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A Simplified Method for Determining Settling Time

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Abstract: *A simplified method for determining settling time of electrical system is presented. Unlike in the method presented by Yildiz, we have not involved the clumsy matrix relations to obtain the CE.*

Keywords: *Settling time, stability, loop method, node method, characteristic equation*

I. INTRODUCTION

The time-constants (TCs) are important parameters in evaluating transient response and determining the duration of the transient-state. They are obtained from the eigenvalues or poles of the transfer function (TF) of the circuits. They are particularly useful in the design of feedback systems in which relative stability, dynamics and other response characteristics are important functions [1]-[6].

In general, TCs of a circuit are obtained from the state space formulation in the form of differential equations [7]-[10], but it has some restrictions in obtaining these equations. TCs can also be computed from nodal and mesh methods in which the system equations are algebraic in nature and are easily obtained. These methods provide a systematic framework in terms of algebraic equations. The TCs are related to the eigenvalues in time-domain and the poles of TFs in *s*-domain. Alternative method used to determine the TCs is based on obtaining the poles of TFs of the circuit.

Cochran and Grabel [11] calculated TCs associated with each reactive element under different combinations of shorting and opening of other reactive elements. A method for determination of TFs of RC circuits using a combination of the TCs and low frequency TFs under different combinations of shorting and opening of the capacitors is developed in [12]. The TF of a first-order system is determined using the extra element theorem Middlebrook [13].

The approach was generalized to *N* extra elements by Middlebrook, Vorperian and Lindal [14]. Haley [15] introduced a modification-decomposition (MD) method to compute poles and zeros of TF. A method for determining poles and zeroes of TFs of linear active circuit is described by Haley and Hurst [16]. Hauksdottir and Hjaltadottir [17] gave closed-form expressions, for real and/or complex eigenvalues of TFs responses. Hagiwara [18] used the eigenvalue approach to calculate the zeros of the system. In Hajimiri [19], the TFs of circuits are expressed in terms of time and TCs calculated under different combinations of shorted and opened inductors and capacitors.

A systematic and generalized method to compute the TCs of linear circuits from nodal and mesh analysis was given by Yildiz [20]. In [21][22], nodal and mesh analysis methods with virtual sources for some special cases in circuit analysis are used. However, it is shown in [23][24], that these virtual sources are not necessary at all.

Students and teachers are very much familiar to solve the electrical circuits by loop and node methods of analysis. However, it is not commonly known that the TCs of the system can be determined from the determinant *A* of the circuit [20].

II. METHODS OF ANALYSIS

A. Method of Loop Analysis

In an N-loop circuit, loop equations can be expressed as

$$\begin{bmatrix} z_{11} & z_{12} \cdots & z_{1N} \\ z_{21} & z_{22} \cdots & z_{2N} \\ \vdots & \vdots & \vdots \\ z_{N1} & z_{N2} \cdots & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (1)$$

where

z_{ii} = self impedance of loop i

z_{ij} = mutual loop impedance betweeen the loops i and j

I_i = Loop current in loop i

V_i = Sum of all the Loop voltages (dependent and independent) in loop i.

If there are dependent current sources, they all should be converted into dependent voltage sources before writing (1). All these dependent sources should be expressed in terms of the loop currents. Now separating the right-hand side into dependent and independent sources as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} + \begin{bmatrix} z_{11} & z_{12}\dots & z_{1N} \\ z_{21} & z_{22}\dots & z_{2N} \\ \vdots & \vdots \quad \dots & \vdots \\ z_{N1} & z_{N2} & z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \qquad (2)$$

Now (1) can be expressed as

$$A(s) \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} \qquad (3)$$

where

$$A(s) = \begin{bmatrix} Z_{11} & Z_{12}\dots & Z_{1N} \\ Z_{21} & Z_{22}\dots & Z_{2N} \\ \vdots & \vdots \quad \dots & \vdots \\ Z_{N1} & Z_{N2} & Z_{NN} \end{bmatrix} = \Delta_n \qquad (4)$$

and $Z_{ij} = z_{ij}$-z_{ij}.

Equation (3) can be solved for various loop currents. This is illustrated with Example 1.

Example 1: Consider the circuit shown in Figure 1(a) where loop currents are marked. Find I_o.

The equivalent circuit after removing the coupling is shown in Fig. 1(b). Note that there are two dependent voltage sources. The loop equations can be written as

$$\begin{bmatrix} R_1 + \frac{1}{sC} & -R_1 & -\frac{1}{sC} \\ -R_1 & R_1 + R_2 + sL_1 & -sL_1 \\ -\frac{1}{sC} & -sL_1 & sL_1 + sL_2 + \frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$= \begin{bmatrix} E_1 \\ E_2 - sMI_3 \\ -sMI_2 + 2sMI_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -sM \\ 0 & -sM & 2sM \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

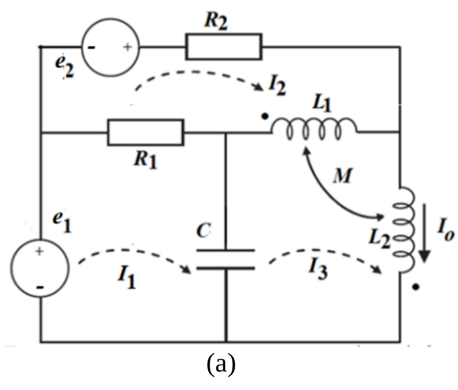
$$\rightarrow A(s) \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ 0 \end{bmatrix} \qquad (5)$$

where

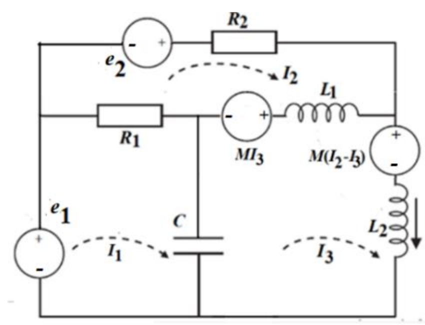
$A(s)$

$$= \begin{bmatrix} R_1 + \frac{1}{sC} & -R_1 & -\frac{1}{sC} \\ -R_1 & R_1 + R_2 + sL_1 & -sL_1 + sM \\ -\frac{1}{sC} & -sL_1 + sM & sL_1 + sL_2 + \frac{1}{sC} - \end{bmatrix} \qquad (6)$$

Substituting the values of components in (5),



(a)



(b)

Figure 1. (a) Circuit for Example 1, R_1 = 4 Ω, R_2 = 5 Ω, L_1 = 1 H, L_2 = 2 H, M = 0.5 H, C = 0.1 F, e_1 = 5 sin ωt, e_2 = 0 V, (b) Equivalent circuit

$$\begin{bmatrix} 4 + \frac{10}{s} & -4 & -\frac{10}{s} \\ -4 & 9 + s & -\frac{1}{2}s \\ -\frac{10}{s} & -\frac{1}{2}s & 2s + \frac{10}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1(s) \\ 0 \\ 0 \end{bmatrix} \qquad (7)$$

where $E_1(s)$ is the Laplace transform of 5 sin ωt

$$I_o = I_3 = \frac{\begin{bmatrix} 4 + \frac{10}{s} & -4 & \frac{5\omega}{s^2+\omega^2} \\ -4 & 9+s & 0 \\ -\frac{10}{s} & -\frac{1}{2}s & 0 \end{bmatrix}}{\begin{bmatrix} 4 + \frac{10}{s} & -4 & -\frac{10}{s} \\ -4 & 9 + s & -\frac{1}{2}s \\ -\frac{10}{s} & -\frac{1}{2}s & 2s + \frac{10}{s} \end{bmatrix}}.$$

$$= \frac{\left(\frac{5\omega}{s^2+\omega^2}\right)\frac{(4s^2+20s+180)}{2s}}{\frac{(14s^3+115s^2+360s+400)}{2s}} \qquad (8)$$

Note that the denominator does not depend on any of the voltage sources present in the circuit. Now let us take the determination of TF $H(s)$ of the system. In such a case there will be *one input source and one output response*. Without any loss of generality, let us assume that the single input source (E_1) belongs exclusively to loop 1, and the response current I_o is the current of N-th loop. Then

$$I_o = I_N = \frac{\begin{bmatrix} z_{11} & z_{12}\cdots & E_1(s) \\ z_{21} & z_{22}\dots & 0 \\ \vdots & \vdots & \\ z_{N1} & z_{N2}\dots & 0 \end{bmatrix}}{A(s)}$$

$$\frac{I_o}{E_1} = H(s) = \frac{\begin{bmatrix} Z_{21}\dots & Z_{2,N-1} \\ \vdots \quad \dots & \vdots \\ Z_{N1} & Z_{N,N-1} \end{bmatrix}}{A(s)} = \frac{\Delta_{n-1}}{\Delta_n}. \qquad (9)$$

$Z_{ij}(s)$ can be expressed in general terms

$$Z_{ij} = \left(R_{ij} + sL_{ij} + \frac{1}{sC_{ij}}\right). \qquad (10)$$

Therefore, each term in Δ_n will be product of such N terms. Hence

$$\Delta_n = \frac{N(s)}{D(s)} = \frac{\sum a_k s^k}{ms^l} \qquad (11)$$

where k and l are positive integers and a_k and m are positive numbers. Similar expression will be for Δ_{n-1}.
The location of the poles of $H(s)$ decides the T_s, i.e., the roots of $N(s) = \sum a_k s^k = 0$ (called the characteristic equation (CE) of the system).

Example 2: Consider again Example 1 and find the settling time (T_s) of the system.
Taking the value of L_1 = 4 H as given in [1], we do not get the same CE as given in therein However, if we take L_1 = 1 H, we get the same CE as shown below.
Here,

$$\Delta_3 = \frac{N_A}{D_A} = \frac{(14s^3 + 115s^2 + 360s + 400)}{2s} \quad (12)$$

CE is

$$14s^3 + 115s^2 + 360s + 400 = 0, \quad (13)$$

roots of which are

-2.85 ± j1.8, -2.5 (14)

TCs are

$$\tau_{1,2} = \left|\frac{1}{-2.85}\right| = 0.35 \text{ s}, \ \tau_3 = \left|\frac{1}{-2.5}\right| = 0.4 \text{ s}$$

$T_s = 5\tau_{\max} = 5 \times 0.4 = 2$ s.

B. Method of Node analysis

Similar to the method of loop analysis, one can develop the same for the node analysis. This is demonstrated with an example.

Example 3: Consider the circuit shown in Figure 2 with all the nodes identified.
Assuming ideal Op Amp, i.e.,

$$I_p = I_n = 0, V_2 - V_3 = 0, \quad (15)$$

node equations can be written as

$$\begin{bmatrix} G_1 + G_2 + sC_2 & -G_2 & 0 \\ -G_2 & G_2 + sC_1 & 0 \\ 0 & 0 & G_3 + G_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_iG_1 + V_4sC_2 \\ 0 \\ V_4G_3 \end{bmatrix} \quad (16)$$

Using the constrained relation of (15), we can modify (16) as

$$\begin{bmatrix} G_1 + G_2 + sC_2 & -G_2 & 0 & 0 \\ -G_2 & G_2 + sC_1 & 0 & 0 \\ 0 & 0 & G_3 + G_4 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_iG_1 + V_4sC_2 \\ 0 \\ V_4G_3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} V_iG_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & sC_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2+2s & -1 & 0 & V_i \\ -1 & 1+2s & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_iG_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

Since there are no inductors, we have
$Y_{ij} = (G_{ij} + sC_{ij})$.
All the determinants will, therefore, have no denominator (*i.e.*, denominator = 1).
Substituting the values,

$$\begin{bmatrix} 2+2s & -1 & 0 & -2s \\ -1 & 1+2s & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

Using Cramer's rule,

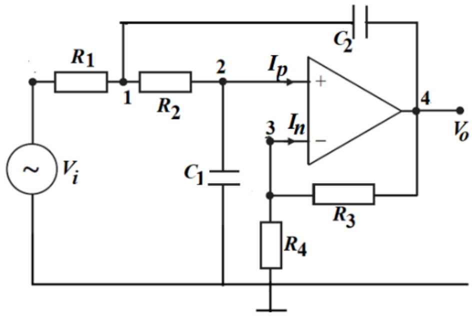


Figure 2 Circuit for example 3, $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$, $C_1 = C_2 = 2$ F.

$$V_4 = \frac{\begin{vmatrix} 2+2s & -1 & 0 & V_i(s) \\ -1 & 1+2s & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{vmatrix}}{\begin{bmatrix} 2+2s & -1 & 0 & -2s \\ -1 & 1+2s & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}} \quad (19)$$

$$\frac{V_4}{V_i} = H(s) = \frac{\begin{vmatrix} -1 & 1+2s & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{bmatrix} 2+2s & -1 & 0 & -2s \\ -1 & 1+2s & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}}$$

$$= \frac{2}{4s^2 + 2s + 1} \quad (20)$$

CE is

$$4s^2 + 2s + 1 = 0, \quad (21)$$

roots of which are

-0.25 ± j0.433. (22)

TCs are

$$\tau_{1,2} = \left|\frac{1}{-0.25}\right| = 4 \text{ s}. \quad (23)$$

$T_s = 5\tau_{\max} = 20$ s.	(24)

Example 4: Determine the T_s for the circuit shown in Fig. 3.

Applying KCL, we get

$$\begin{bmatrix} G_2 + sC_2 & -sC_2 & -G_2 \\ -sC_2 & s(C_1 + C_2) + G_3 & 0 \\ -G_2 & 0 & G_1 + G_2 + sC_3 \end{bmatrix} \begin{bmatrix} V_o \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ sC_1V_i(s) \\ G_1V_i(s) \end{bmatrix} \quad (25)$$

where $G_i = 1/R_i$.

Then

$$H(s) = \frac{\begin{vmatrix} 0 & -sC_2 & -G_2 \\ sC_1 & s(C_1 + C_2) + G_3 & 0 \\ G_1 & 0 & G_1 + G_2 + sC_3 \end{vmatrix}}{\begin{vmatrix} G_2 + sC_2 & -sC_2 & -G_2 \\ -sC_2 & s(C_1 + C_2) + G_3 & 0 \\ -G_2 & 0 & G_1 + G_2 + sC_3 \end{vmatrix}}$$

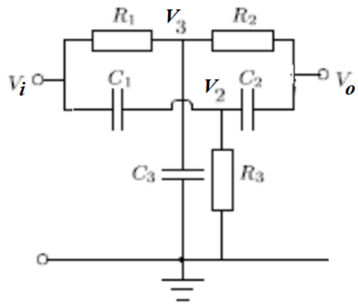


Figure 3. Circuit for Example 4.

$$= \frac{s^3 + \frac{G_1 + G_2}{C_3}s^2 + \frac{(C_1 + C_2)G_1G_2}{C_1C_2C_3}s + \frac{G_1G_2G_3}{C_1C_2C_3}}{s^3 + \left\{\frac{(C_1 + C_2)G_2}{C_1C_2} + \frac{G_1 + G_2}{C_3} + \frac{G_3}{C_1}\right\}s^2 + \left\{\frac{(C_1 + C_2)G_1G_2}{C_1C_2C_3} + \frac{(G_1 + G_2)G_3}{C_1C_3} + \frac{G_2G_3}{C_1C_2}\right\}s} \quad (26)$$

Let us assume

$$\frac{C_1 + C_2}{C_3} = \frac{G_3}{G_1 + G_2}. \quad (27)$$

Under this condition, (28) reduces to

$$H(s) = \frac{\left(s + \frac{G_1 + G_2}{C_3}\right)\left[s^2 + \frac{G_1G_2G_3}{(G_1 + G_2)C_1C_2}\right]}{\left(s + \frac{G_1 + G_2}{C_3}\right)\left[s^2 + \left\{\frac{(C_1 + C_2)G_2}{C_1C_2} + \frac{G_3}{C_1}\right\}s + \frac{G_1G_2G_3}{(G_1 + G_2)C_1C_2}\right]} \quad (28)$$

Let

$$R_1 = R_2 = 2R_3 = 1;\ C_1 = C_2 = \frac{1}{2}C_3 = 1. \quad (29)$$

These values satisfy the condition of (27). Sub-stituting the values in (28), we get

$$H(s) = \frac{(s+1)[s^2+1]}{(s+1)[s^2+4s+1]}. \quad (30)$$

CE is

$$(s+1)[s^2+4s+1] = 0.$$

The roots are

$s_{1,2,3}$= -0.27, -1, -3.73.

Therefore

$$\tau_{max} = \left|\frac{1}{-0.27}\right| = 3.704 \text{ s} \quad (31)$$

C. Cancellation of Pole and Zero

There are two possibilities: (i) Pole and zero of $H(s)$ may not cancel as in Examples 1 and 2, and (ii) there may be cancellation of only one real pole and real zero under some specific condition as in Example 4. Under this condition, one has to know which pole has cancelled with corresponding zero. This will require the knowledge of the numerator of $H(s)$. However, it can be overcome as follows. Let this pole be $s_k = -\sigma_k$, k increases from right to left starting from s = 0. There are two possible cases: (a) s_k is not the first pole, ($k \neq 1$), (b) s_k is the first pole (k = 1). In case (a), corresponding TC will be less than the largest value. Hence this will not decide the T_s and, therefore, can be ignored. In case (b), the TC will be the largest, and will fix up $T_s = 5(1/\sigma_1)$. In practice, the condition of pole-zero cancellation may not be satisfied exactly, and, therefore, we may assume that there is no pole zero cancellation. Now T_s will be greater than that ignoring σ_1 because of cancellation. However, one is assured that this will result into a longer T_s, but safer.

III. CONCLUSION

A simplified method for determining T_s is presented. Though we have restricted to the electrical system, it can be applied to any one which has a system equation similar as (3). Unlike in [1], we have not involved the matrix relations to obtain the CE.

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