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Minimal Realizations of Logic Functions

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Abstract - *In the conventional method, a truth table (TT) is prepared from the specified logic function. Then it is expressed as the sum of minterms corresponding to the rows in which 1 appears. Finally, this function is further reduced using the Boolean identities. Thus, all the simplifications are concentrated at one place after the TT. This procedure does not always lead to minimal realization. This paper deals with the minimal realization of the logic function using TT in which TT is reduced successively by one variable at a time till all the variables are exhausted. Instead, the simplification is carried out at the end of the TT at the end of each step of TT reduction. The method is shown to be systematic and leads to minimal function. It is simpler in operation than based on only Boolean identities, Karnaugh map, and Quine-McClusky methods and can handle any number of variables. It is explained with several examples. It is worth introducing as an improvement over the classical TT method in classroom teaching.*

Keywords - *Minimal Realizations, Logic Functions, Truth Table Method, Digital Circuits*

I. INTRODUCTION

The minimal expression for a logic function is the one that has the least possible number of terms and also the least possible total number of literals [1].

Several methods are available for minimizing logic expressions, for example [1]-[18]. Some of them do not always lead to minimal realization, and some are convenient when the number of variables is restricted to 6 [3], while some require a very much lengthy tabulation procedure [1]. The map method developed recently [16] is systematic and yields all the minimal realization(s). It is faster, simpler, and more convenient than Prasad's method [17][18]. In the present work we

1. show that the TT representation of a logic function is a special case of a map,
2. extend a conventional TT into two dimensional TT,
3. obtain minimal realizations of single variable logic functions and then of any number of variables,
4. get back the original function from the given minimal realization.

II. Map Representation of Logic Functions

Various maps of a 4 variable logic function $f = (A, B, C. D)$ are shown in Fig. 1. Note that the map of Fig. 1(e) is the same as the TT, except that the table is written differently than the normal one shown in Fig. 1(f). The latter is arranged in increasing order of decimal numbers simply for convenience not to forget any number. We can conclude that the TT of logic functions is a special case of map representation. TT can be drawn both in horizontal (Fig. 1(d)) and vertical (Fig. 1(e)) directions. Finally, we can draw the TT by distributing the variables in vertical and horizontal directions, such as Fig 2. However, Fig. 1(a) only represents the K-map and gives minimal realization; others do not.

The method given in [17] is based on TT shown in Fig. 1(f) but applies to Fig. 2.

CD→ / AB↓	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

(a)

D → *ABC*↓	0	1
000	0	1
001	2	3
011	6	7
010	4	5
110	12	13
111	14	15
101	10	11
100	8	9

(b)

*A**BCD*	000	001	011	010	110	111	101	100
1	0	1	3	2	6	7	5	4
1	8	9	11	10	14	15	13	12

(c)

ABCD	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
→	0	1	3	2	6	7	5	4	12	13	15	14	10	11	9	8

(d)

ABCD↓	
0000	0
0001	1
0011	3
0010	2
0011	6
0001	7
0011	5
0010	4
0100	12
0101	13
0111	15
0110	14
1000	10
1001	11
1011	9
1010	8

(e)

ABCD↓	
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

(f)

Fig. 1 Various map representations of a 4-variable (a) 2×2, (b) 3×1 (c) 1×3 (d) 0×4, (e) 4×0 (f) Normal TT table

CD→ *AB*↓	00	01	10	11
00	0	1	2	3
01	4	5	6	7
10	8	9	10	11
11	12	13	14	15

Fig. 2 Two dimensional TT

Example 1: Let $f = \sum$ 0, 2, 3

The minimal realizations are obtained as shown in Fig. 3 by simplification through TT and K-map. Note that TT requires simplification while K-map does not. The next section will show how *minimal* realizations can be obtained by the method of [16].

AB↓	*f*
00	1
01	0
10	1
11	1

$$\begin{aligned} f &= ab + Ab + AB \\ &= ab + A(b + B) \\ &= ab + A \\ &= b + A \end{aligned}$$

(a)

A↓ *B*→	0	1
0	1	0
1	1	1

$f = b+A$

(b)

Fig. 3 Minimal realization of the function of Example 1 (a) using TT (b) using K-map

III. Minimal Realizations of Single Variable Logic Functions

A single variable logic function can be shown by the K-map or TT, as shown in Fig. 4.

x	*f*
0	*y*
1	*z*

Fig. 4 K-map reprenation of a single variable logic function

Then

$$f = xy + Xz \tag{1}$$

Here, X is the complement of x. Using eqn (1) and simplifying, minimal realizations of f for various values of y and z are given in Table 1, where y_n (z_n) represent the number of literals in y (z).

From table 1, we note the following properties:

1. From rows 1 and 4, the Total number of literals is 0 (minimum) when y, z are both 0 or 1.
2. From rows 2 and 3, the total number of literal is 1, if y, z = 0, 1, or 1, 0.
3. From rows 5, 6, 8, and 9, the total number of literals is $1 + y_n$ or $1 + z_n$ when either of y, z is 0 or 1.
4. From row 7, the number of literals is z_n if $y = z$.
5. In rows 4 and 7, there is a reduction in the number of literals by $z_n + 2$ due to the property $a + A = 1$
6. In the last four rows, there is a reduction in the number of literals by 1 due to property $a+Ab = a + b$.
7. All the functions are minimal as no further reduction is possible.

Table 1: Minimal realizations of single variable logic function

Row No	*y*	*z*	Minimal function *f*	Total number of Literals, *n*
1.	0	0	0	0
2.	0	1	X	1
3.	1	0	x	1
4.	1	1	$x + X = 1$	0
5.	y	0	xy	$1 + y_n$
6.	0	z	Xz	$1 + z_n$
7.	z	z	$xZ+XZ = z$	z_n
8.	1	z	$x+XZ= x+z$	$1 + z_n$
9.	y	1	$xy+X= y+X$	$1 + y_n$
10.	y	wy	$xy+Xwy = xy+wy$	$1 + 2y_n+w_n$
11.	wz	z	$xwz+Xz = wz+Xz$	$1 + 2z_n+w_n$

The method yields the minimal realization of the properties of Table 1 are used. This is illustrated with the following example.

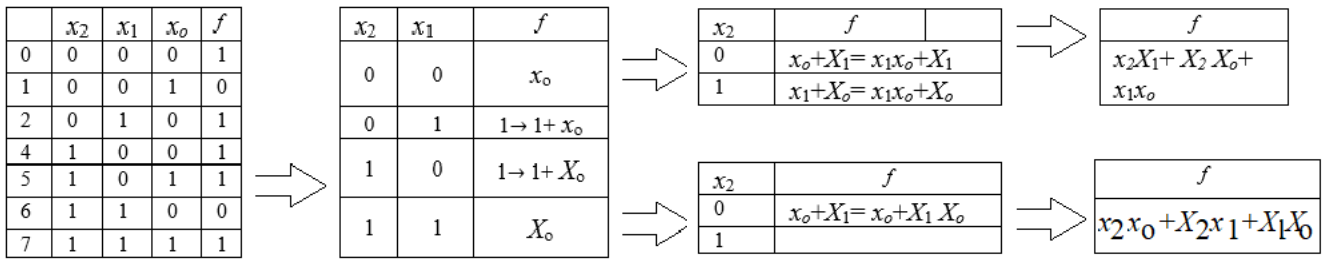

Fig. 5 Minimal realizations of *f* given in eqn (2)

Table 2 Procedure for obtaining the original function from its minimal function

1	2	3	4	5	6	7	8	9	10	11	12
f	x_4		x_4x_3		$x_4x_3x_1$		$x_4\underline{x_3}\,\underline{x_1}\,\underline{x_2}$		$x_4x_3x_1x_2x_0$		
$x_4x_3x_1x_0+x_4x_3x_2x_1+X_4x_3X_2X_0+x_3X_1X_0+X_3X_2X_0$	0	$x_3x_1x_0+x_3x_2x_1+x_3X_1X_0+X_3X_2X_0$	0	$x_1x_0+x_2x_1+X_1X_0$	0	x_0+x_2	0	$1+x_0=1$	0	1	0
									1	1	1
							1	x_0	0	1	4
									1	0	5
					1	X_0	0	X_0	0	0	2
									1	1	3
							1	X_0	0	0	6
									1	1	7
			1	X_2X_0	0	X_2X_0	0	0	0	0	8
									1	0	9
							1	X_0	0	0	12
									1	1	13
					1	X_2X_0	0	0	0	0	10
									1	0	11
							1	X_0	0	0	14
									1	1	15
	1	$x_3X_2x_0+x_3X_1X_0+X_3X_2X_0$	0	$X_2x_0+X_1X_0$	0	X_2x_0	0	0	0	0	16
									1	0	17
							1	x_0	0	1	20
									1	0	21
					1	$X_2x_0+X_0=X_2+X_0$	0	X_0	0	0	18
									1	1	19
							1	$1+X_0=1$	0	1	22
									1	1	23
			1	X_2X_0	0	X_2X_0	0	0	0	0	24
									1	0	25
							1	X_0	0	0	28
									1	1	29
					1	X_2X_0	0	0	0	0	26
									1	0	27
							1	X_0	0	0	30
									1	1	31

Example 2: Simplify

$$f = \sum (0,2,3,4,5,7) \qquad (2)$$

Solution: The solution is given in Fig. 5. Note that there are two possible minimal functions. This is because there are two ways of making common terms, either x_1x_o or X_1X_o.column are minimal since they are obtained using the minimal functions of Table 1. Now all the preceding basic blocks are also minimal as they are derived using the minimal functions of Table 1. Hence the final function f obtained is minimal. This inference can be extended to a function of any number of logic variables.

IV. To Obtain the Original Logic Function from Its Minimal Realization

The procedure is explained with the minimal function derived in Table 2 and shown in Table 3. Column 1 represents the minimal function given. Since there are 5 variables, the table is drawn with 32 rows as marked. The Upper (lower) basic block in column 3 corresponding to 0 (1) in column 2 is obtained as the sum of all the terms which have x4 (X4) in column 1 dropping x4 (X4). Similarly, column 5 corresponding to 0 (1) in column 4 are obtained as the sum of all the terms which have x3 (X3) in column 3 dropping x3 (X3). There are no common terms in column 3; hence no more terms are added in column 5. The same procedure is repeated till we arrive at column 11. Please note that simplifications are carried out in columns 7 and 9 before proceeding as per the above steps. From the last two columns, one can write (after arranging in increasing decimal numbers)

$$f = \sum (0, 1, 3, 4, 7, 13, 15, 19, 20, 22, 23, 29, 31).$$

V. Conclusion

It is shown that the TT representation of a logic function is a special case of a map. The conventional method of TT is extended to two dimensional TT. Miimal realization is obtained for a single variable logic function and then to any number of variables. It is proved that Rathore's method is simpler, more convenient, and straightforward than that of Prasad. It gives all the possible minimal realizations. It is illustrated how to get back the original transfer function from a given minimal function.

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