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A Fast Convergent Interference Alignment Algorithm for Multiple Interfering Channels

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Abstract

Interference alignment is an efficient technique to improve the performance of MIMO systems. A number of interference alignment algorithms were proposed to obtain interference alignment solutions in the K-user multiple-input multiple-output (MIMO) interference channel (IC). Most of the proposed algorithms are based on either alternating minimization or steepest descent method. These methods requiresa large number of iterations to converge, which results in high computational time. In this paper we propose a faster convergent interference alignment algorithm based on first order gauss-newton method.Our numerical results show that, in addition to systematically converging to a zero interference leakage point (in feasible scenarios) regardless of the initialization point, theproposed method provides remarkable computation time savings when compared to the well-known alternating minimization (AM) or steepest-descent (SD) algorithms.

Keywords –*Alternating minimization, Gauss Newton, interference alignment, interference channel, steepest descent.*

I. INTRODUCTION

Multiple input- multiple output (MIMO) wireless systems are wireless networks in which the transmitters and receivers may have more than one antenna. They offer the possibility of spatial multiplexing of data streams in addition to time and frequency multiplexing. Spatial multiplexing leads to capacity increase linear to min(M,N), where M and N are the number of transmitter and receiver antennas, respectively[1]. The challenge in designing such MIMO systems, however, is in dealing with interference from concurrent signal transmissions. In such systems, wireless interference shows up among different sender antennas at each receiver. Interference alignment is among several techniques which has been recently used to overcome this problem.

When the interference is strong, the interfering signal can be decoded as well as the desired signal [2].This may be at the cost of degradation in the user's sending rates. On the other hand, in case the interference is very weak, it can betreated as noise. More recently interference alignment[2] has been suggested for the cases when the strength of the interference is comparable to the strength of the desired signal. Interference alignment techniques involve the use of suitable encoding and decoding matrices, at the transmitter and receiver respectively, such that at each receiver the interference caused by all undesired transmitters is projected on to a separate interference subspace. It has been shown that while the per-user sum rate for a K-user interference channel without interference alignment is $\frac{1}{K}\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$, where SNR is the signal-to-noise ratio, the sum-rate per user can be increased to $\frac{1}{2}\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$ with interference alignment [2].

Most of the work in the field of interference alignment is related to determining the maximum possible degrees of freedom (DoF) as well as studying its feasibility and achievability through finding optimal encoding and decoding matrices in a (M,N) system with K users (where K is as small as 2,3) such that the interference is minimum at undesired receivers.

The effectiveness of interference alignment in fully connected wireless networks with more than two users is considered [3,4]. [5] provide a distributed approach using reciprocity of wireless networks to make the local channel knowledge adequate. The achievability of high DoF in wireless networks when instantaneous channel state information is not available is studied [6]. The feasibility of interference alignment in MIMO systems has been considered in [7] by relating it to the problem of determining the solvability of a multivariate polynomial system.

Using interference alignment technique the throughput of MIMO local area networks can be almost doubled [8]. Interference alignment techniques have also been exploited for downlink cellular systems where the channel state information exchange is not required across base stations of different cells [9]. Alternating minimization [10]-[12] and steepest descent algorithms [13]-[15] are the prominent examples for interference alignment optimization. Many other algorithms with different cost functions and optimization techniques have been developed [16]-[18] and they are compared in [19].The algorithm which we propose is based on [20]

In this paper we propose a fast convergent interference alignment algorithm which converges non-monotonically.

II. SYSTEM MODEL

Consider the K-user wireless MIMO interference channel depicted in Fig.1

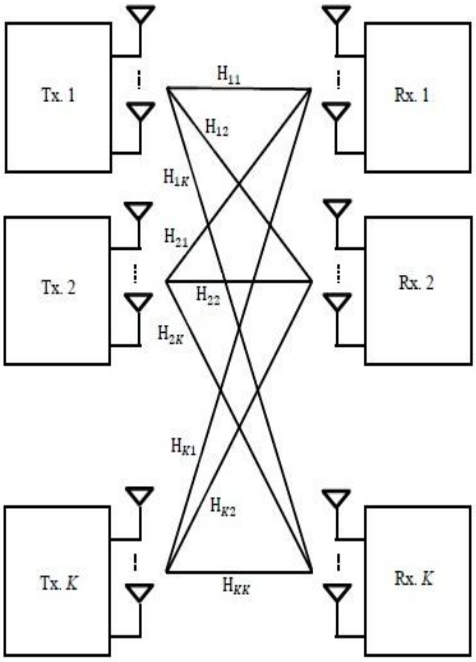

Fig. 1. Network Topology for Wireless MIMO System with K Users

We assume that the transmitter and receiver corresponding to the l^{th} user pair are equipped with M_l and N_l antennas respectively. Each transmitter wishes to send a message to its corresponding receiver, and creates interference at all other receivers. We denote H_{pq} to be the $\mathrm{N}_p \times \mathrm{M}_q$ channel matrix between the q^{th} transmitter and the p^{th} receiver. We consider a quasi-static channel, ,i.e., the channel realization remains fixed throughout the duration of transmission. For ease of exposition, we assume a flat fading environment, while noting that our results may be extended to the frequency selective case in straight forward fashion.

Each transmitter modulates its message onto a d_l dimensional vector s_l, and transmits the signal $\mathrm{V}_l\mathrm{s}_l$, where V_l is the $\mathrm{M}_l \times \mathrm{d}_l$ precoding matrix with columns comprising of d_l suitably chosen linearly independent beamforming vectors. Then the $\mathrm{N}_l \times 1$ received signal at the l^{th} receiver is [21],

$$y_l = \mathrm{H}_{ll}\mathrm{V}_l\mathrm{s}_l + \sum_{i\neq l} \mathrm{H}_{il}\mathrm{V}_i\mathrm{s}_i + \mathrm{n}_l, \quad l = 1, \dots, K, \ (1)$$

where n_l denotes the receiver thermal noise, modeled as i.i.d. zero mean, unit variance complex Gaussian random variables. An input power constraint $|\mathrm{V}_l\mathrm{s}_l|^2 \leq \mathrm{SNR}$ is imposed on each user, where SNR denotes the signal to noise ratio.

A. Interference Alignment and Degrees of freedom (DOF)

A system is said to achieve a DOF of d if the pre-log factor associated with the achievable rate of transmission is d, i.e.,

$$\lim_{SNR\to\infty} \frac{\mathrm{R}}{\log \mathrm{SNR}} = d$$

where R is the rate of transmission. Characterizing the achievable degrees of freedom for the K- interference channel when SNR goes to infinity has attracted considerableinterest lately. The technique of interference alignment has been shown to achieve the maximum DoF of K/2for single antenna nodes and channels varying across time or frequency [4], and has also been applied to X channel [5]. However, even though the best achievable DoF is unknown for a channel that has only a finite number of signalling dimensions available (such as the one in Fig. 1), interference alignment can be applied to improve the performance as well as to provide a lower bound
on the achievable DoF.

The basic idea behind interference alignment is the following [4]. Based on the channel realizations, transmitter *l* chooses an $\mathrm{M}_l \times \mathrm{d}_l$precoding matrix V_l, and receiver *l* chooses an $\mathrm{N}_l \times (\mathrm{N}_l - d_l)$interference receiving matrix U_l. Ideally, we would like to design them such that [4]

- At receiver *l*, the subspace spanned by the interfering signals $\mathrm{H}_{i,l}\mathrm{V}_l$, I $\neq l$ belongs to its interference receiving subspace spanned by the columns of $\mathrm{U}_{\boldsymbol{l}}$
- The subspace corresponding to the "useful signal" atreceiver *l*, viz., $\mathrm{H}_{ll}\mathrm{V}_l$is linearly independent of U_l.

The above will be referred to subsequently as the "interference alignment conditions." This can be written in equation as

$$\mathbf{U}_l^*(\mathrm{t})\mathbf{H}_{lk}(\mathrm{t})\mathbf{V}_k(\mathrm{t}) = 0, \forall \ k,l \in \beta, \ k \neq l (2)$$

$$\mathrm{rank}(\mathbf{U}_k^*(\mathrm{t})\mathbf{H}_{kk}(\mathrm{t})\mathbf{V}_k(\mathrm{t}) = d_k, \forall \mathrm{k} \in \beta (3)$$

Given this, user *l* can achieve a DoF of $\dim(\mathrm{H}_{ll}\mathrm{V}_l)$, which is equal to d_l when H_{ll} is full rank. One possible decoding strategy is that each receiver first projects its received signal onto $\mathrm{U}_l^{\wedge}$, the orthogonal complement of U_l, to zero-force the interference; the message is subsequently decoded.

III. FAST CONVERGENT INTERFERENCE ALIGNMENT ALGORITHM (FCIA)

In any interference alignment algorithm we need to find the precoding matrix such that the interference is projected in to separate subspace. Any

iterative algorithm begins with a random starting point. At each iteration the precoding matrix is updated. This can be done by the well-known gauss-newton method. The update is obtained by second order derivative of the cost function. Here the cost function is the interference leakage.
Let us define the vector containing all the optimization Variables as :

$$Y = [\mathrm{vec}(V_1)^T, \ldots, \mathrm{vec}(V_K)^T, \mathrm{vec}(\mathrm{U}_i^{\mathrm{H}})^T, \ldots, \mathrm{vec}(\mathrm{U}_K^{\mathrm{H}})^T]^T$$

Where vec(X) denotes the vector obtained by stacking the columns of matrix below one another. Let b(x) be the function evaluating the residuals of the equations in (2) and is given by b(x) $=[\mathrm{b}_{21}^{\mathrm{T}}, \ldots, \mathrm{b}_{(K-1)K}^{\mathrm{T}}]^T$ where $\mathrm{b}_{\mathrm{ij}} = \mathrm{vec}(\mathrm{U}_i^{\mathrm{H}} \mathrm{H}_{ij} \mathrm{V}_j)$. At the i-th iteration of Newton-like methods, the variables are updated according to the rule $y_{i+1} = y_i + \Delta y_i$

Where the update vector Δy_i is obtained through the second order approximation of the cost function f(x). The interference leakage cost function is given by:

$$f(x) = \mathrm{b}(x) b(x)^{\mathrm{H}} \longrightarrow \mathrm{R}$$

The second order approximation of any cost function f(x)at $y_0$is given by

$$f(x) = f(x_0) + \Delta x_0^T \mathrm{J}_x f(x_0) + \frac{1}{2} \Delta x_0^H \mathrm{H}_x f(x_0) \Delta x_0 \quad (4)$$

Where $\Delta x_0 = x - x_0$, $\mathrm{J}_x f(x_0)$ denotes the complex gradient of the scalar function $f(x)$ at x_0 , J_x is the jacobian of the function f(x) and $\mathrm{H}_x f(x_0)$ denotes the Hessian matrix of $f(x)$ at x_0. Eqn (4) can also be written as :

$$f(x) = f(x_0) + 2\mathrm{R}\{\mathrm{b}(\mathrm{J}_x)\mathrm{b}(x_0))\Delta x_0\}$$

$$+ \Delta x_0^{\mathrm{H}} \left(\mathrm{J}_x \mathrm{b}(x_0)\right)^{\mathrm{H}} (\mathrm{J}_x \mathrm{b}(x_0)) \Delta x_0 \qquad (5)$$

the Jacobian matrix of b can be written as:

$$\mathrm{J_X b} \triangleq \frac{\partial b(x)}{\partial \mathrm{X}^{\mathrm{T}}} = \left[\frac{\partial b(x)}{\partial x^{\mathrm{T}}} \frac{\partial b(x)}{\partial x^{\mathrm{H}}}\right] = [\mathrm{J}_x b \;\; \mathrm{J}_{x^*} b] \qquad (6)$$

Finally, the update is obtained when the derivative of (5) with respect to Δx_0 equals zero:

$$\frac{\partial f(x)}{\partial \Delta x_0} = 2\left(\mathrm{J}_x b(x_0)\right)^{\mathrm{H}} \mathrm{b}(x_0)$$

$$+2(\mathrm{J}_x b(x_0))^{\mathrm{H}} (\mathrm{J}_x b(x_0)) \Delta x_0 = 0 \qquad (7)$$

The updates for precoder and decoder can be given as:

$$\mathrm{V}^{n+1} = \mathrm{V}^n + \Delta \mathrm{V}_n \,(8)$$
$$\mathrm{U}^{n+1} = \mathrm{U}^n + \Delta \mathrm{U}_n \,(9)$$

The entire algorithm is summarized in algorithm 1.

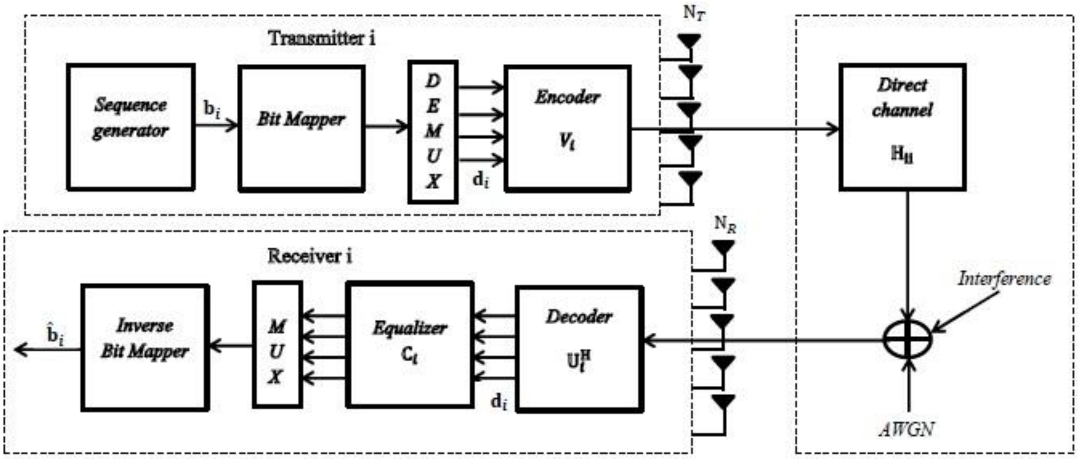

Fig. 2. Network Topology for Wireless MIMO System with K users

ALGORITHM 1 FCIA algorithm

Choose a tolerance level,δ, initial point $\{\mathrm{V}_{i,0}, \mathrm{U}_{i,0}\}_{i=0}^{\mathrm{K}}$, lying on the Stiefel manifold.

Set n = 0.

repeat

1. Construct x_n, $\mathrm{b}(x_n)$,$\mathrm{J}_x(x_n)$
2. Solve (7) to find $\Delta \mathrm{x}_n$
3. Construct $\{\Delta \mathrm{V}_n\}$, $\{\Delta \mathrm{U}_n\}$ from $\Delta \mathrm{x}_n$

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    4.  Update
        $V^{n+1} = V^{n} + \Delta V_{n}$
        $U^{n+1} = U^{n} + \Delta U_{n}$
    5.  n=n+1
Until $f(y_{i+1}) \leq \delta$
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IV. SIMULATIONS AND FURTHER DISCUSSIONS

The algorithm is implemented using matlab software.The simulation set up is shown in Fig 2. All the channels are generated according tothe independent and identically distributedRayleigh distribution i.e., each entry of the channel matrices is generated independently from a complex Gaussian distribution with zero mean and unit variance. In the results to bepresented, the algorithm is terminated when all the total interference power is less than 10^{-6}.

The interference leakage evolution of the proposed FCIA algorithm is shown in “Fig.3” The algorithm is compared with the well-known alternating minimization and steepest descent method.The evolution of interference leakage for the scenario $(5 \times 5,2)^{4}$ and $(12 \times 12,4)^{5}$is shown in “Fig.3”.and “Fig.4”. Our results are averaged over 100 iterations. From the “Fig.3” and “Fig.4” it can be seen that the proposed algorithm converges to the optimum value with lesser number of iterations, when compared to Alternating Minimization and the steepest descent method.

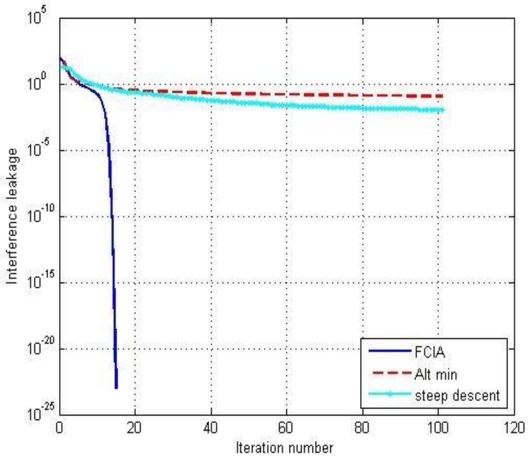

Fig. 3. Interference Leakage with the Evolution of Iteration Number for the Scenario $(5 \times 5,2)^{4}$

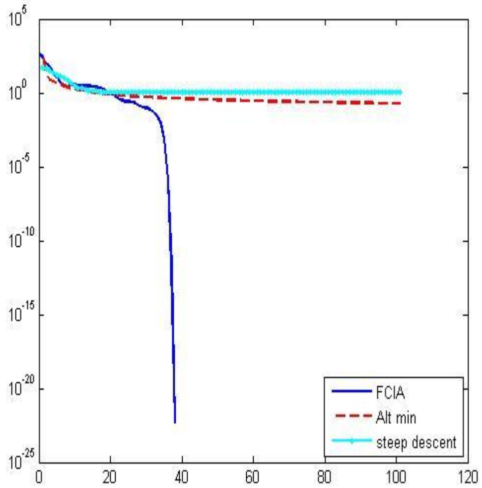

Fig. 4. Interference Leakage with the Evolution of Iteration number for the Scenario $(12 \times 12,4)^{5}$

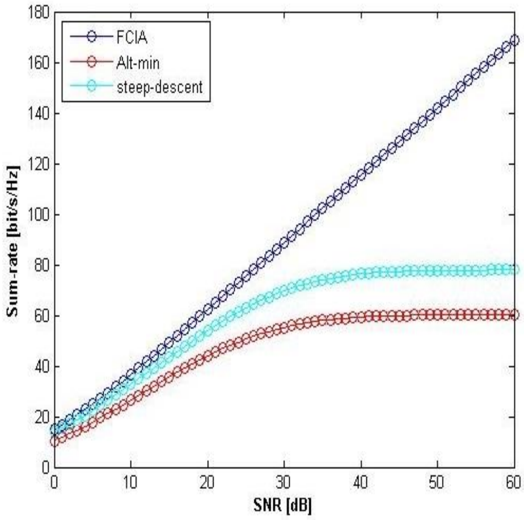

Fig. 5. Sum Rate Performance of the Scenario (5 × 5,2)4

The sum rate performance of proposed algorithm for two different scenario $(5 \times 5,2)^{4}$ and $(12 \times 12,4)^{5}$ shown in “Fig 5” and “Fig 6”.From the figure it is obvious that the proposed algorithm outperforms the alternating minimization and steepest descent method in terms of sum rate. Moreover the algorithm converges non-monotonically.

The median number of iterations to reach an interference leakage of 10^{-6} and the average time per iteration is given in Table I.

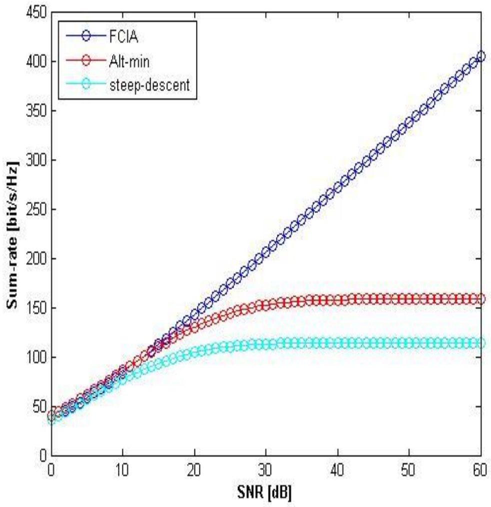

Fig. 6.Sum Rate Performance of the Scenario$(12 \times 12{,}4)5$

Table I : Number of Iterations to Reach an Interference Leakage Of 10^{-6}.

Scenario	Median number of iterations		
	FCIA	AltMin	SD
$(5 \times 5{,}2)^4$	22	6402	2109
$(12 \times 12{,}4)^5$	44	33910	∞

It can be seen from the “Fig.3” and “Fig.4” , as the number of users increases it takes more time to converge to a optimum value. As compared to alternating minimization and steepest descent method the fast convergent interference alignment algorithm converges more faster. For the considered scenarios, the SD algorithm is always slower than AltMin and,in fact, fails to converge (stagnating in local minima) in the $(12 \times 12{,}4)^5$scenario. On the other hand, both AltMin and GNhave always converged to a zero-leakage solution.

Similarly the sum rate performance also decreases with the number of users. The FCIA algorithm achieves better sum rate in both scenarios, when compared to steepest descent and the alternating minimization algorithm. The FCIA algorithm has reduced computational time.

V. CONCLUSION

The interference alignment solutions are mostly achieved by using iterative algorithms based on alternating minimization. However these algorithms require high computation time and have slow convergence. In this paper we propose a new algorithm for IA problem in K-user MIMO interference channel. The proposed algorithm has low computation time and achieves fast convergence. The convergence properties of the proposed approach have been validated through exhaustive numerical simulations. The simulation results show that the algorithm converges to optimum value with less number of iterations, as compared to the steepest descent and alternating minimization algorithm. The simulation results also reveal that the proposed algorithm outperforms the existing algorithms in terms of sum rate.

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