Design and Analysis of MIMO Radar GLRT Detector

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Abstract

In this work, a detector is designed and analyzed for MIMO Radar Network with two transmitters and three receivers. The detection is carried out for each bi-static pair of the network using Linear Frequency Modulated (LFM) signal individually. The detector design is divided into Signal Model and Generalized Likelihood Ratio Test termed as GLRT. Signal Model design deals with the construction of MIMO Radar Network and GLRT deals with the detection of target considered for detection. Simulation is carried out by varying number of samples in the transmitted signal, number of receivers and direct to noise ratio. Simulation results shows that the detection sensitivity depends explicitly on number of samples in transmitted signal, number of receivers in the network and direct to noise ratio. Further the work is extended to detect and analyze swerling type targets.

Keywords – *MIMO Radar, LFM Signal, Signal Model, GLRT, Swerling Targets*

I. INTRODUCTION

MIMO is a system of multiple antennas where each antenna radiates an arbitrary waveform independent of other transmitting antennas.The receiver receives each transmitted signal. Fig 1 shows principle of a MIMO radar system. MIMO radars utilizes spatially diverse receivers and transmitters, which has the potential to provide better tolerance to fading and increases the detection sensitivity. Consider a radar network as shown in Fig 1. The network comprises two transmitters and three receivers with a stationary point target. Each transmitter and receiver contains a single isotropic antenna for transmission and reception respectively.

The approach to detection of point target in MIMO radar is decentralized which means each bi-static pair performs detection independently. The resulting detection of each pair is fused in Cartesian space to localize target [1]–[5]. The direct path (i.e. transmitter-to-receiver) and target path (i.e. transmitter-to-target-to-receiver) signals are isolated into reference and surveillance channel respectively. This is done by using directional antennas at receivers and transmitters respectively [6]-[7], which is one of the distinctive operation in this network. In case of multichannel system, digital beam forming is used [8]-[9].

This work considers detection of point and swerling targets in MIMO radar network in Cartesian space. A Generalized Likelihood Ratio Test termed as GLRT is used to distinguish real and false alarm targets. Finally the simulation is carried out by varying dependent parameters and analysis is done for the detection sensitivity of the detector by varying each parameters separately while the other parameters are kept constant.

In section II, the generated LFM waveform is described. In section III, model for the target is discussed. In section IV, the signal model is proposed for the MIMO radar network. The section V describes the GLRT used for detection. Section VI analyzes the simulation results.

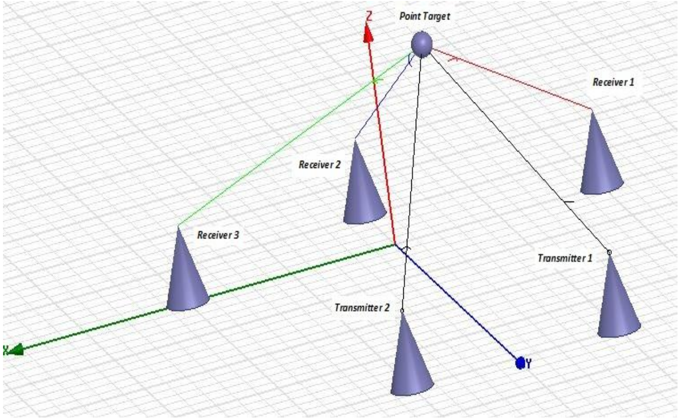

Fig 1 MIMO Radar Network with two transmitters and three receivers

II. WAVEFORM MODEL

Frequency or phase modulated waveforms can be used to achieve much wider operating bandwidths. Linear Frequency Modulation (LFM) is commonly used. In this case, the frequency is swept linearly across the pulse width, either upward (up-chirp) or downward (down-chirp). The sweep bandwidth is proportional to matched filter bandwidth, and is independent of the pulse width.A typical LFM waveform can be expressed as

$$S(t) = Rect\left(\frac{t}{\tau}\right) e^{j2\pi\left(f_0 t + \frac{\mu}{2}t^2\right)} \qquad (1)$$

Where f_0 is the radar center frequency, $Rect\left(\frac{t}{\tau}\right)$ denote a rectangular pulse of width τ, $\mu = B/\tau$ is the LFM coefficient, B is the bandwidth of the signal.

III. TARGET MODEL

A Non-Fluctuating point target of radar cross section 1 square meter is created using Phased.RadarTarget object of Phased Array Toolbox in MATLAB. This object gives the liberty to the designer to select different type of targets such as swerling targets. For detection and analysis of swerling targets, swerling properties had been chosen while modelling the target. The reflected signal from the target is given by the following equation

$$\mathrm{Y} = \sqrt{\mathrm{G}}.\mathrm{X} \qquad (2)$$

Where
X is the incident signal
G is the target gain factor, a dimensionless quantity given by

$$\mathrm{G} = \frac{4\pi\sigma}{\lambda^2} \qquad (3)$$

σ is the mean RCS of the target given by

$$\sigma = \frac{4\pi A_t^2}{\lambda^2} \qquad (4)$$

λ is the wavelength of incident signal.

IV. SIGNAL MODEL

Consider a MIMO Radar network with n_t transmitters and n_r receivers similar to that of Fig 1. Fig2 gives signal environment and geometry of a single bi-static pair. Bi-static pair is the combination of a transmitter and receiver signal environment.Let the position of the ith transmitter is denoted by t^i, and the position of the jth receiver is denoted by r^j. The position of the target is denoted by T. In the MIMO Radar environment, each reflected echo from the target is received by all the receivers in the environment. It is shown that the detection in MIMO Radar network is formulated in terms of surveillance channel and reference channel, where clutter can be mitigated and it will become negligible. Thus clutter-free environment suggest that the detector is upper bound to compare with clutter-suppressing detectors. Hence only reference channel and surveillance channel signals are considered here.The overall network is divided into 6 bi-static pairs. For all bi-static pairs the position of the target is same. Each transmitter contains only oneantenna. The operationofeach bi-static pair is carried individually and theresultingdetection is fused across Cartesian space to localize target.

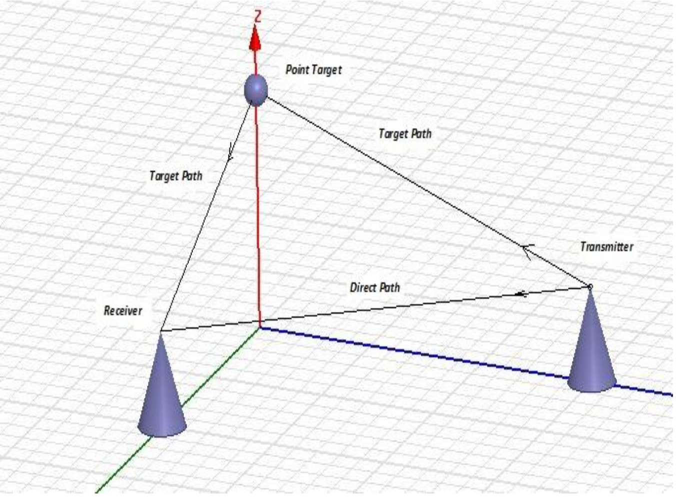

Fig 2 Bi-Static Radar Pair

The position of the target denoted by T has Cartesian values $[T_x T_y T_z]$. The position of the ith transmitter denoted by t^i has Cartesian values $[t_x^i t_y^i t_z^i]$ and similarly the position of the jth receiver denoted by r^j has Cartesian values $[r_x^j r_y^j r_z^j]$.Assume that the signal transmitted from the transmitters are non-overlapping with respect to frequency having a common bandwidth. The received signals at the receiver is the collection of reference channel signal and surveillance channel signal. Since the target is stationary, the Doppler Effect coefficients will not come into consideration and the scenario is assumed to be cluster free environment. Now the transmitted signal vector V^{ij} is given by the following matrix.

$$V^{ij} = [S(t)_1^{ij} S(t)_2^{ij} \dots S(t)_{N_r}^{ij}] \qquad (5)$$

Where S(t) is the transmitted signal. Thus the received signal at the receiver can be written as

$$Y^{ij} = \alpha^{ij} V^{ij} + n^{ij} \qquad (6)$$

Where n^{ij} is additive white gaussian noise, α^{ij} indicates α_S^{ij} in case of surveillance channel and α_R^{ij} in case of reference channel. Hence signal received by the receiver from reference and surveillance channel can be written as

$$Y_r^{ij} = \alpha_r^{ij} V^i + n^{ij} \qquad (7)$$

$$Y_s^{ij} = \alpha_s^{ij} V^i + n^{ij} \qquad (8)$$

Reference channel and surveillance channel signals are very important in further proceedings as the two will be required to determine whether the target is present or absent from the received signal at the receiver.

V. GLRT

The detection problem is defined in presence of target and absence of target under two hypotheses. The presence of target is called as real hypothesis and the absence of target is called null hypothesis. Thus the real and null hypothesis are as follows.

$$H_1 : Y_S^{ij} = \alpha_S^{ij} V_S^{ij} + n_S^{ij}$$

$$Y_R^{ij} = \alpha_R^{ij} V_R^{ij} + n_R^{ij} \qquad (9)$$

$$H_0 : Y_S^{ij} = n_S^{ij}$$

$$Y_r^{ij} = \alpha_r^{ij} V_r^{ij} + n_r^{ij} \qquad (10)$$

Where H_1 is the real hypothesis and H_0 is the null hypothesis. The surveillance channel co-efficientα_s^{ij} , the reference channel coefficient α_r^{ij} and coefficients of transmitting signal vector V^i are unknown parameters and are replaced by maximum likelihood estimates. The maximum likelihood estimate of unknown parameters is obtained by partial differentiating eqn. (9) and eqn. (10). It yields

$$V^i = \sqrt{L} \qquad (11)$$

$$\alpha_{rs}^{ij} = \frac{V^i * y_{rs}^{ij}}{(\|V^i\|)^2} \qquad (12)$$

The detection algorithm is derived using Gram Matrix. Gram Matrix is the possible combination of all the possibilities of bi-static signal environment in the radar network under consideration. Thus Gram Matrix is given by

$$g_1^i = \begin{bmatrix} g_{ss}^i & g_{sr}^i \\ g_{rs}^i & g_{rr}^i \end{bmatrix} \qquad (13)$$

The coefficients of eqn. (13) are the product of reference and surveillance channel given by

$$g_{ss}^i = Y_S^{ij} {Y_S^{ij}}^H \qquad (14)$$

$$g_{rs}^i = Y_r^{ij} {Y_S^{ij}}^H \qquad (15)$$

$$g_{sr}^i = Y_S^{ij} {Y_r^{ij}}^H \qquad (16)$$

$$g_{rr}^i = Y_r^{ij} {Y_r^{ij}}^H \qquad (17)$$

From eqn. (17) it is clear that g_{rr}^i does not contains echo signals reflected from the target. The MLE of eqn. (17) is calculated and subtracted with MLE of eqn. (13). The resulting valueiscompared with the threshold value. Thus the detection equation is written as

$$\xi_{pd} = \frac{1}{\sigma^2} \sum_{i=1}^{N_t} (\lambda_1(|g_1^i|) - \lambda_1(|g_{rr}^i|)) \gtrless_{H0}^{H1} T \qquad (18)$$

Where $\lambda_1(.)$ indicates MLE of g_1^i and g_{rr}^i.

VI. RESULTS AND ANALYSIS

The Simulation is carried out by varying one parameter, while the other parameters kept constant.

A. *Analysis by Varying Number of Samples*

In this simulation, number of samples is varied and all other parameters are kept constant. The analysis is carried out for 3X2MIMO Radar network with L varying from 10 to 30. For L = 10, the Probability of Detection (Pd) reaches 0.9 at -0.1724 dB SNR. Similarly for L = 20 it reaches at -4.0555 dB SNR and for L = 30 it reaches at -5.9694 dB SNR. Fig 3 shows the detection curves of L = 10, 20 and 30 samples. From Table 1 it is observed that as number of samples of the transmitted signal increases, the detection sensitivity of MIMO Radar detector also increases.

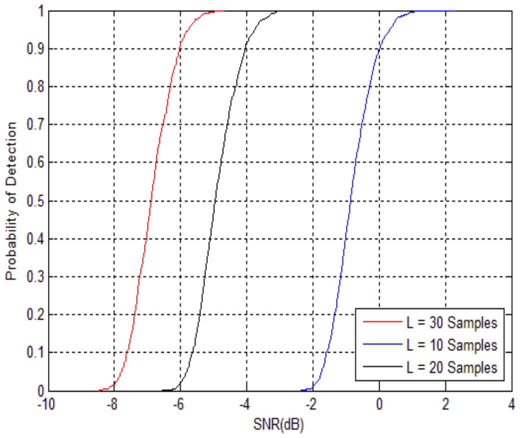

Fig 3 Detection Curves for L = 10, 20 and 30 Samples

Table 1 Detection Analysis of L=10, 20 and 30 Samples

Number of Samples (L)	Probability of Detection (Pd)	SNR in dB
10	0.9	-0.1724
20	0.9	-4.0555
30	0.9	-5. 9694

B. Analysis by Varying Number of Receivers

In this simulation, number of receivers is varied from 2 to 5 while keeping number of transmitters constant to 3. The other parameter such as L is kept to 30 and PFA to 1e-12. For n_r = 2, Pd reaches 0.9 at -2.4023 dB of SNR, for n_r = 3, Pd reaches 0.9 at -5.9694 dB of SNR. Similarly for n_r = 4 and 5, Pd reaches 0.9 at -8.4921 dB of SNR and -10.4414 dB of SNR respectively. From table 2 it is observed that as number of receivers in the MIMO Radar network increases, the sensitivity of MIMO Radar detector increases. Fig 4 shows detection curves of varied values of n_r.

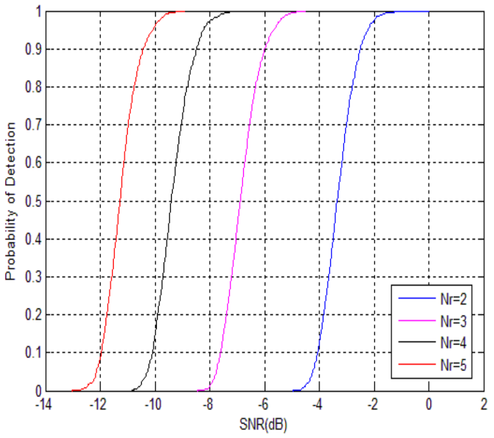

Fig 4 Detection Curves for n_r= 2, 3, 4 and 5

Table 2 Detection Analysis of n_r = 2, 3, 4 and 5

Number of Receivers	Probability of Detection (Pd)	SNR in dB
2	0.9	-2.4023
3	0.9	-5.9694
4	0.9	--8.4921
5	0.9	-10.4414

C. Analysis by Varying Direct to Noise Ratio

SNR of the direct signal or reference channel signal is termed as direct to noise ratio. The analysis is carried out by varying the values of DNR of reference channel from -30 dB to -15 dB. All other parameters are kept constant. The value of Pd reaches 0.9 at -5.9694 dB of SNR for -30 dB of DNR. For -25 dB of DNR, Pd reaches 0.9 at -8.4988 dB of SNR. Similarly for -20 dB and -30 dB of DNR, Pd reaches 0.9 at -10.4785 dB and -12.0901 dB of SNR respectively. From table 3 it is observed that as the value of DNR increases, the sensitivity of detector also increases. Figure 5 shows detection curves of varied DNR.

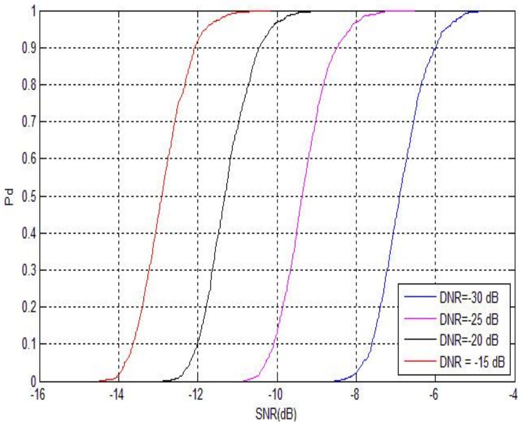

Fig 5 Detection Analysis of DNR

Table 3 Detection Analysis of DNR

DNR (dB)	Probability of Detection (Pd)	SNR (dB)
-30	0.9	-5.9694
-25	0.9	-8.4988
-20	0.9	-10.4784
-15	0.9	-12.0901

D. Analysis of Swerling Targets

This analysis considers the detection of swerling type target by keeping all other parameters to constant. Due to its variation in RCS, the probability of detection varies. The detector reaches 0.9 Pd at 6.0212 dB of SNR for swerling 1 target. For swerling 2 target, Pd reaches 0.9 at -2.1226 dB of SNR. Similarly for swerling 3, 4 and 5 targets, the detector reaches 0.9 Pd at 2.0412 dB, -2.5123 dB and -1.1623 dB of SNR respectively. Figure 6 shows detection curves of swerling targets. From table 4 it is observed that the detector works better for swerling 4 target when compared to other swerling targets.

Table 4 Detection Analysis of Swerling Targets

Target	Probability of Detection (Pd)	SNR in dB
Swerling 1	0.9	6.0212
Swerling 2	0.9	-2.1226
Swerling 3	0.9	2.0412
Swerling 4	0.9	-2.5123
Swerling 5 or 0	0.9	-1.1623

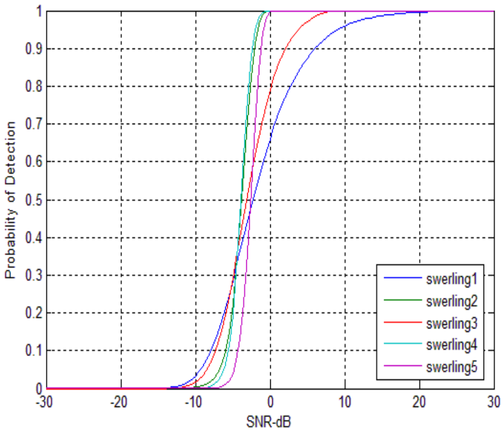

Fig 6 Detection Curves of Swerling Targets

VII. CONCLUSIONS

This work presents the design and analysis of MIMO Radar GLRT detector. It was found that the designed detector depends on number of samples, number of receivers and direct to noise ratio. A detailed analysis on the dependency of the designed detector on these parameters reveals that, as any one of the parameter increases, the detection sensitivity of the detector increases. The detector gives better sensitivity for swerling targets when compared to point target. Among swerling targets, the designed detector gives better sensitivity for swerling 4 type target.

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