VSS-LMS Algorithms for Multichannel System Identification using Volterra Filtering

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Abstract

This paper propounds the adaptive polynomial filtering deploying the multifarious variable step-size least mean square (VSS-LMS) algorithms for the nonlinear Volterra multichannel system identification, and all are compared with a fixed step-size Volterra least mean square (VLMS) algorithm, under the various noise constraints comprising an individual signal-to-noise ratio (SNR). The VSS-LMS algorithm corroborates steady behaviour during convergence, and the step-size of the adaptive filter is altered in compliance with a gradient descent algorithm delineated to abate the squared estimation error in the course of each iteration, and it also revamps tracking rendition in the smoothly time-varying environments to the choice of the parameters and the boundaries of adaptive filter. In multitudinous practical implementations, the autocorrelation matrix of the input signal has the immense eigenvalue spread in the manifestation of nonlinear Volterra filter than in respect of the linear impulse response filter. In such circumstances, an adaptive step-size is a pertinent option to mitigate the unpropitious effects of eigenvalue spread on the convergence of VLMS adaptive algorithm. The simulation results are exhibited to reinforce the analysis, which compare the VSS-LMS algorithms with fixed step-size of the second-order Volterra filter, and also substantiate that the VSS-LMS algorithms are more robust than the fixed step-size algorithm when the input noise is logistic chaotic type.

Index Terms - *least mean square (LMS), minimum mean square error (MMSE), system identification, variable step-size least mean square (VSS-LMS), Volterra filter.*

I. INTRODUCTION

In the field of signal processing, the linear filters have played a key role because of its inherent simplicity. However, there are numerous practical situations, where nonlinear processing models are needed as the behavior of linear filters is unacceptable and also gives misleading results [1]. These systems don't follow superposition theorem and thus exhibit certain degree of nonlinearity. Thus to deal with nonlinearity problems, nonlinear filters, equalizers, or controllers are used. The most familiar used techniques are polynomial filters, order statistics filters, homomorphic filters, and morphological filters [3, 4] and out of these most widely used polynomial filters are the Volterra filters [1, 5]. Adaptive Volterra filters have turn out to be a compelling option for various nonlinear adaptive applications, such as acoustic echo cancelling [2], active control of nonlinear noise processes [6,10-14], and diminution of distortions on loudspeaker systems, among others [7]. Recently, the literature [6, 11-14] suggests several techniques for the development of active noise control (ANC) of nonlinear processes for a single channel case. The Volterra system model shows similar behavior to the Taylor series, but it has an ability to capture 'memory' effects [8]. In [1], the LMS second order adaptive Volterra filter has been introduced using truncated Volterra series expansions. Li Tan *et al.* [6] and Das *et al.* [9] proposed a Volterra filtered-x LMS and filtered-s LMS algorithms hinged on a multichannel structures, which were illustrated for feedforward ANC of nonlinear processes, but for a fixed step-size. Although, these methods execute well under certain conditions, in which noise can deteriorate their performance. However, this paper focuses on the variable step-size algorithms for multichannel system identification using adaptive nonlinear Volterra filtering system [7, 8, 16, 25].

The LMS algorithm has been profusely used in many applications because of its simplicity, intelligibility and robustness [15]. Step-size is the key parameter in LMS algorithm. As, if the step-size is large, the convergence rate will be expeditious, but the steady state MSE will increase and vice-versa. Thus, the step-size provides an accord between the steady state MSE and convergence rate of the LMS algorithm. An instinctive way to enhance the performance of the LMS algorithm is to make the step-size variable in lieu of fixed, i.e., select large step-size values during the inceptive period of the convergence and use small step-size when the system is in close proximity to its steady state. Thus, results in VSS-LMS algorithms [16].

Many VSS-LMS algorithms have been propounded during recent years [17-22]. In [17] and [18], Karni *et al.* and Kwong *et al.* respectively have proposed a new convergence factor, which enables the adaptive filter to exhibit very minute misadjustment and an exorbitant convergence rate for the non-stationary inputs. Mathews and Xie [19] have presented an adaptive step-size gradient filter, whose step-size is changed with respect to a gradient descent algorithm to minimize the squared estimation error in each iteration, which performed better than [17] and

[18]. Aboulnasr andMayyas have proposed a VSS-LMS adaptive algorithm in [20], in which the step-size of this algorithm is acclimatized according to the square of estimated time averaged of the auto-correlation of present and past estimation error, i.e., $\overline{e}(n)$and $\overline{e}(n-1)$respectively. In [21], Pazaitis *et al.* have proposed a new variable step-size LMS algorithm, which differs from previous techniques in terms of the time varying step-size series, which utilizes the kurtosis of the estimation error signal to adjust its convergence factor, but it exhibits a small increase in complexity. WeePeng and Farhang-Boroujeny have proposed a new class of VSS-LMS algorithms, which is a simplified version of the algorithms discussed above, and thus reduces complexity without observable loss in performance [22]. All the aforestated VSS-LMS algorithms are achieved using the linear filtering perspective.

In this paper, we propose adaptive nonlinear Volterra based VSS-LMS algorithms, which are better than the fixed step-size VLMS algorithm under certain noise conditions with different SNRs on a multichannel structure by using the convergence factor as discussed in [17-22]. This paper is organized as follows. In Section II, we first elucidate a single and multichannel system identification model for Volterra filter and give particulars about the mathematical formulation. In the next part, a concise overview of prevailing VSS-LMS algorithm is provided. In Section III, different methods of VSS-LMS algorithms are explained for a multichannel system. In Section IV, computer simulations are analysed validating mathematical analysis. Finally, conclusions and future scope are presented in Section V.

II. NONLINEAR ADAPTIVE VOLTERRAFILTERING SYSTEM MODEL

A. Mathematical Formulation for Volterra Filters

The Volterra system is the most preferred paradigm among polynomial system models because of its nonlinear relationship between input-output signals. However, its output is linear in the context of kernels. There are numerous time-varying nonlinear wireless communication channels, which need to be estimated by the nonlinear polynomial adaptive filtering [22].

An input-output relationship of a causal Volterra filter [1, 6] is given by the following equation:

$$y(n) = \sum_{\mathrm{u}=1}^{\mathrm{U}} \tilde{y}_{\mathrm{u}}(n) \tag{1}$$

where, $\tilde{y}_{\mathrm{u}}(n)$ is further expressed for an U-th order discrete filter and memory length M as:

$$\tilde{y}_{\mathrm{u}}(n) = h_0 + \sum_{m_1=0}^{M-1} \sum_{m_2=m_1}^{M-1} \cdots \sum_{m_k=m_u-1}^{M-1} h_u(n; m_1, \ldots\ldots, m_u). \prod_{t=1}^{u} x_{in}(n - m_t) \tag{2}$$

Here, $x_{in}(n)$ and $y(n)$ represent the input and output of Volterra filter, whereas h_u are the u-th order Volterra kernels, and n indicates the time index. Fig.1. shows the schematic diagram of an adaptive Volterra filter. For simplicity, let us contemplate a second-order (U=2) Volterra series expansion and thus input-output relationship is given as:

$$\tilde{y}_{\mathrm{u}}(n) = h_0 + \sum_{m_1=0}^{M-1} h_1(n; m_1)\, x_{in}(n - m_1) + \sum_{m_1=0}^{M-1} \sum_{m_2=m_1}^{M-1} h_2(n; m_1, m_2)\, x_{in}(n - m_1) x_{in}(n - m_2) \tag{3}$$

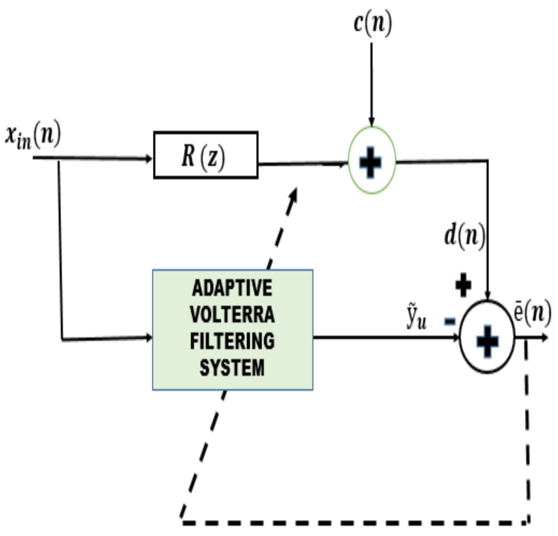


Fig. 1. A Block Diagram of an Adaptive Volterra Filter.

In this case, the adaptive filter would try to estimate the desired signal $d(n)$ using above second-order truncated Volterra series expansion. Here, $h_1(n; m_1)$ and $h_2(n; m_1, m_2)$ are the first-order and second-order adaptive Volterra kernels, respectively that are iteratively updated. These are updated at each time, so as to decrease some convex function of the error signal designated as:

$$\overline{e}(n) = d(n) - \tilde{y}_u(n). \tag{4}$$

Further, desired signal $d(n)$ is denoted as:

$$d(n) = X_{in}^T(n)\, r(n) + c(n) \tag{5}$$

where, X_{in} is the input signal vector, $r(n)$ signifies the impulse response of primary path $R(z)$ as shown in Fig. 1 and $c(n)$ indicates additive white Gaussian noise (AWGN) with $N(0, V_{min})$, in which V_{min} denotes variance.

Therefore, the estimated output signal is indicated by:

$$y'(n) = H^T(n)X_{in}(n) \qquad (6)$$

where, the adaptively approximated Volterra filter coefficient vector, H, may be represented by:

$$H(n) = [h_1(n;0), h_1(n;1), \dots h_1(M-1;n), h_2(n;0,0), h_2(n;0,1), \dots \dots, h_2(n;0,M-1),\ h_2(n;1,1), \dots \dots, h_2(n;M-1,M-1)]^T \qquad (7)$$

Here, $(.)^T$ denotes matrix transpose operator.

For second-order Volterra filter, the input signal, X_{in}, is given by:

$$X_{in}(n) = [x_{in}(n), x_{in}(n-1), \dots \dots, x_{in}(n-M+1), x_{in}^2(n), \dots, x_{in}(n)x_{in}(n-M+1), \dots \dots, x_{in}^2(n-M+1)]^T \qquad (8)$$

In the nonlinear system, the final purpose is to determine the time-varying Volterra kernels $h_u(n; m_1, \dots \dots, m_u)$ in (2) and thus, filter coefficients are updated at each time using a steepest descent algorithm for minimizing $\bar{e}^2(n)$ at every point. These updated filter coefficients can be expressed as [1, 6, 23]:

$$H(n+1) = H(n) + \mu' X_{in}(n)\bar{e}(n) \qquad (9)$$

More specifically, equations for updated filter coefficients for second-order Volterra filter can be written as:

$$h_1(n+1; m_1) = h_1(n; m_1) - \frac{\mu_1'}{2}\frac{\partial \bar{e}^2(n)}{\partial h_1(n; m_1)} \qquad (10)$$

$$= h_1(n; m_1) + \mu_1' \bar{e}(n)X_{in}(n-m_1) \qquad (11)$$

and

$$h_2(n+1; m_1, m_2) = h_2(n; m_1, m_2) - \frac{\mu_2'}{2}\frac{\partial \bar{e}^2(n)}{\partial h_2(n; m_1, m_2)} \qquad (12)$$

$$= h_2(n; m_1, m_2) + \mu_2' \bar{e}(n)X_{in}(n-m_1). X_{in}(n-m_2) \qquad (13)$$

where, μ_1' and μ_2' are the step-sizes that control the steady state and speed of convergence properties of the filters, and these convergence constant are chosen such that $0 < \mu_1', \mu_2' < 2/\lambda_m$, where, λ_m is the maximum eigenvalue of the matrix $\vec{R}_{x_{in}x_{in}} = E[\vec{x}_{in}(n)\vec{x}_{in}^T(n)]$.

Hence, the estimation error at the receiver output from Eq. (4) and (6) is computed by:

$$\bar{e}(n) = d(n) - H^T(n)X_{in}(n) \qquad (14)$$

This error signal starts from an initial guess that depends on the prior information available to the system, and then this is fed back to the self-designing filter. Thus, it converges finally to the optimal solution.

As described in [6, 24], we write the following expression from Eq. (2) by placing variables $m_t = m + s_{t-1}$, for $t = 1, \dots, u$ with $t_0 = 0$, that is

$$\tilde{y}_u = \sum_{m=0}^{M-1}\sum_{s_1=0}^{M-1-m}\sum_{s_2=t_1}^{M-1-m} \dots \dots \sum_{s_{u-1}=s_{u-2}}^{M-1-m} h_u(m, m+s_1, \dots, m+s_{u-1}). x_{in}(n-m)\prod_{t=1}^{u-1} x_{in}(n-m-s_t). \qquad (15)$$

Interchanging the order of summation in (15), as shown in [6, 24], we get:

$$\tilde{y}_u = \sum_{s_1=0}^{M-1}\sum_{s_2=s_1}^{M-1} \dots \dots \sum_{s_{u-1}=s_{u-2}}^{M-1} \left\{\sum_{m=0}^{M-1-s_{u-1}} h_u(m, m+s_1, \dots, m+s_{u-1}). \left(x_{in}(n-m)\prod_{t=1}^{u-1} x_{in}(n-m-s_t)\right)\right\} \qquad (16)$$

Simplifying Eq. (16),

$$\tilde{y}_u = \sum_{s_1=0}^{M-1}\sum_{s_2=s_1}^{M-1} \dots \sum_{s_{u-1}=s_{u-2}}^{M-1} h_u(n, n+s_1, \dots, n+s_{u-1}) * x_{in\,s_1,\dots\dots,s_{u-1}}(n) = H_u^T(n)X_{in\,u}(n) \qquad (17)$$

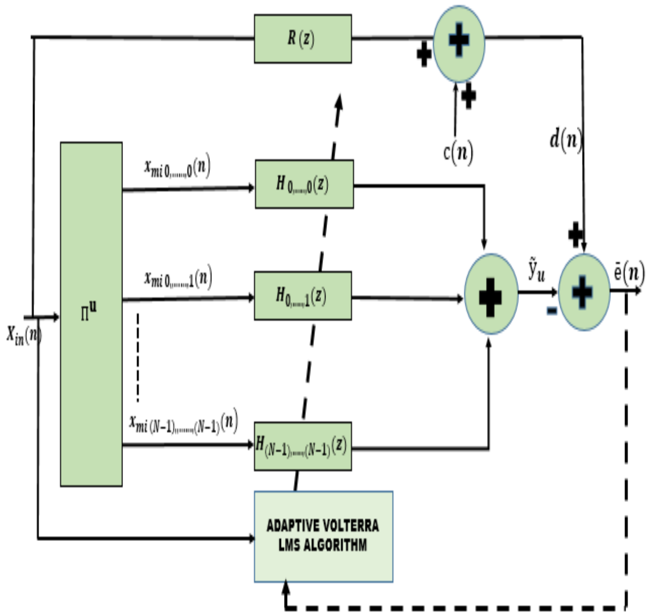
Fig. 2. Block diagram of a multichannel implementation of Volterra filter.

where, * denotes linear convolution, and further $x_{in_{s_1,\ldots,s_{u-1}}}(n)$is the distinct channel input with the u^{th}-order given by

$$x_{in_{s_1,\ldots,s_{u-1}}}(n) = x_{in}(n) \prod_{t=1}^{u-1} x_{in}(n - s_t) \tag{18}$$

where, $x_{in_{i_0}}(n) = x_{in}(n)$.

Taking Z transform on Eq. (17), we write:



$$\bar{Y}_u(z) = \sum_{s_1=0}^{M-1} \sum_{s_2=s_1}^{M-1} \ldots$$

$$\ldots \sum_{s_{u-1}=s_{u-2}}^{M-1} H_{s_1,\ldots,s_{u-1}}(z) X_{in_{s_1,\ldots,s_{u-1}}}(z) \tag{19}$$

where, $H_{s_1,\ldots,s_{u-1}}(z)$ represents FIR channel transfer function having memory size of $M - 1 - s_{u-1}$, which is further given by

$$H_{s_1,\ldots,s_{u-1}}(z) = \sum_{m=0}^{M-1-s_{u-1}} h_u(m, m + s_1, \ldots \ldots$$

$$\ldots, m + s_{u-1})\, z^{-m} \tag{20}$$

and $X_{in_{s_1,\ldots,s_{u-1}}}(z)$ implies the Z transform of the analogous channel input $x_{in_{s_1,\ldots,s_{u-1}}}(n)$, and $\bar{Y}_u(z)$ is the Z transform of $\tilde{y}_u(n)$. Eq. (19) can be represented through block diagram as shown in Fig. 2, and it can be realized by $D(u, M)$ parallel FIR filters [24], which is calculated as:

$$D(u, M) = \sum_{s=1}^{M} D(u - 1, s)$$

with $D(1, M) = 1$, and the closed form is given by:

$$D(u, M) = \frac{(u + M - 2)!}{(u - 1)!\,(M - 1)!} = \binom{u + M - 2}{u - 1} \tag{21}$$

B. An Overview of Variable Step-Size LMS Algorithm

For ease of explanation, the LMS algorithm is framed firstly within the milieu of a system identification model, where real signals are taken into consideration.
The mathematical expression is as follows [16, 23]:

$$d(n) = X_{in}^T(n)\, r(n) + c(n) \tag{22}$$

where, $d(n)$ is the zero mean desired signal and is a filtered genre of the input signal vector $X_{in}(n)$ corrupted by the noise $c(n)$, and $r(n)$ is the impulse response of primary path $R(z)$. Also, n signifies time index, and $(.)^T$ is the transpose operator.

The error signal is the difference between the desired signal and the output signal of the adaptive filter and is given by:

$$\bar{e}(n) = d(n) - X_{in}^T(n)\bar{w}(n) \tag{23}$$

where, $\bar{w}(n)$represents weights of the adaptive filter. The LMS algorithm for updating the weights is given:

$$\bar{w}(n + 1) = \bar{w}(n) + \mu' \bar{e}(n) X_{in}(n) \tag{24}$$

where, μ' is the step-size.

In the VSS-LMS algorithms, Eq. (24) can be written as:

$$\bar{w}(n + 1) = \bar{w}(n) + \mu'(n) \bar{e}(n) X_{in}(n) \tag{25}$$

i.e., μ' is replaced by a variable parameter$\mu'(n)$.

The adaptive step-size algorithm is delineated to eliminate the “postulate work” involved in stipulation of the step-size parameter, and at the same time gratify the following requirements : a) Fast convergence speed; b) the steady state misadjustment data should be minute, when operating in a stationary scenario; c) under non-stationary environment, the algorithm should be proficient to sense the pace, at which the peerless coefficients are changing, and step-size should be selected in such a way that it can result in close estimation in mean-squared-error sense.

Several VSS-LMS algorithms have been propounded to refine the performance of the LMS algorithm by using massive step-sizes at the primary stage of the process and minute step-sizes at the later stage when system approaches convergence.

Distinctive methods for adaptive step- sizes can be found in [17-22].

III. VSS-LMS ALGORITHMS FOR MULTICHANNEL VOLTERRA FILTER

As discussed in Section II, a multichannel system for the Volterra filter is given by Eq. (17), i.e.,

$$\tilde{y}_u = \sum_{s_1=0}^{M-1} \sum_{s_2=s_1}^{M-1} \dots \sum_{s_{u-1}=s_{u-2}}^{M-1} h_u(n, n+s_1, \dots$$

$$\dots, n+s_{u-1}) * x_{in\, s_1,\dots,s_{u-1}}(n)$$

$$= H_u^T(n) X_{in_u}(n) \quad (26)$$

Let p be the number of inputs and k be the number of channel outputs of an adaptive Volterra filter, and thus the updated filter coefficients from Eq. (9) can be expressed as:

$$H_{pk}(n+1) = H_{pk}(n) + \mu'_{pk}(n) X_{in_p}(n) \bar{e}_k(n) \quad (27)$$

In context to second order Volterra filter, updated filter coefficients from Eq. (10-13) can be written as:

$$h_{pk1}(n+1; m_1) = h_{pk1}(n; m_1) - \frac{\mu'_{pk1}(n)}{2} \frac{\partial \bar{e}_k^{\,2}(n)}{\partial h_{k1}(n; m_1)} \quad (28)$$

$$= h_{pk1}(n; m_1) + \mu'_{pk1}(n) \overline{e_k}(n) X_{in_p}(n - m_1) \quad (29)$$

and

$$h_{pk2}(n+1; m_1, m_2) = h_{pk2}(n; m_1, m_2) - \frac{\mu'_{pk2}}{2} \frac{\partial \bar{e}_k^{\,2}(n)}{\partial h_{pk2}(n; m_1, m_2)} \quad (30)$$

$$= h_{pk2}(n; m_1, m_2) + \mu'_{pk2}(n) \bar{e}_k(n) X_{in_p}(n - m_1) X_{in_p}(n - m_2) \quad (31)$$

In the above equations, variable step-size $\mu'_{pk}(n)$ is used, which refines the performance of VLMS algorithms and can obtain a fast convergence speed and a small steady state mean square error. In [17-22], different VSS-LMS algorithms are used and all these methods are executed on the basic equations of the LMS algorithms, where the input and the noise signals are deduced to be statistically stationary. In this section, we will discuss all the mathematical formulation of the VSS-LMS algorithms for a multichannel system identification process.

A. *Methods for Variable Step-Size VLMS Algorithms for Multichannel System*

1) *Karni's Method (KVSS-VLMS)*

In [17], Karni *et. al.* have proposed a new convergence factor, which is time-varying, and are applied on adaptive LMS algorithm. In this paper, the variable step-size is applied on a multichannel system identification of the Volterra filter and is given by:

$$\mu'_{pk}(n) = \mu'_{max\,pk} \cdot \left(1 - \bar{e}_k^{\,-\alpha \left\| \bar{e}_k(n) X_{in_p}(n) \right\|^2}\right) \quad (32)$$

where, $\mu'_{max\,pk} = \frac{1}{(L+1)\sigma^2_{X_{in_p}}}$, $\sigma^2_{X_{in_p}}$ is the variance of the input signal vector X_{in_p} for p inputs, L is the length of the filter vector, α is the damping parameter and should be greater than zero, $\overline{e_k}(n)$ is the error signal for k outputs. $\|.\|^2$ indicates the operation of squared Euclidean norm of the abrupt gradient vector $\overline{e_k}(n) X_{in_p}(n)$, which controls the step-size.

When $\|\overline{e_k}(n) X_{in_p}(n)\|$ is large, $\mu'_{pk}(n) = \mu'_{max\,pk}$, i.e., the process is in its fast convergence state and vice-versa. For non-stationary input, the abrupt change of the input induces $\|\overline{e_k}(n) X_{in_p}(n)\|$ to become large and thus, brings the process back to the rapid convergence state automatically.

2) *Kwong's Method (KwVSS-VLMS)*

In [18], Kwong and Johnston have proposed a VSS-LMS algorithm for the trailing of time-varying order-I Markovian channels. Now, for a multichannel Volterra system variable step- size algorithm is given by:

$$\mu'_{pk}(n+1) = \beta \mu_{pk}(n) + \gamma \bar{e}_k^2(n) \quad (33)$$

with $0 < \beta < 1, \quad \gamma > 0$

and

$$\mu_{pk}(n+1) = \begin{cases} \mu_{max\,pk} & if\ \ \mu'_{pk}(n+1) > \mu_{max\,pk} \\ \mu_{min\,pk} & if\ \ \mu'_{pk}(n+1) < \mu_{min\,pk} \\ \mu'_{pk}(n+1) & otherwise \end{cases} \quad (34)$$

where, $0 < \mu_{min\,pk} < \mu_{max\,pk}$, and for the guaranteed bounded mean square error:

$$\mu_{max\,pk} \le \frac{2}{3\, tr\left(E\left(X_{in_p} X_{in_p}^{\,T}\right)\right)} \quad (35)$$

The input X_{in_p} is presumed to be a zero mean independent sequence. Initially $\mu_0 = \mu_{max}$, although the algorithm is not fragile to the choice. The step-size is controlled by the parameters β, γ and the square of the prediction error, $\overline{e_k^2}(n)$. If the prediction error is large, it increases the step-size, and hence provides faster tracking. Similarly, smaller prediction error will decrease the step-size and reduce the misadjustment.

3) *Mathews' Method (MVSS-VLMS)*

Mathews and Xie in [19], have presented an adaptive step-size LMS algorithm for adaptive filter, which is changed with respect to a gradient descent algorithm. For multichannel Volterra filter adaptive step-size is given as:

$$\mu'_{pk}(n) = \mu'_{pk}(n-1) + \rho\overline{e_k}(n)\overline{e_k}(n-1) X_{in_p}^T(n)X_{in_p}(n-1) \quad (36)$$

where, ρ is a positive constant usually small that controls the behaviour of the adaptive step- size $\mu'_{pk}(n)$. To reduce the squared estimation error, a gradient descent algorithm is used at each time, which is swayed by the inner product between adjacent gradient vectors.

4) *Aboulnasr's Method (AVSS-VLMS)*

In [20], Aboulnasr and Mayyaas have proposed a VSS-LMS adaptive algorithm, in which step- size is controlled by the square of time-averaged autocorrelation of errors at instantaneous and past times. For a multichannel Volterra system, VSS-LMS algorithm is given as:

$$\mu'_{pk}(n) = \alpha\mu'_{pk}(n-1) + \gamma\, \vartheta_{pk}^2(n) \quad (37)$$

where,$0 < \alpha < 1$,$\gamma > 0$,and the approximation is a time average of $\overline{e_k}(n)\overline{e_k}(n-1)$, which is described as:

$$\vartheta_{pk}(n) = \beta\, \vartheta_{pk}(n-1) + (1-\beta)\overline{e_k}(n).\ \overline{e_k}(n-1) \quad (38)$$

where, β is a positive constant, which lies between 0 and 1 and it is an exponential weighting specification that governs the quality of the time estimation.

$\vartheta_{pk}(n)$ is used in the update of $\mu'_{pk}(n)$, which serves two main purposes. First, the error autocorrelation is commonly a proficient measure of the contiguity to the optimum. Second, it spurned the effect of the uncorrelated noise signal on the step-size update.

5) *Pazaitis' Method (PVSS-VLMS)*

The step-size in [21] is adjusted by the specimen fourth order cumulant of the$\overline{e_k}(n)$, i.e., instantaneous error, and if the adaptive step-size is extended to multichannel Volterra system, it can be described as:

$$\mu'_{pk}(n) = \mu'_{max\ pk}\left(1 - \overline{e}_k^{-\alpha\, C_{pk}(n)}\right) \quad (39)$$

where,

$$C_{pk}(n) = \varsigma_{pk}(n) - 3\vartheta_{pk}^2(n) \quad (40)$$

is the kurtosis of the error, $\mu'_{max\ pk}$ in Eq. (39) can be selected as the maximum value of μ'_{pk} that supports good convergence, α is a positive constant and the estimation of the second- and the fourth- order error moments are given as:

$$\vartheta_{pk}(n) = \beta\, \vartheta_{pk}(n-1) + (1-\beta)\overline{e_k^2}(n) \quad (41)$$

$$\varsigma_{pk}(n) = \beta\varsigma_{pk}(n-1) + (1-\beta)\overline{e_k^4}(n) \quad (42)$$

where, β is the forgetting factor that controls the system memory and should be selected accordingly.

6) *Wee-Peng's Method (WPVSS-VLMS)*

In [22], Wee-Peng *et.al.* have proposed a new class of variable step-size algorithms, which outperforms other algorithms with reduced complexity. The adaptive step-size for a multichannel Volterra system by following the approach of [22] can be given as:

$$\mu'_{pk}(n) = \mu'_{pk}(n-1) + \gamma\,\overline{e_k}(n)X_{in_p}^T(n)\acute{g}_{pk}(n) \quad (43)$$

where, γ is a constant, which is used to update the step-size, and further

$$\acute{g}_{pk}(n) = \beta\, \acute{g}_{pk}(n-1) + \overline{e_k}(n-1) X_{in_p}(n-1) \quad (44)$$

where, β is very small positive constant proximity to one. When β is set equal to zero, Mathews' algorithm is obtained as seen in Eq. (36), and thus Mathews' algorithm is improved by the Wee-Peng's algorithm by utilizing smooth operation on one gradient vector to diminish the measured noise, and can be observed in Fig. 3.

IV. SIMULATION RESULTS

In this section, three cases are effectuated on noise processes in a multichannel nonlinear system identification system to analyse the performance of second-order VLMS based algorithm by using the different variable step-size algorithms under different conditions of SNR and thus comparing it with fixed step-size VLMS algorithm. In these entire cases, the memory size *M* is selected to be 10, for which length of the adaptive filter is 65. According to the Monte-Carlo simulations, the behaviour of adaptive algorithms is differentiated on the basis of calculated performance appraisal factor as:

$$\hat{G}(n) = \sum_{g=1}^{G} \frac{\bar{e}_k^2(n)}{G} \qquad (45)$$

where, G is the ensemble average of square error, $\bar{e}_k^2(n)$, for k channels of 700 independent experiments, which are plotted in semi-logarithmic scale w.r.t iterations to evaluate the performance. In this experiment, number of iterations are taken to be 2500. Let the number of input noise signal, p=1 and the number of channels, k =4.

In this experiment, logistic chaotic noise signal is considered and is given by the recursive equation [6, 9, 13, 14]:

$$x_{in}(n+1) = \lambda\, x_{in}(n)\left[\left(1 - x_{in}(n)\right)\right] \qquad (46)$$

where, λ=4 and initialize x_{in} with 0.9. This nonlinear method is then normalized to possess unity signal power.

Four linear primary paths are used in this experiment, and there transfer functions are:

$$R_{1,1}(z) = z^{-5} - 0.3z^{-6} + 0.2z^{-7}$$
$$R_{1,2}(z) = z^{-5} - 0.2z^{-6} + 0.1z^{-7}$$
$$R_{1,3}(z) = z^{-5} - 0.3z^{-6} + 0.1z^{-7}$$
$$R_{1,4}(z) = z^{-5} - 0.2z^{-6} + 0.2z^{-7}.$$

Three different cases are taken into consideration for comparison between the fixed step-size VLMS algorithm and variable step-size VLMS algorithms under different SNR conditions.

Case 1:

In this case, three algorithms (fixed VLMS, MVSS-VLMS, and WPVSS-VLMS) are taken into consideration, and are compared under low noise condition of SNR, which is equal to 10dB.The step-sizes and positive constants used are a) Fixed step VLMS:$\mu_1 = 0.0002, \mu_2 = 0.00005$; b) MVSS-VLMS: $\mu_{1k}(0) = 0.02,\ \mu_2 = 0.00005, \rho = 10^{-5}$;c) WPVSS-VLMS:$\gamma = 4\text{x}10^{-2},\ \beta = 0.9995, \mu_{1k}(0) = 0.02, \mu_2 = 0.00005$. The variable step-size sways the issue of eigenvalue spread, and inevitably leads to the intensified convergence rate in the presence of chaotic input noise signal. It is evident from Fig. 3 that the performance of WPVSS-VLMS algorithm and MVSS-VLMS algorithm outperform the fixed step VLMS algorithm under the SNR of 10 dB.

Case 2:

In this simulation, convergence characteristics of three algorithms, i.e., fixed step VLMS algorithm, AVSS-VLMS, and PVSS-VLMS algorithms are analogized under the medium noise condition of SNR equivalent to 15 dB as shown in Fig. 4. Akin to the methodology opted in [20, 21], the constant parameter values of the above adaptive algorithms are stipulated to process a comparable level of misadjustment. The values of these parameters are: a) Fixed step VLMS:$\mu_1 = 0.0002, \mu_2 = 0.00005$; b) AVSS-VLMS:$\alpha = 0.97, \gamma = 10^{-3},\ \mu_{1k}(0) = 0.02,\ \mu_2 = 0.00005, \beta = 0.9999$; c) PVSS-VLMS: $\mu_2 = 0.00005,\ \alpha = 0.997, \beta = 0.999$,and $\mu'_{max\ pk} = 0.048$ which is the maximum value of μ'_{pk} that supports good convergence. The initial adaptive step-size for PVSS-VLMS is set to zero. It is very much apparent from Fig.4 that PVSS-VLMS algorithm has good convergence, and outperforms the fixed step VLMS algorithm. Also, AVSS-VLMS algorithm has better convergence than the fixed step VLMS algorithm under medium noise condition.

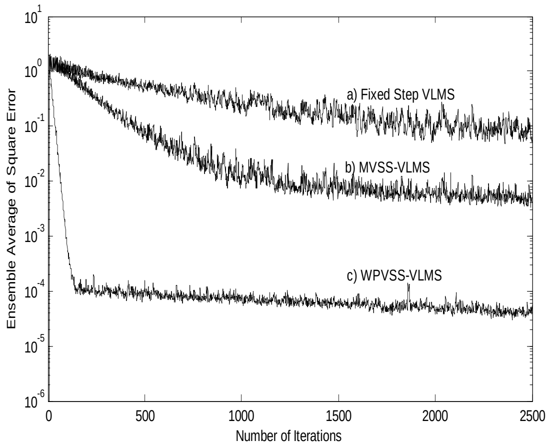


Fig.3. Comparison of convergence attributes with a logistic chaotic input noise signal between fixed step, MVSS-VLMS, and WPVSS-VLMS for second order Volterra filter.

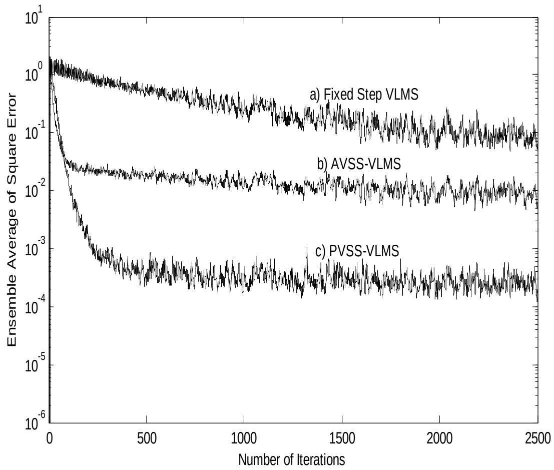


Fig.4. Comparison of convergence attributes with a logistic chaotic input noise signal between fixed step, AVSS-VLMS and PVSS-VLMS for second order Volterra filter.

Case 3:

Now, in this case comparison is drawn between fixed step VLMS, KVSS-VLMS, and KwVSS-VLMS algorithms under the high noise condition of SNR 20 dB. The step-sizes and positive constants used are: a) Fixed step VLMS: $\mu_1 = 0.0002, \mu_2 = 0.00005$; b) KVSS-VLMS:$\mu_2 = 0.00005, \alpha = 5$; c) KwVSS-VLMS: $\beta = 0.9997$, $\gamma = 5.8\text{x}10^{-4} \mu_{1k}(0) = 0.0054, \mu_2 = 0.00005$. In KVSS-VLMS [17], a proper damping parameter α should be selected for a fast convergence rate and very small misadjustment. From Fig. 5, it can be observed that KwVSS-VLMS, and KVSS-VLMS outperform the fixed step VLMS and thus they yield a fast convergence rate.

It should be noted that value of step-size μ_2 is fixed in all the above cases as it will lead to misadjustment error if it is updated at each time. Thus, adaptive step-size is applied only on the linear portion of the adaptive filter.

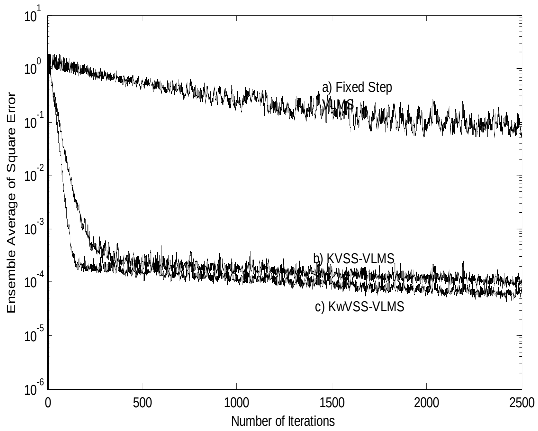


Fig. 5. Comparison of convergence attributes with a logistic chaotic input noise signal between fixed step, KVSS-VLMS, and KwVSS-VLMS for second order Volterra filter.

V. CONCLUSIONS

This paper confers on an adaptive step-size LMS algorithms for the second-order Volterra filter based on a multichannel structure and compared with a fixed step-size VLMS algorithm. Expounded mathematical formulation of a multichannel system identification, and adaptive step-size algorithms are also depicted. It is discerned from simulation results that variable step-size algorithms outplay and have superior rendition to the fixed step size VLMS algorithm in controlling all sorts of nonlinear noise processes in high, medium, and low SNR environments as the variable step-size diminishes the susceptivity of the misadjustment to the degree of non-stationarity and also lessen the accordbetween misadjustment and trailing ability of the fixed step-size LMS algorithm.

The simulation results exhibit that the preliminary convergence rate of the adaptive Volterra filters is very rapid and after an initial span when the step-size proliferates, the step-size starts dwindling slowly and smoothly and yields to a small misadjustment errors. But, in case of nonstatic environments, the algorithms seek to calibrate the step-sizes in such a manner, so as to obtain proximate to the foremost possible performance. Variable step-size algorithm also controls the unpropitious effects of spreading of an eigenvalue of the autocorrelation matrix of the input signal.

There is passably substantial amount of research activity progressing in this domain at present. The fine properties and the computational intelligibility related with the algorithm makes us anticipate to see massive number of advance techniques being evolved, and will be a potential quantum leap with significant impact on practical applications like nonlinearly magnified analog as well as digital communication signal processing [25], biomedical engineering [26], equalization of nonlinear communication medium [27].

REFERENCES

[1] V. John Mathews, Adaptive Polynomial Filters, IEEE Signal Process. Mag., vol. 8, pp. 10-26, Jul. 1991.

[2] A. Stenger, L. Trautmann, and R. Rabenstein, Adaptive Volterra filters for nonlinear acoustic echo cancellation, University of Erlangen, Germany, 1999.

[3] L.F.C. Pessoa, MRL-Filters: A general class of nonlinear systems and their optimal design for image processing, IEEE Trans. On Image Process., vol. 7, no. 7, pp. 966-978, Jul. 1998.

[4] E.J. Coyle and J.H. Lin, Stack filters and the mean absolute error criterion, IEEE Trans. Acoust. Speech Signal Process., vol. 36, pp. 1244-1254, Aug. 1988.

[5] A.K. Kohli and A. Rai, Numeric variable forgetting factor RLS algorithm for second-order Volterra filtering, Circuit Syst. Signal Process., vol. 32, no.1, pp. 223-232, Feb. 2013.

[6] L. Tan and J. Jiang, Adaptive Volterra filters for active control of nonlinear noise processes,IEEE Trans. Signal Process., vol. 49, no. 8, pp. 1667-1676, Aug. 2001.

[7] V.J. Mathews and G.L. sicuranza, Polynomial Signal Processing, John Wiley & Sons Inc., 2000.

[8] M. Schetzen, The Volterra and Wiener Theories of Nonlinear System, Wiley, New York, 1980.

[9] D.P. Das, S.R. Mohapatra, A. Routray and T.K. Basu, Filtered-s LMS algorithm for multichannel active control of nonlinear noise processes, IEEE Trans. on Speech and Audio Process., vol. 14, no. 5, pp. 1875-1879, Sep. 2006.

[10] S.M. Kuo and D.R. Morgan, Active noise control systems-Algorithms and DSP implementations, New York: Wiley, 1996.

[11] P. Strauch and B. Mulgrew, Active control of nonlinear noise processes in a linear duct,IEEE Trans. Signal Process., vol. 46, no. 9, pp. 2404-2412, Sep. 1998.

[12] M. Bouchard, B. Paillard, and C. T. L. Dinh, Improved training of neural networks for the nonlinear active noise control of sound and vibration,IEEE Trans. Neural Netw., vol.10, no. 2, pp. 391-401, Mar. 1999.

[13] D.P. Das and G. Panda, Active mitigation of nonlinear noise processes using a novel filtered-s LMS algorithm,IEEE Trans. Speech Audio Process., vol. 12, no. 3, pp. 313-322, May 2004.

[14] G.L. Sicuraranza and A.C. Carini, Filtered-X affine projection algorithm for multichannel active noise control using second-order Volterra filters,IEEE Signal Process. Lett., vol. 11, no. 11, pp. 853-857, Nov. 2004.

[15] B. Farhang- Boroujeny, Adaptive Filters: Theory and Applications, John Wiley & Sons, New York, NY, USA, 1998.

[16] Y. Zhang, N. Li, J. A. Chambers, and Y. Hao, New gradient-based variable step-size LMS algorithms, Journal on Advances in Signal Process., vol. 1, Feb.2008.

[17] S. Karni and G. Zeng, A new convergence factor for adaptive filters, IEEE Trans. on Signal Process., vol. 40, no. 7, pp. 1633-1642, Aug. 1989.

[18] R. H. Kwong and E. W. Johnson, A variable step-size LMS algorithm,IEEE Trans.on Signal Process., vol. 40, no. 7, pp. 1633-1642, Jul. 1992.

[19] V. J. Mathews and Z. Xie, A stochastic gradient adaptive filter with gradient adaptive step-size,IEEE Trans. on Signal Process., vol. 41, no. 6, pp. 2075-2087, Jun. 1993.

[20] T. Aboulnasr and K. Mayyas, A robust variable step-size LMS-type algorithm: analysis and simulations,IEEE Trans. on Signal Process., vol. 45, no. 3, pp. 631-639, Jan. 1997.

[21] D. I. Pazaitis and A. G. Constantinides, A novel kurtosis driven variable step-size adaptive algorithmIEEE Trans. on Signal Process., vol. 47, no. 3, pp. 864-872, Mar. 1999.

[22] W. P. Ang and B. Farhang- Boroujeny, A new class of gradient adaptive step-size LMS algorithms,IEEE Trans. on Signal Process., vol. 49, no. 4, pp. 805-810, Apr. 2001.

[23] A. Rai and A. K. Kohli, Adaptive polynomial filtering using generalised variable step-size least mean pth power (LMP) algorithm,Circuit System and Signal Process., vol. 33, no. 12, pp. 3931-3947, Dec. 2014.

[24] G. V. Raz and B. V. Veen, Baseband Volterra filters for implementing carrier based nonlinearities, IEEE Trans. Signal Process., vol. 46, pp. 103-114, Jan. 1998.

[25] T. Ogunfunmi, Adaptive nonlinear system identification: Volterra and Weiner model approches, Springer, Berlin, 2007.

[26] D. T. Westwick and R. E. Kearney, Identification of Nonlinear Physiological Systems, Wiley, New York, 2003.

[27] T. Ogunfunmi and T. Drullinger, Equalization of nonlinear channels using a Volterra-based nonlinear adaptive filter, in Proceedings of IEEE International Midwest Symposium on Circuits and Systems, Seoul, South Korea, pp. 1-4, Jan. 2011.