# Novel Realizations of Current Mode Transfer Functions 

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#### Abstract

Jitendra Mohan has proposed a new first order all-pass filter with its application in realizing an oscillator. The fitler consists of one single active device (MODXCCII), and (grounded) 1 resistor and 1 capacitor. In this paper, we present a systematic procedure to realize a wider range of functions where the all-pass filter realization is a special case.


Keywords - Current mode, All-pass filter, MO-DXCCII, Current transfer function, Synthesis

## I. INTRODUCTION

There has been a lot of interest in realizing all-pass functions since long [1]-[20]. While Rathore [1] deals with an $n$th order all-pass functions, others first or second order only. Some realizations focus on a minimum number of active and passive components, while some have all grounded passive elements. Some of them are voltage mode (VM), while others are current-mode realizations.

Current-mode (CM) circuits are attractive because of their wider bandwidth, wider dynamic range, and lower power consumption than VM counterparts [19]. A current differencing buffered amplifier (CDBA) [14] and a current operational amplifier (COA) [15] based first-order CM allpass filter (APF) configurations have been proposed. Kacar and Mahmut [19] have presented a CM circuit using one DVCC for first-order APF only.

Jitendra Mohan [20] has proposed a CM APF employing one multi-output dual-X second-generation current conveyor (MO-DXCCII), a grounded resistor and a grounded capacitor. The symbol for the MO-DXCCII is shown in Fig. 1. Its termincl characteristics are given as

$$
\left[\begin{array}{c} 
\pm V_{y}  \tag{1}\\
I_{y} \\
I_{z \pm} \\
I_{w \pm}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
V_{x \pm} \\
V_{w} \\
I_{x \pm} \\
I_{x \pm}
\end{array}\right]
$$



Fig. 1 Symbol of MO-DXCCII


Fig. 2 Circuit configuration
The circuit is ideal for CM cascading due to its lowinput and high-output impedances. The use of grounded passive components makes the circuit, ideal for IC implementation. The theoretical results have been validated through PSPICE simulation program using $0.35 \mu \mathrm{~m}$ CMOS process parameters.

In this paper, the same configuration, shown in Fig. 2 with admittances $Y_{1}$ and $Y_{2}$, is exploited to syntheisze a more general class of CM transfer functions. We present two synthesis methods.

## II. SYNTHESIS

## A. Partial fraction expansion method

Analysis of the circuit shown in Fif. 2 gives

$$
\begin{equation*}
T(s)=\frac{I_{o}}{I_{i}}=K \frac{N(s)}{D(s)}=\frac{Y_{1}-Y_{2}}{Y_{1}+Y_{2}} \tag{2}
\end{equation*}
$$

where $K$ is a gain constant, $N^{\circ} \leq D^{\circ}$

$$
\begin{equation*}
\frac{Y_{1}-Y_{2}}{Y_{1}+Y_{2}}=K \frac{\prod_{j=1}^{m}\left(s+z_{j}\right)}{\prod_{i=1}^{n}\left(s+y_{i}\right)}=K \frac{\frac{N(s)}{Q(s)}}{\frac{D(s)}{Q(s)}} \tag{3}
\end{equation*}
$$

where $Q(s)$ is defined in (5) and $m \leq n$. In eqn. (2), let

$$
\begin{equation*}
Y_{1}+Y_{2}=\frac{D(s)}{Q(s)} \text { and } Y_{1}-Y_{2}=K \frac{N(s)}{Q(s)} \tag{4}
\end{equation*}
$$

Since $Y_{1}+Y_{2}$, being the sum of two RC DPAs, is also an RC DPA, it must have poles and zeros on the negative real axis, interlaced and the lowest (highest) critical frequency a zero (pole). With these restrictions,

$$
\begin{equation*}
Q(s)=\prod_{k=1}^{n-1}\left(s+p_{k}\right) \tag{5}
\end{equation*}
$$

where $Y_{k+1}>p_{k}>y_{k}, k=1,2, \ldots, n-1$.
A factor $\left(s+p_{n}\right)$ such that $p_{n}>y_{n}$ could be added in $Q(s)$, but the above choice is made to have a smaller number of elements in $Y_{1}$ and $Y_{2}$. Equation (4) can be expressed as, by partial fraction expansions,

$$
\begin{equation*}
Y_{1}+Y_{2}=\left[A_{\infty} s+A_{o}+\sum_{k=1}^{n-1} \frac{A_{k} s}{\left(s+p_{k}\right)}\right] \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
Y_{1}-Y_{2}=\left[K B_{\infty} s+K B_{o}+K \sum_{k=1}^{n-1} \frac{B_{k} s}{\left(s+p_{k}\right)}\right] \tag{7}
\end{equation*}
$$

where

$$
\begin{gathered}
A_{\infty}= \begin{cases}0, & m<n \\
\left.\frac{D(s)}{s Q(s)} \right\rvert\, s \rightarrow \infty, & m=n,\end{cases} \\
B_{\infty}= \begin{cases}0, & m<n \\
s Q(s) \\
s Q(s) & m \rightarrow \infty\end{cases} \\
\left.A_{o}=\frac{D(s)}{Q(s)} \right\rvert\, s=0
\end{gathered}, \begin{aligned}
& \left.B_{o}=\frac{K N(s)}{Q(s)} \right\rvert\, s=0 \\
& \left.A_{k}=\frac{\left(s+p_{k}\right) D(s)}{s Q(s)} \right\rvert\, s=-p_{k} \\
& \left.B_{k}=\frac{\left(s+p_{k}\right) N(s)}{s Q(S)} \right\rvert\, s=-p_{k}
\end{aligned}
$$

$A_{k}$, being the residues at the poles of an RC DPA, will be positive real. Thus,

$$
A_{k}>0 .
$$

From eqns. (6) and (7).

$$
Y_{1}=\left(\frac{1}{2}\right)\left[\begin{array}{l}
s\left(A_{\infty}+K B_{\infty}\right)+\left(A_{o}+K B_{o}\right)  \tag{8}\\
+\sum_{k=1}^{n-1} \frac{\left(A_{k}+K B_{k}\right) s}{s+p_{k}}
\end{array}\right]
$$

and

$$
Y_{2}=\left(\frac{1}{2}\right)\left[\begin{array}{l}
s\left(A_{\infty}-K B_{\infty}\right)+\left(A_{o}-K B_{o}\right)  \tag{9}\\
+\sum_{k=1}^{n-1} \frac{\left(A_{k}-K B_{k}\right) s}{s+p_{k}}
\end{array}\right]
$$

For $Y_{1}$ and $Y_{2}$ to be RC DPAs, the residues at the poles must be positive real, i.e.,

$$
\begin{equation*}
A_{k}+K B_{k} \geq 0, \quad k=0,1,2, \ldots n-1, \infty \tag{10}
\end{equation*}
$$ and

$$
\begin{equation*}
A_{k}-K B_{k} \geq 0, \quad k=0,1,2, \ldots n-1, \infty \tag{11}
\end{equation*}
$$

Thus, $K$ must be chosen such that eqns. (10) and (11) satisfy for both $Y_{1}$ and $Y_{2}$ to be RC realizable, that is,

$$
\begin{equation*}
K \leq \min \left[\frac{A_{k}}{B_{k}^{-}}, \frac{A_{k}}{B_{k}^{+}}\right], k=0,1,2, \ldots n-1, \infty \tag{12}
\end{equation*}
$$

where $\mathrm{B}_{\mathrm{k}}{ }^{-}$is $-B_{k}$ and $B_{k}{ }^{+}$is $B_{k}$. Note from eqn. (2) that the poles of $T(s)$ are the zeros of the RC DPA $\left(Y_{1}+Y_{2}\right)$. Hence, the method can realize the current transfer functions with distinct negative real poles only.

## Realization of biquadratic functions

Consider

$$
\begin{equation*}
T(s)=\frac{K\left[s^{2}+a_{1} s+a_{o}\right]}{\left[s^{2}+b_{1} s+b_{o}\right]} \tag{13}
\end{equation*}
$$

where $a$ s and $b s$ are constants. Choosing as per eqn. (5), $Q(s)=(\mathrm{s}+\alpha), p_{1} \leq \alpha \leq p_{2}$, we identify

$$
\begin{aligned}
& \left(Y_{1}-Y_{2}\right)=\frac{K\left[s^{2}+a_{1} s+a_{o}\right]}{(s+\alpha)}=K\left[s+\frac{a_{o}}{\alpha}+\frac{\left(a_{1} \alpha-\alpha^{2}-a_{o}\right) s}{\alpha(s+\alpha)}\right] \\
& \left(Y_{1}+Y_{2}\right)=\frac{\left[s^{2}+b_{1} s+b_{o}\right]}{(s+\alpha)}=\left[s+\frac{b_{o}}{\alpha}+\frac{\left(b_{1} \alpha-\alpha^{2}-b_{o}\right) s}{\alpha(s+\alpha)}\right]
\end{aligned}
$$

From these relations,

$$
\begin{align*}
& Y_{1} \\
& =\frac{(1+K)}{2} s+\frac{b_{o}-K a_{o}}{2 \alpha}  \tag{14}\\
& +\frac{\left[\left(b_{1} \alpha-\alpha^{2}-b_{o}\right)+K\left(a_{1} \alpha-\alpha^{2}-a_{o}\right)\right] s}{2 \alpha(s+\alpha)} \\
& Y_{2} \\
& =\frac{(1-K)}{2} s+\frac{b_{o}+K a_{o}}{2 \alpha}  \tag{15}\\
& +\frac{\left[\left(b_{1} \alpha-\alpha^{2}-b_{o}\right)-K\left(a_{1} \alpha-\alpha^{2}-a_{o}\right)\right] s}{2 \alpha(s+\alpha)}
\end{align*}
$$

Both $Y_{1,2}$ to be RCDPAs, all the terms in $Y_{1,2} \geq 0$. Hence

$$
\begin{equation*}
K \leq 1, \frac{b_{1} \alpha-\alpha^{2}-b_{o}}{\alpha^{2}+a_{o}-a_{1} \alpha}, \frac{b_{1} \alpha-\alpha^{2}-b_{o}}{a_{1} \alpha-\alpha^{2}-a_{o}} \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\rightarrow \leq \min \left\{1, \frac{b_{1} \alpha-\alpha^{2}-b_{o}}{\alpha^{2}+a_{o}-a_{1} \alpha}\right\} \tag{17}
\end{equation*}
$$

Since, there are several possible locations of two zeros, to demonstrate the method, we take the case of an APF for which $a_{1}=-b_{1}, a_{o}=b_{o}$. Then eqns. (14) and (15) become, respectively,

$$
\begin{align*}
& Y_{1} \\
& =\frac{(1+K)}{2} s+\frac{(1+K) b_{o}}{2 \alpha}  \tag{18}\\
& +\frac{\left[\left(b_{1} \alpha-\alpha^{2}-b_{o}\right)-K\left(\alpha^{2}+b_{o}+b_{1} \alpha\right)\right] s}{2 \alpha(s+\alpha)}
\end{align*}
$$

$$
\begin{align*}
& Y_{2} \\
& =\frac{(1-K)}{2} s+\frac{(1-K) b_{o}}{2 \alpha}  \tag{19}\\
& +\frac{\left[\left(b_{1} \alpha-\alpha^{2}-b_{o}\right)+K\left(b_{1} \alpha+\alpha^{2}+b_{o}\right)\right] s}{2 \alpha(s+\alpha)}
\end{align*}
$$

## Example 1

Realize the APF

$$
\begin{equation*}
T(s)=K \frac{(s-1)(s-3)}{(s+1)(s+3)} \tag{20}
\end{equation*}
$$

Let $\alpha=2$. Then from eqn. (16), we get

$$
K \leq\left(\frac{1}{15}, 1\right)
$$

Choosing $K=1 / 15$, we get from eqns. (18) and (19),

$$
Y_{1}=\frac{8}{15} s+\frac{4}{5}
$$

and

$$
Y_{2}=\frac{7}{15} s+\frac{3.5}{5}+\frac{0.5 s}{s+2}
$$

Although the method yields a realization that uses only one active device, and all the capacitors grounded, no matching of the componenets; it requires too many components.

## Realiation of bilinear functions

Let the function be expressed as

$$
\begin{equation*}
T(s)=K \frac{(s-z)}{(s+p)} \tag{21}
\end{equation*}
$$

Choosing $Q(s)=1$ (there is only one pole on the negative real axis, we don't need $Q(s)$ at all), we


Fig. 3 Realization of $\mathbf{T}(\mathbf{s})$ given by eqn. (20)


Fig. 4 Realization of function given in (20).

$$
\begin{gather*}
\left(Y_{1}-Y_{2}\right)=K(s-z), \quad \text { and } \quad\left(Y_{1}+Y_{2}\right)  \tag{22}\\
=(s+p) .
\end{gather*}
$$

Solving we get

$$
\begin{align*}
Y_{1}= & \frac{1}{2}[(1+K) s+(p-K z)]  \tag{23}\\
Y_{2} & =\frac{1}{2}[(1-K) s+(p+K z)] \tag{24}
\end{align*}
$$

In general, $2 C$ and $2 R$ elements will be required. The number can be reduced by choosing suitable value for $K$.
Case 1: $z>p$
Obviously, for $Y_{1,2}$ to be RCDPAs, $K \leq 1$. Chooswing $K=1$ (this redeuces $1 C$ in $Y_{2}$ ). Equation (23)-(24) give

$$
\begin{equation*}
Y_{1}=s+\frac{p-z}{2}, \quad Y_{2}=\frac{p+z}{2} . \tag{25}
\end{equation*}
$$

Case 2: $z<p$
Obviously, for $Y_{1,2}$ to be RCDPAs, $K \leq p / z$. Choosing $K=p / z$ (this redeuces $1 R$ in $Y_{1}$ ), (23)-(24) give.

$$
\begin{equation*}
Y_{1}=\frac{1}{2}\left(1+\frac{p}{z}\right) s, \quad Y_{2}=\frac{1}{2}\left(1-\frac{p}{z}\right) s+p \tag{26}
\end{equation*}
$$

Case 3: $z=p$
Equation (25) or (26) give

$$
Y_{1}=s, \quad Y_{2}=p
$$

It is interesting to note that, in this case, we require only $1 C$ and $1 R$. The complete realization of $T(s)$ of eqn. (21) for this case is shown in Fig. 4.

## B. ZY method

Equation (2) can be expressed as

(d)

Fig. 5 Admissible pole-zero patterns

$$
\begin{equation*}
K \frac{N(s)}{D(s)}=\frac{1-Z_{1} Y_{2}}{1+Z_{1} Y_{2}} \tag{27}
\end{equation*}
$$

Solving, we get

$$
\begin{equation*}
Z_{1} Y_{2}=\frac{D(\mathrm{~s})-K N(s)}{D(\mathrm{~s})+K N(s)} \tag{28}
\end{equation*}
$$

If poles and zeros of $Z_{1} Y_{2}$ follows any of the patterns shown in Fig. 5, $Z_{1}$ and $Z_{2}$ can be identified as RC DPIs.

## Example 3

Consider the transfer function given by eqn. (20).
In this case,

$$
\begin{equation*}
Z_{1} Y_{2}=\frac{(1-K) s^{2}+4 s(1+K)+3(1-K)}{(1+K) s^{2}+4 s(1-K)+3(1+K)} \tag{29}
\end{equation*}
$$

To fit into any one pole-zero pattern of Fig. 5, both the zeros and poles must be negative real. This requires

$$
K\left\{\begin{array}{c}
\leq 1  \tag{30}\\
16(1+K)^{2} \geq 12(1-K)^{2} \\
16(1-K)^{2} \geq 12(1+K)^{2}
\end{array}\right.
$$

These relations require (considering the positive value)

$$
\begin{equation*}
-\frac{1-a}{1+a} \leq K \leq \frac{1-a}{1+a}=0.0718 \tag{31}
\end{equation*}
$$

where $a=\frac{\sqrt{3}}{2}$.
Choosing

$$
K=0.0718
$$

we get from (28)

$$
\begin{equation*}
Z_{1} Y_{2}==\frac{\{s+1.73\}^{2}}{\{(s+0.78)+(s+3.84)\}} \tag{32}
\end{equation*}
$$

Thus, the poles and zeros satisfy the pattern (d) in Fig. 5. Identifying

$$
Z_{1}=\frac{s+1.73}{s+0.78} \text { and } Y_{2}=\frac{s+1.73}{s+3.84}
$$

The dcomplete realization of trtanfer function is shown in Fig. 6. Minimum 6 elements ( $2 C, 4 R$ ) are required.

## Synthesis of bilinear functions



Fig. 6 . Realization of transfer function given by eqn. (12).

Let the function be expressed as

$$
\begin{equation*}
T(s)=K \frac{(s-z)}{(s+p)} \tag{33}
\end{equation*}
$$

From eqn. (28)

$$
\begin{gather*}
Z_{1} Y_{2}=\frac{(s+p)-K(s-z)}{(s+p)+K(s-z)}  \tag{34}\\
=\frac{(1-K) s+(p+K z)}{(1+K) s+(p-K z)} \tag{35}
\end{gather*}
$$

For pole and zero to be negativfe real,

$$
K \leq\left\{\begin{array}{l}
\frac{1}{z} \\
z
\end{array}\right.
$$

Case 1: $z<p$
In this case, choose $K=1$. Then

$$
\begin{equation*}
Z_{1}=\frac{(p+z)}{2 s+(p-z)}, \quad Y_{2}=1 \tag{36}
\end{equation*}
$$

Case 2: $z>p$
In this case, choose $K=p / z$. Then

$$
\begin{equation*}
Z_{1}=\frac{(z-p) s+2 p}{(z+p) s}, \quad Y_{2}=1 \tag{37}
\end{equation*}
$$

Case 1: $z=p$
In this case, choose $K=1$. Then

$$
\begin{equation*}
Z_{1}=\frac{p}{s}, \quad Y_{2}=1 \tag{38}
\end{equation*}
$$

Complete realization of transfer function is the same as given in Fig. 4. Thus, the APF requires (i) only one active device, (ii) $1 C$ and $1 R$, (iii) both $R$ and $C$ are grounded, (iv) gain constant $K=1$, (v) no matchg of compoenets, by both the methods. Therefore, this realization is the best amonst all those reported in the past, except that given by Jitedra Mohan.

## III. CONCLUSION

The circuit configuration suggested by Jitendra Mohan has been exploited to realize a wider range of functions. Two synthesis methods are given. Both yield the same relaizaiton for the first order APF. This filter is novel in the sense that it uses only one active device and minimum number of passive elements ( $1 C$ and $1 R$ ) and both are grounded. No matching of components is required. Moreover, the gain constant is also 1 . To the best of author's knowledge, this was not reported earlier except the one by Jitendra Mohan which has been systematicfally derived in this paper.

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