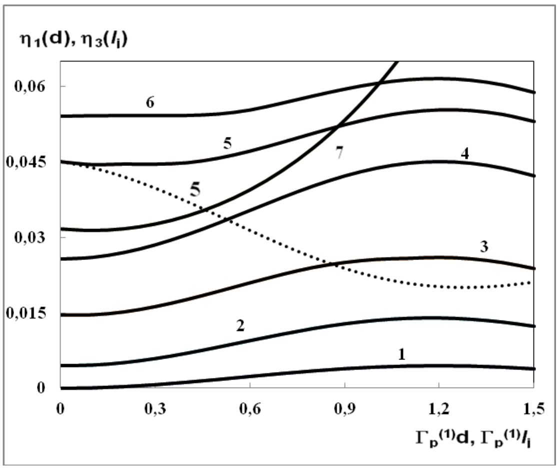

Figure 1: Dependences of intracavity reduced efficiency $\eta_1(d) = I_1(d) / I_{p0}$ and $\eta_3(l_j) = I_3(l_j) / I_{p0}$ ($j = 1 \div 6$) on the given lengths of layers $\Gamma_p^{(1)} d$ (curve 7) and $\Gamma_p^{(1)} l_j$ (curve 1-6), respectively, calculated in the constant-intensity approximation for $\lambda_j l_{j,opt} = \pi / 2$ ($j = 1 \div 4$), $\delta_3^{(2)} = \delta_1^{(2)} + \delta_2^{(2)}$, $\delta_3^{(2)} / \Gamma_p^{(1)} = 0.1$, $I_{p0} / I_{30} = 10^4$, $I_{p0} / I_{10} = 10^3$, $\gamma_3^{(2)} \gamma_p^{(2)} / \gamma_1^{(1)} \gamma_2^{(1)}$ =6, $\Delta_1 / 2\Gamma_p^{(1)}$ =0.5, $\Delta / 2\Gamma_p^{(1)}$ =1. Here $\psi = \psi_{opt}$ (solid curve 5), 0 (dashed curve 5).

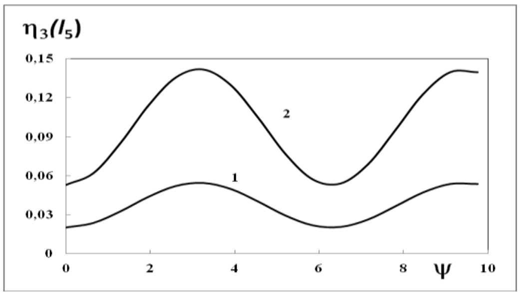

Figure 2: Dependences of intracavity conversion efficiency $\eta_3(l_5)$ in the fifth domain on phase ratio ψ for $\lambda_j l_{j,opt} = \pi / 2$ ($j = 1 \div 4$) at $\Delta / 2\Gamma_p^{(1)}$ =1, $\Delta_1 / 2\Gamma_p^{(1)}$ =0.5, $\delta_3^{(2)} = \delta_1^{(2)} + \delta_2^{(2)}$, $\delta_3^{(2)} / \Gamma_p^{(1)} = 0$ (curve 2) and 0.1 (curve 1).

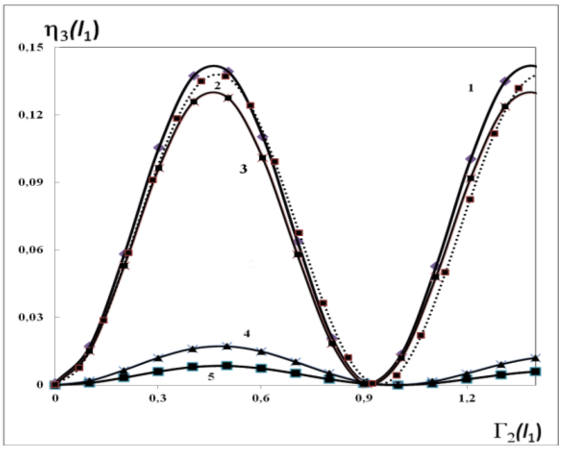

Figure 3: Dependences of reduced efficiency $\eta_3(l_1) = I_3 / I_{p0}$ on the given length of first layer $\Gamma_2 l_1$ calculated in the constant-intensity approximation for $\Delta / 2\Gamma_1^{(2)}$=3.2, $\delta_3^{(2)}$=0 at cascade parametric interaction ($\omega_p = \omega_1 + \omega_2$, $\omega_p + \omega_1 = \omega_3$) (curves 1-3) and traditional conversion of frequency upwards ($\omega_p + \omega_1 = \omega_3$) (curves 4 and 5). Here $I_{10} / I_{p0} = 10^{-4}$ (curves 1 and 3), $2 \cdot 10^{-4}$ (curve 2), 0.1 (curve 5), 0.2 (curve 4); I_{30} / I_{p0}=0 (curves 4 and 5), $2 \cdot 10^{-4}$ (curves 2 and 3), 10^{-3} (curve 1).

V. Conclusion

The work presents the results of theoretical investigations of the process of quasi-phase-matched intracavity parametrical interaction of optical waves in active–nonlinear crystals with a regular domain structure. Analytical expression for the optimum phase ratio between interacting waves has been obtained. It is shown that by the choice of optimum values of pump intensity and phase correlation it is possible to increase considerably the conversion efficiency in comparison with the case of lack of cavity. The numerical estimations gave the expected conversion efficiency in conditions of real experiment, much more exceeding frequency conversion efficiency increase by a traditional way. With the elaboration of the effective frequency parametric converters at a low frequency pump on the basis of active-nonlinear semi-conductor structures it is possible to work out highly efficient quasi-phase-matched converters of frequency.

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