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Easier Node Analysis of Circuits with Non-convertible Voltage Sources

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Abstract: *An easier method of solving the circuits, when non-convertible voltage sources are present, by node analysis is presented. It is independent of whether the current through or voltage across the resistance in series with an ideal voltage source is the controlling variable for any controlled source in the circuit.*

Keywords: *Node analysis, non-convertible voltage sources, source transformation, Controlled sources.*

I. INTRODUCTION

Chatzarakis and Tortoreli [1] have defined two types of non-convertible voltage (NCV) sources: Type 1 is a voltage source (independent or dependent) that *does not have series resistance*, and Type 2 is a voltage source (independent or dependent) which *has a series resistance, but the current through or voltage across this resistance controls the dependent source in the circuit.* They have replaced such sources with virtual current sources so that node equations can be written. However, in the next step, the node equations corresponding to these current sources have been removed by the known relations of the controlling variables. This means it is not necessary to replace the voltage sources with current sources at all. Moreover, one should not include the equations corresponding to the controlled variables as they are known in terms of other node voltages. We suggest the modified method to overcome these drawbacks and also an alternative but the easier method of analysis based on already known principles. The latter method is independent of the second type of NCV source.

In section 2, the method proposed in [1] is described shortly, followed by the method proposed in Section 3. Both the methods are illustrated with examples. Section 4 concludes the findings of this paper.

II. CHATZARAKIS AND TORTORELI METHOD

In [1], four cases are dealt with.

Case a: Circuits which have no voltage sources and only convertible current sources. This is well-known and will not be considered anymore here.

Case b: Planar or nonplanar circuits, with independent (current or/and voltage) sources but with at least one NCV source.

The steps involved in the method [1] are the following.

Step 1: NCV sources are replaced by the *virtual current Sources*, with values equal to the corresponding current values flowing through the non-convertible voltage sources of the given circuit.

Step 2: All the n nodes are identified, out of which one node is taken as a reference node. The voltages of other nodes are defined with respect to the reference node.

Step 3: The KCL equations are written in matrix form. Assuming the virtual current sources, help in writing the KCL equations.

Step 4: The virtual currents are now eliminated from the KCL equations by using constraints on the node voltages and/or by solving the equations which involve the virtual current sources. Therefore, it is a useless effort to first assuming the virtual current sources and then eliminating them.

Step 5: These equations are solved simultaneously for all h node voltages.

Step 6: The voltages across and currents through all the elements are calculated.

Example 1: Find all the node voltages in the circuit shown in Fig. 1(a) [1] and current I_x by node analysis. Nodes are identified with numbers in circles on the figure [1].

$R_1 = 4$, $R_2 = 2$, $R_3 = 2$, $R_4 = 8$, $V = 10$, $I_1 = 2$, $I_2 = 5$, $I_3 = 4$. The circuit has V as a Type 1 NCV source.

This example is solved in [1] by replacing the V NCV source with a virtual current source i_x. The authors have chosen 4 nodes in addition to a reference node. Therefore, they have to solve the following 4 KCL equations [1].

$$\begin{bmatrix} 0.75 & -0.5 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 0.625 & -0.125 \\ 0 & 0 & -0.125 & 0.125 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 2 - i_x \\ 5 \\ i_x \\ -4 \end{bmatrix}$$

There are two KCL equations involving i_x. The equation corresponding to the first row has been replaced by the constraint between the node voltages $V_3 - V_1 = V = 10$ V

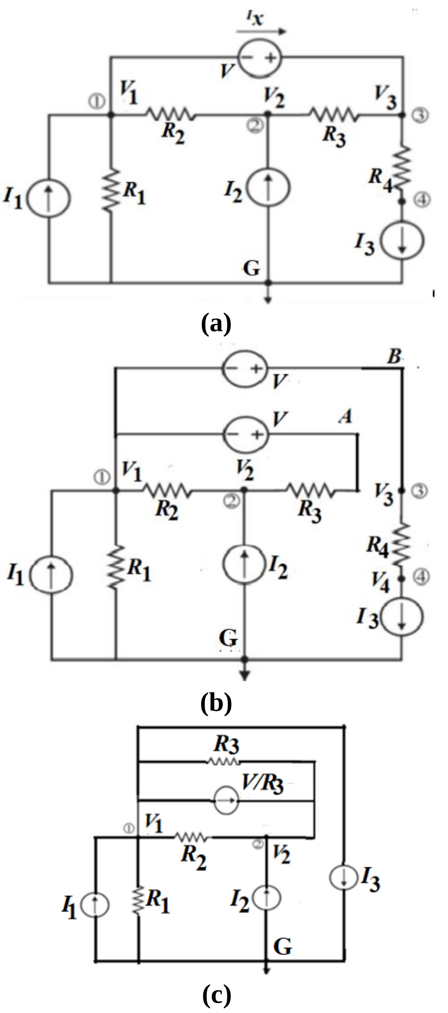


Figure 1. (a) Given circuit with nodes marked, (b) Circuit after breaking the node 3, (c) Equivalent circuit

The equation corresponding to the third row has been manipulated by solving equations of 1st row and 3rd row. This is an additional calculation step. Now the equations become

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & 0 & -0.125 & 0.125 \\ 0.75 & -1 & 0.625 & -0.125 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ -4 \\ 2 \end{bmatrix}$$

The equations are reordered [why? It is not clear]. Solution of these equations give

$V_1 = 12$ V, $V_2 = 22$ V, $V_3 = 22$ V, $V_4 = -10$ V.

III. PROPOSED METHOD

Our method differs from that in [1] as follows.

1. We choose the total number of nodes (n) given by.[2]
n = Number of branches (b) – number of loops (l) +1. Therefore, assuming one of the nodes as a reference node, the number of unknown node voltages are

$$n - 1 = b - l$$

2. We do not assume virtual current sources at all. In fact, we convert the NCV sources into CV sources by breaking the nodes [2] and then write KCL equations. Therefore, there is no need to eliminate these virtual current sources from the KCL equations. This is illustrated by solving the same example by our method.

1) Taking node G as reference node, we need to write the KCL equations for nodes $b - l = 7 - 4 = 3$. Noe that node 4 is not an essential node.
2) The V becomes a CV source if we break node 3 into two nodes, A and B, as shown in Fig. 1(b) [2]. The upper V source has an R_4 series resistance, and the lower V source has R_3 series resistance. Both the sources are now CV sources.
3) The upper V source and R_4 series resistance become redundant being in series with an ideal I_3 current source, they are removed.
4) converting the voltage sources into equivalent current sources, the circuit reduces to that shown in Fig. 1(c).
5) The KCL equations are

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\left(\frac{1}{R_2}+\frac{1}{R_3}\right) \\ -\left(\frac{1}{R_2}+\frac{1}{R_3}\right) & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - \frac{V}{R_3} - I_3 \\ I_2 + \frac{V}{R_3} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{1}{4}+\frac{1}{2}+\frac{1}{2} & -\left(\frac{1}{2}+\frac{1}{2}\right) \\ -\left(\frac{1}{2}+\frac{1}{2}\right) & \frac{1}{2}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2-4-5 \\ 5+5 \end{bmatrix} \qquad (1)$$

From (1), we get

$V_1 = 12$ V, $V_2 = 22$ V. (2)

Once these node voltages are known, all other voltages and currents can easily be determined from Fig. 1(a) as follows.

$V_3 = V_1 + V = 12 + 10 = 22$ V (3)

the voltage across I_3 current source

$V_3 - R_4 I_4 = 22 - 8 \times 4 = -10$ V. (4)

By KCL at node 3,

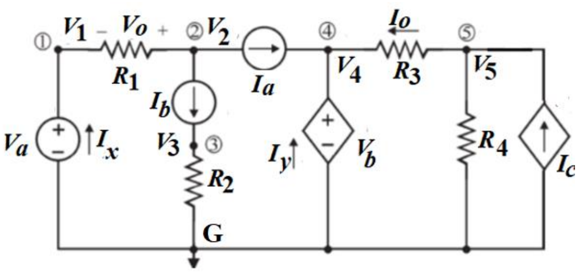
$i_x = (V_3 - V_2)/R_3 + I_3 = (22 - 22)/2 + 4 = 4$ A. (5)

Case (c): Circuits with dependent or independent current sources but all voltage sources are convertible types. This case is also well-known and will not be considered anymore here.

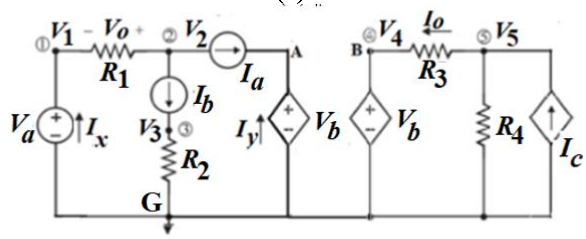
Case d: Circuits with dependent or independent current sources but at least one dependent or independent NCV source.

Example 2: Find all the node voltages and currents I_x and I_y in the circuit shown in Fig. 2(a). $R_1 = 20$, $R_2 = 10$, $R_3 = 15$, $R_4 = 5$, $V_a = 10$, $V_b = 0.5V_o$, $I_a = 0.5$, $I_b = 1$, $I_c = 3I_o$. The nodes are marked as in [1]. As per the authors of [1], dependent V_b source is Type 1 NCV source and independent V_a is a Type 2 NCV source.

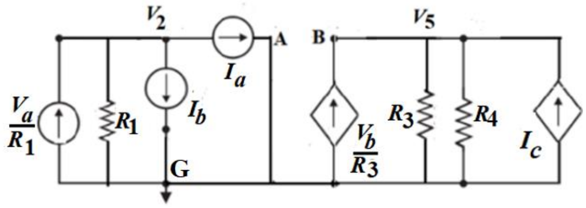
This is solved in [1] by assuming the virtual current sources and writing KCL equations, including at nodes 1 and 4, which are not necessary. Hence the order of the matrix is 5, greater than the minimum necessary 3.



(a)



(b)



(c)

Figure 2. (a) Given a circuit, (b) Circuit after breaking the node, (c) equivalent circuit

Solution by our method

The nodes are identified as 2, 4, and 5, and G as ground.

We have the following relations relating to controlled variables in terms of node voltages.

$$V_o = V_2 - V_a = V_2 - 10, \qquad (6)$$

$$I_o = (V_5 - V_4)/R_3 = (V_5 - V_4)/15. \qquad (7)$$

Breaking node 4 into two nodes A and B, the circuit becomes as shown in Fig. 2(b). Note that voltage source V_b at node A and the 10-Ω resistance become redundant. Removing these elements and converting voltage sources into current sources, the circuit reduces to that shown in Fig. 2(c). Note that the circuit has divided into two parts. Each has only one unknown voltage. Therefore, we need to solve just one node equation for each part which is extremely easier. Node equations are

$$\left[\frac{1}{R_2}\right][V_2] = \left[\frac{V_a}{R_1} - I_a - I_b\right]$$

$$= \left[\frac{1}{20}\right][V_2]\left[\frac{10}{20} - 0.5 - 1\right] \qquad (8)$$

$$(9)$$

$$\rightarrow\ V_2 = -20\ \text{V}.$$

$$\left[\frac{1}{R_3} + \frac{1}{R_4}\right][V_5] = \left[\frac{V_b}{R_3} + I_c\right]$$

$$\rightarrow \left[\frac{1}{15} + \frac{1}{5}\right][V_5]\ \ \left[\frac{0.5V_o}{15} + 3I_o\right]. \qquad (10)$$

Substituting for V_o and I_o from (7) and (8), and solving, we get

$$V_5 = 30\ \text{V}. \qquad (11)$$

Now from Fig. 2(a),

$V_1 = 10$ V, $V_3 = 1 \times 10 = 10$ V,

$$V_4 = 0.5V_o = -15\ \text{V}. \qquad (12)$$

Note that we need to solve only single node equations, which is easier than solving for 5 node equations in [1]. Unlike in [1], we do not care for the second type of NCV source (10 V).

$$I_x = \frac{V_1 - V_2}{R_1} = \frac{10 - (-20)}{20} = 1.5\ \text{A}. \qquad (13)$$

$$I_y = -I_a - I_o = -0.5 - \frac{V_5 - V_4}{R_3}$$

$$= -0.5 - \frac{30 + 15}{15} = -3.5\ \text{A} \qquad (14)$$

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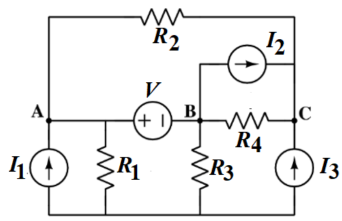
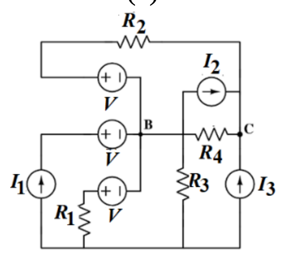
Example 3: Determine voltages of nodes A, B and C and current through 2 Ω in the circuit shown in Fig. 3(a) by node analysis. $R_1 = 2$, $R_2 = 4$, $R_3 = 3$, $R_4 = 4$, $V = 25$, $I_1 = 30.5$, $I_2 = 3$, $I_3 = 5$. This circuit has two current sources: I_2 is a converting type and I_3 is a non-converting type. The V is a floating NCV source.

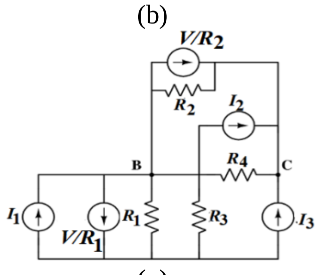
To convert the V source into a current source, first push the V source in three branches, as shown in Fig.



(a)



(b)



(c)

Figure 3. (a) Circuit for Example 3, (b) Pushing the 25-voltage source (c) Circuit after converting the voltage sources into current sources.

3(b). The V source in series with I_1 current source can be removed. Now converting all the voltage sources into current sources, the circuit becomes as shown in Fig. 3(c). Routine node method of analysis gives

$$V_B = \frac{\begin{bmatrix} I_1 - \frac{V}{R_1} - \frac{V}{R_2} - I_3 & -\frac{V}{R_1} - \frac{V}{R_2} \\ I_2 + I_3 + \frac{V}{R_2} & \frac{V}{R_1} + \frac{V}{R_2} \end{bmatrix}}{\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{V}{R_1} - \frac{V}{R_2} \\ -\frac{V}{R_1} - \frac{V}{R_2} & \frac{V}{R_1} + \frac{V}{R_2} \end{bmatrix}} \tag{15}$$

$$= \frac{\begin{bmatrix} 30.5 - \frac{25}{2} - \frac{25}{4} - 3 & -\frac{1}{4} - \frac{1}{4} \\ 3 + 5 + \frac{25}{4} & \frac{1}{4} + \frac{1}{4} \end{bmatrix}}{\begin{bmatrix} \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} & -\frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} - \frac{1}{4} & \frac{1}{4} + \frac{1}{4} \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} \frac{35}{4} & -\frac{1}{2} \\ \frac{57}{4} & \frac{1}{2} \end{bmatrix}}{\begin{bmatrix} \frac{4}{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}} = \frac{138}{5} \text{ V.} \tag{16}$$

and

$$V_C = \frac{\begin{bmatrix} \frac{4}{3} & \frac{35}{4} \\ -\frac{1}{2} & \frac{57}{4} \end{bmatrix}}{\begin{bmatrix} \frac{4}{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}} = \frac{561}{10} \text{ V.} \tag{17}$$

Thus

$V_A = V_B + 25 = 263/5$ V, $I = (263/5)/2 = 26.3$ A. (18)

Instead of pushing the voltage source on the left side, it can be done on the right side also.

IV. CONCLUSION

An easier method for node analysis of circuits that contain NCV sources has been outlined. Unlike the Chatzarakis and Tortoreli method [1], a smaller number of node equations are to be handled, and hence less calculation burden. Type 2 NCV sources are treated just like any other voltage source.

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