

# Computational Complexity of Adaptive Algorithms in Echo Cancellation

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**ABSTRACT:** *The proportionate normalized least-mean-squares (PNLMS) algorithm is a new scheme for echo canceller adaptation. On typical echo paths, the proportionate normalized least-mean-squares (PNLMS) adaptation algorithm converges significantly faster than the normalized least-mean-squares (NLMS) algorithm. Two proportionate affine projection sign algorithms (APSAs) are also proposed for network echo cancellation application where the impulse response is often real-valued with sparse coefficients and long filter length. The proportionate-type algorithms can achieve fast convergence rate in sparse impulse responses and low steady-state misalignment. The new algorithms are more robust to impulsive interferences and colored input signals than the proportionate least mean squares algorithm, normalized sign algorithm and the robust proportionate affine projection algorithm. The computational complexity of the new algorithms is lower than the affine projection algorithm (APA) family due to the elimination of the matrix inversion. The computational complexity of the proportionate APSAs are compared with that of conventional algorithms in terms of the total number of additions, multiplications, divisions, comparisons, and square-roots.*

**Keywords** - Adaptive filter, Proportionate filters, Proportionate adaptive algorithm, Sparse channel identification

## I. Introduction

For decades people have been using the telephone as a means of distant voice communication. As the coverage area and number of subscribers increased, the telephone system has become more and more sophisticated. Long distance connections and the sophisticated systems introduced a lot of challenging problems. One of the major challenges in the telephone system is the “Echo Effect”. Echo is an undesired phenomenon experienced by a user over a phone call when he/she hears his/her own voice back after a delay. Echo generation can be characterized as Network/Line Echo and Acoustic Echo. An echo canceller is a simple adaptive filter with self adjusting coefficients to cancel out the echo. This adaptive filter uses different algorithms such as LMS, NLMS, PNLMS and affine projection algorithms. As the required adaptive filter lengths grow, the conventional normalized least mean squares (NLMS) algorithm exhibits a slower convergence rate. This slow convergence

rate becomes noticeable in that echo is often heard, especially in the first few seconds of a connection. The proportionate NLMS (PNLMS) has been designed to ameliorate this situation by exploiting the sparse nature of the NIR. By selecting a proportionate matrix at each iteration, PNLMS updates each coefficient in the weight vector proportionate to its magnitude. This results in very fast initial convergence for sparse NIRs relatively independent of their length. However, the drawback of PNLMS is that, though it has fast initial convergence for sparse NIRs, it has slower convergence than NLMS for non-sparse NIRs. This problem has been addressed by several modifications to PNLMS. The first is PNLMS which has two versions, one where the adaptation algorithm alternates between both PNLMS and NLMS in successive sample periods and another where both updates are combined in each sample period. The resulting convergence is generally the better of the two algorithms. That is, PNLMS’s convergence is like PNLMS’s for sparse NIRs and like NLMS’s for dispersive NIRs. Another modification to PNLMS is the improved PNLMS (IPNLMS) which has the feature of being optimal for a given NIR sparseness. This feature has later been exploited in a class of sparseness-controlled (SC) algorithms which measure the sparseness of the developing coefficients on-the-fly. Another approach is to use adaptive combination of proportionate filters which adaptively mix the outputs of two independent adaptive filters together based on IPNLMS. In addition, the  $\mu$ -law PNLMS (MPNLMS) is an optimal step-size algorithm modified from PNLMS. A number of proportionate algorithms are also developed for the affine projection algorithm (APA) which is well known for its better convergence than NLMS for colored input. These proportionate algorithms include proportionate APA (PAPA), improved proportionate APA (IPAPA) and “memory”-IPAPA (MIPAPA). The IPAPA extends the proportionate matrix of IPNLMS directly to APA; while the MIPAPA designs an efficient matrix to reduce computational complexity. Both algorithms improve convergence rate over that of PNLMS in

practical NEC applications where the inputs are speech and the NIRs are sparse. However, they have higher complexity than PNLMS algorithms because they generally require a matrix inversion where the size of the matrix is the order of the projection. In practice, the projection orders are typically around ten and direct matrix inversion of this size may be too expensive.

### 1.1 Echo Canceller

Telephone companies use a device called an echo canceller (EC) to overcome the echo problem. An echo canceller as in Fig 1.1 is a simple adaptive filter with self adjusting coefficients to cancel out the echo. Every echo has an echo path and it is characterized by an impulse response. The echo canceller adapts its filter coefficients to the network echo path such that it cancels out the echo. This process can be visualized with an echo canceller unit as in Fig 1.1.

## II. Adaptive Filters for Echo Cancellation

In this section we discuss two types of adaptive filters. The first is a stochastic gradient algorithm called least mean squares (LMS) and the second is a derivative of LMS, normalized least mean squares (NLMS).

### 2.1 The LMS Algorithm

LMS is the most widely used adaptive filtering algorithm in the world. It is used in various applications like system identification problems (e.g. echo cancellation), speech coding and channel equalization problems. Although, its speed of convergence is often slower than desired, it is popular because of its robust performance, low cost of implementation and simplicity. LMS is a stochastic gradient algorithm. The derivation of the algorithm is similar to the steepest descent method. However, the steepest descent uses a deterministic gradient to reach the Wiener solution, whereas LMS uses a stochastic gradient for recursive updates which tends to achieve the Wiener solution[15]. LMS's application to echo cancellation can be visualized from figure 1. Let us make the following definitions, input is the excitation signal, often called the far-end signal given by equation (1) is the excitation vector. and equation(2) is the desired signal, it is the summation of the echo and near-end background noise/signal. The impulse response of the system is given by equation (3). The a priori error or residual echo is given by equation (4). The performance index (cost function) of LMS is the mean squared error and is given by equation(5). The computational complexity of LMS algorithm

for one sample period is summarized in the below Table1. The total memory required for LMS implementation is about  $2L$  where,  $L$  is the length of the adaptive filter.

### 2.2 The NLMS Algorithm

The step size of LMS is restricted by its region of stability which is determined by the energy in the excitation signal. For signals that have time-varying short-time energy, like speech, a constant step size means the speed of convergence will vary with the short-time energy. NLMS overcomes this problem by normalizing the step size every update with the squared Euclidian norm of the excitation vector. NLMS can be derived by considering a sample-by-sample cost function given by equation (6) that minimizes the size of the coefficient update under the constraint that the a posteriori error (the error after the coefficient update) for that sample period is zero. In equation (6),  $r^T(n)$  is the coefficient update vector at sample period  $n$ ,  $e(n)$  is the a posteriori error, and  $\delta$  is a weighting factor between the size of the update and the a posteriori error. The a posteriori error can be expressed using equation (8). Thus, the cost function can be given by equation (6). NLMS algorithm can be written in the two steps of its usual implementation using equation (7) and (9). In equation (9) step-size parameter,  $\mu_{NLMS}$  has been added as a relaxation factor and the stability range of  $\mu_{NLMS}$  for NLMS is  $0 < \mu_{NLMS} < 1$ . The parameter  $\delta$  in the NLMS coefficient update is also known as the regularization parameter[8]. It is seen that when  $\delta$  is non-zero (it is always non-negative) the coefficient update is prevented from becoming unstable. The coefficient update is given by equation (10). The computational complexity of N LMS algorithm for one sample period is summarized in the Table2.

### 2.3 Proportionate NLMS Algorithm

The PNLMS algorithm tries to accelerate the convergence of the filter by adapting faster the weights corresponding to the active region of the sparse echo path. The advantage of PNLMS over ES-NLMS is that it does not assume any other a priori knowledge about the echo channel but its sparsity. PNLMS algorithm exploits the sparseness of impulse responses to achieve significantly faster adaptation than the conventional normalized least-mean-squares (NLMS) algorithm. The proportionate NLMS algorithm (PNLMS) makes the adaptation step for each tap proportional to the current absolute value of the estimated weight[1]. The advantage of PNLMS is that it does not assume any other a priori knowledge about the echo channel but its sparsity. The PNLMS algorithm differs from the NLMS algorithm in that

the available adaptation “energy” is distributed unevenly over the N taps. The weight update is given by equation (11). PNLMS under this kind of complexity analysis is 50% more complex than either NLMS or LMS[17]. The computational complexity of PNLMS algorithm for one sample period is summarized in the Table3.

**2.3.1 Improved Proportionate NLMS Algorithm**

The coefficients of an IPNLMS filter are adapted according to the equation (12).  $\mu$  is the step size of the filter and is given by equation (13). The gain  $g_m(n)$  for each weight is calculated using Equation (14) where  $\epsilon$  is a small positive constant, and  $k$  is a constant between -1 and 1, which establishes a tradeoff between the standard NLMS filter ( $k=-1$ ) and basic PNLMS( $k=1$ ). From the above equations it is evident that the step size associated to each coefficient increases with the absolute value of that coefficient. Consequently, like in the PNLMS, IPNLMS spends more energy adapting the active coefficients, thus converging faster than NLMS.

**2.4 APA Algorithm:** A number of proportionate algorithms developed for the affine projection algorithm (APA) which is well known for its better convergence than NLMS for colored input. These proportionate algorithms include proportionate APA (PAPA), improved proportionate APA (IPAPA) and “memory”-IPAPA (MIPAPA). The IPAPA extends the proportionate matrix of IPNLMS directly to APA; while the MIPAPA designs an efficient matrix to reduce computational complexity. Both algorithms improve convergence rate over that of PNLMS in practical NEC applications where the inputs are speech and the NIRs are sparse. However, they have higher complexity than PNLMS algorithms because they generally require a matrix inversion where the size of the matrix is the order of the projection. In practice, the projection orders are typically around ten and direct matrix inversion of this size may be too expensive. The computational complexity of PNLMS algorithm for one sample period is summarized in the Table4.

**2.5 Computational Complexity**

The APSA achieves faster convergence and lower steady-state normalized misalignment than NLMS, APA, and NSA under impulsive interference. This is achieved without the need for a matrix inversion as in APA. The computational complexity of the proportionate APSAs are compared with that of conventional algorithms in terms of the total number of additions, multiplications, divisions,

comparisons, square-roots, and direct matrix inversions. With filter length and projection order , the complexities are shown in above tables.. Although APA behaves better than NLMS, APA has higher complexity because the number of multiplications and the size of the DMI increase proportionately to  $M^2$ . In contrast, the APSA does not require matrix inversion thus the projection order  $M$  does not affect the number of multiplications, and the number of additions is only linearly dependent on  $M$  . Note that APSA does not require matrix inversion, thus the APSAs are more efficient than APA. With a modest increase in the proportionate matrix computation, the proportionate APSAs behave much better than the original APSA, especially for sparse NIRs. This APA algorithm takes into account the history of the proportionate factors (the last values) for each filter coefficient. This algorithm achieves a lower computational complexity due to the recursive implementation of the “proportionate history”, providing both faster tracking and lower misadjustment. However, IPAPA algorithms have higher complexity than PNLMS algorithms because they generally require a matrix inversion where the size of the matrix is the order of the projection.

**III. INDENTATIONS AND EQUATIONS**

$$X(n) = [x(n), x(n - 1), \dots, x(n - L + 1)]^T \dots\dots\dots(1)$$

$$s(n) = X^T(n)h_{true} + y(n) \dots\dots\dots (2)$$

$$h(n) = [h_0(n), h_1(n), \dots, h_{L-1}(n)]^T \dots\dots(3)$$

$$e(n) = s(n) - X^T(n)h(n - 1) \dots\dots\dots(4)$$

$$J(h(n)) = E\{e^2(n)\} = E\{[s(n) - X^T(n)h(n - 1)]^2\} \dots\dots\dots(5)$$

$$J_n = \delta r^T(n)r(n) + [e(n) - X^T(n)r(n)]^T \dots\dots\dots(6)$$

$$\hat{e}(n) = s(n) - X^T(n)h(n) \dots\dots\dots(7)$$

$$\hat{e}(n) = s(n) - X^T(n)[h(n - 1) + r(n)] \dots\dots\dots(8)$$

$$h(n) = h(n - 1) + \mu_{NLMS}[X^T(n)X(n) + \delta]^{-1}X(n)e(n) \dots\dots\dots(9)$$

$$h(n) = h(n - 1) + \frac{\mu_{NLMS}}{\|X(n)\|^2 + \delta} X(n)[s(n) - X^T(n)h(n - 1)] \dots\dots\dots(10)$$

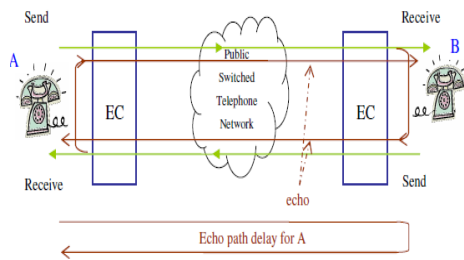
$$w_m(n+1) = w_m(n) + \mu_m(n) \frac{e(n)}{\delta + x^T(n)x(n)} x_m \dots\dots\dots(11)$$

$$w_m(n+1) = w_m(n) + \mu_m(n)e(n)x_m(n) \dots\dots\dots(12)$$

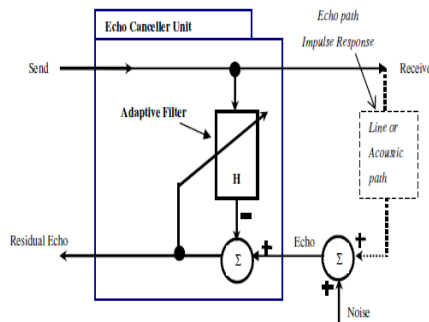
$$\mu_m(n) = \frac{\mu g_m(n)}{\delta + \sum g_k(n)x_k^2(n)} \dots\dots\dots(13)$$

$$g_m(n) = (1 - k) \frac{1}{2M} + (1 + k) \frac{w_m(n)}{\epsilon + 2\|w(n)\|^2} \dots\dots\dots(14)$$

**IV.FIGURES AND TABLES**



**1. Echocanceller**



1.1blockdiagram of echocanceller

**Table -1: Complexity of LMS Algorithm**

Equations	Multiplications
$e(n) = s(n) - X^T(n)h(n - 1)$	L
$h(n) = h(n - 1) + 2\mu_{NLMS}x(n)$	L
Total Complexity	2L

**Table -2: Complexity of NLMS Algorithm**

Equations	Multiplications
$e(n) = [s(n) - X^T(n)h(n - 1)$	L
$\epsilon(n) = [\ X(n)\ ^2 + \delta]^{-1}$	$O_{NLMS}$

$e(n)$	
$h(n) = h(n - 1) + 2\mu_{NLMS}x(n)$	L
Total Complexity	$2L + O_{NLMS}$

**Table -3: Complexity of PNLMS Algorithm**

Equations	Multiplications
$e(n) = [s(n) - X^T(n)h(n - 1)$	L
$\epsilon(n) = [\ X(n)\ ^2 + \delta]^{-1}$	$O_{NLMS}$
$h(n) = h(n - 1) + \mu_{NLMS}x(n)$	L
Total Complexity	$2L + O_{NLMS}$

**Table -4 : Complexity of APA Algorithm**

Equations	Multiplications	Memory
$e(n) = [s(n) - X^T(n)h(n - 1)$	NL	L
$\epsilon(n) = [X^T(n)X(n) + \delta I]^{-1}$	$O_{APA}N^2$	$N^2$
$h(n) = h(n-1) + \mu_{APA} X^T(n) \epsilon(n)$	NL	L
Total Complexity	$2NL + O_{APA}N^2$	$2L + N^2$

**V.Conclusion**

In PNLMS algorithm stability is assured and adaptation quality (misadjustment) is held constant at any desired level. With update gains proportional to current tap weights, very fast convergence on typical echo paths is achieved. Also it can be seen from simulations that NLMS has a lower steady-state error compared to the PNLMS, which can be analyzed through a separate work. Two proportionate affine projection sign algorithm (APSA) have been proposed for the identification of real-coefficient, sparse systems. With a modest increase in computational complexity over that of the original APSA, the proportionate APSAs can

achieve faster convergence rate and lower complexity in a steady-state misalignment in a sparse network echo path, colored input, and impulsive interference environment. The computational complexity of the two proportionate APSAs is lower than the APA family due to elimination of the matrix inversion.

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