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*Original Article*

# Optimization of Thinned Circular Antenna Array Pattern using Dynamic Differential Evolution

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***Abstract*** **-** *Thinning array antennas have become one of the best approaches towards minimizing side lobes and achieving the required FNBW and SLL. Unlike all element arrays, thinning systematically removes some fraction of elements and activates the remaining element to obtain the intended pattern without degrading the performance of the antenna array. An evolutionary algorithm is gaining growth in providing better optimization solutions than other computational algorithms. In this research, Standard Differential Evolution (SDE) and Dynamic Differential Evolution (rDE) utilizing a range of binary mapping possibilities have been used to achieve thinning of Circular Antenna Array (CAA). Experimental result shows that D.E. with dynamic mutation factor technique has outperformed in fulfilling the desired radiation pattern for thinned CAA compared to all the variants of D.E. and also has exhibited finer solution compared to All Element Array (AEA) and Fire Fly Algorithm (FFA).*

***Keywords*** **-** *Antenna array, Circular array, Differential evolution, Side lobe level, Thinning.*

## 1. Introduction

The research in the field of antenna mainly concentrates on enhancing radiation patterns by altering the structure's geometry to minimize the side lobe level while maintaining the main beam gain. Several synthesis methodologies coexist, as the desired pattern varies substantially based on the application. SLL can be lowered using a thinning process by eliminating some elements in an equal-spaced array without degrading the antenna array performance. A few elements attached to the feed are kept 'on', and the elements attached to the matched load are made 'off'. Apart from lowering the side lobe levels, array thinning is also considered a cost-effective approach, as it reduces the array elements and the cost remarkably. Thinning can also be seen as a viable approach to balance lower SLL and narrow beams.

## 2. Related Work

Various synthesis methods use different optimization algorithms to get the desired radiation pattern. [1] has the thinning liner array antenna approach using a dynamic differential evolutionary algorithm, showing that Dynamic DE has outperformed different variants of the D.E. and Firefly algorithms. [2] and [3] discusses different methods for increasing the efficiency of an array aperture by interleaving a thinned linear array. Reduction of the Number of Elements by Matrix Pencil Method and Hybrid Wow/G.A. algorithm in a Linear Antenna Array and design concepts of a linear array antenna are elaborated in [4], [5], and[6], respectively. The use of FFA for CCA thinning has been described in [7, 8]. [9, 10] covers the issue of circular and concentric circular antenna arrays' DOA estimate approaches. [11] discusses the use of ecology-based Flower Pollination Algorithms (FPA) on antenna arrays instead of linear antenna arrays. To suppress the high side lobe level (SLL) with particular half power beam width (HPBW) of the concentric circular antenna array (CCAA) based on the cuckoo search (C.S.) algorithm has been proposed in [12].

The thinning of concentric circular arrays using the Galaxy Based Search Algorithm (GBSA) has been considered in [13]. A firefly-cuckoo search algorithm (FCSA) has been proposed in [14] to remedy the beam sample optimization hassle of the random antenna arrays (RAA). A deterministic approach to synthesizing a standard a-periodic arc or ring antenna array is presented in [15]. The proposed method, based on the auxiliary array pattern (AAP) idea, has aimed toward the semi-analytical determination of the most excellent excitation tapering and array detail angular distribution for mimicking a given radiation sample mask without resorting to any optimization technique. A hybrid approach to minimizing SLL for concentric circular arrays has been proposed in [16].

An improved version of the discrete cuckoo search algorithm has been used for thinning CCAA. In contrast, the cuckoo search-invasive weed algorithm (CSIWA) has been applied to optimize the thinned CCAA further.

The optimization of circular antenna arrays to obtain the lower value of SLL was done in [17] using a flower pollination algorithm inspired by the pollination practice of flowers. Concentric Circular Antenna Arrays (CCAA) can be thinned to minimize SLL while lightening their weight and cost.

Binary Salp Swarm Algorithm (BSSA), a swarm intelligence approach, has been used in [18-20] to synthesize thinner CCAA. For the Fifth Generation (5G) of wireless communication, a technique of radiation pattern synthesis has been presented in [21] for improved SLL reduction, directivity improvement, power waste reduction, and interference reduction. [22] has presented the Opposition-based Gravitational Search Algorithm (OGSA), a reliable search method used to boost the directivity and reduce the SLL of a non-uniform concentric circular antenna array (CCAA), considering mutual coupling.

In [23-25], An improved invasive weed optimization with random mutation and Lévy flight (IWORMLF) method has been presented to synthesize the beam patterns of linear antenna arrays (LAAs) and circular antenna arrays (CAAs) in order to solve the problem of beam pattern synthesis, which has been stated as an optimization problem[26].

## 3. Existing Model

### *3.1. Planar Circular Concentric Array Antenna (PCCAA)*

The Planar Circular Concentric Antenna Array (PCCAA), having M number of circular concentric rings, is illustrated in figure1.

Each $m^{th}$ ring with a radius of $r_m$ comprises $N_m$ isotropic components. Here m=1, 2,..., M. Eq.1 describes the radiation pattern in the free space region[7]

$$E(\theta,\emptyset) =$$

$$\sum_{m=1}^{M}\sum_{n=1}^{N_m} A_{mn}\ e^{jkr_m[sin\theta\ cos(\emptyset-\emptyset_{mn})-sin\theta_0\ cos(\emptyset_0-\emptyset_{mn})]} \quad (1)$$

Where,

$A_{mn}$ : In $m^{th}$ ring, excitation of $n^{th}$ element, $k = 2\pi/\lambda$ (wave number),

$r_m = {}^{N_m d_m}/_{2\pi}$ where $d_m$ is the spacing of elements of $m^{th}$ circle, θ= Zenith angle

∅= Azimuth angle.

The dB- Power pattern (normalized) can be defined as given in Eq.2

$$P(\theta,\emptyset) = 10\, log_{10}\left[\frac{|E(\theta,\emptyset)|}{|E(\theta,\emptyset)_{max}|}\right]^2 = 20\, log_{10}\left[\frac{|E(\theta,\emptyset)|}{|E(\theta,\emptyset)_{max}|}\right] \quad (2)$$

### *3.2. Design Considerations*

Thinning of the concentric circular array is done to get the necessary objectives with fewer array elements for Peak Side Lobe level (PSLL) and FNBW, assuming all the elements have a phase of $0^0$ with the same excitation. Eq. (3) expresses thinning, where the 'on' element is represented by one and the 'off' element is represented by 0

$$A_{mn} = \begin{cases} 1 & if m^{th} ring, n^{th} element is 'on' \\ 0 & if m^{th} ring, n^{th} element is 'off' \end{cases} \quad (3)$$

Objective function

$f$ : minimize {Total Number of array elements}

Subjected to constraints

$$\begin{cases} PSLL_{obtained} \le PSLL_{desired} \\ FNBW_{obtained} \le BW_{desired} \end{cases} \quad (4)$$

The objective function f specified in Eq.4 that must be minimized to attain the fewest possible array elements (NAE) and meet the specified PSLL and FNBW restrictions may be expressed mathematically, as stated in Eq.5.

$$F = K_1(|PSLL_d| - |PSLL_o|)H(T1) + K_2(FNBW_d - FNBW_o)H(T2) + NAE \quad (5)$$

Where $k_1$ and $k_2$ are the penalty constants and should consider as enormous values like 1000 or more. Eq.6 gives the values for the Heaviside step function parameters T1 and T2

$$T1=(|PSLL_d|-|PSLL_o|) \quad (1)$$
$$T2=(FNBW_d-FNBW_o) \quad (6)$$

Eq 7 defines the Heaviside step function H(T)

$$H(T) = \begin{cases} 0 & if\ \ T \le 0 \\ 1 & if\ \ T > 0 \end{cases} \quad (7)$$

## 4. Dynamic Differential Evolution (rDE)

Within evolutionary computation, Differential Evolution (D.E.) is considered the powerful entity in solutions to global optimization problems. The population of Standard DE (SDE) consists of N.P. individuals. According to the dimensions D present in the problem, each N.P. individual has a D-dimensional vector. D.E. uses mutations to produce a D-dimensional donor vector for each vector throughout one generation. A variety of methods can be used to define the donor vector. A method called D.E./rand/1 is used here, given in equation Eq. 8. As indicated in Eq. 9, the trial vector was created in a probabilistic setting using the crossover operator. A crossover operator C.R. represents the probability of generation of trial vectors from the mutant vector within a range [0, 1]. Index jrand is an integer of range [1, N.P.] chosen randomly. Then, using Eq. 10, a greedy selection process selects vectors for the next generation between the corresponding trial vectors and the target.

$$V_i^{(G)} = X_{r1}^{(G)} + F * \left(X_{r2}^{(G)} - X_{r3}^{(G)}\right) \tag{8}$$

$$u_{ij}^{(G)} = \begin{cases} v_{ij}^{(G)} & if\ \ rand(0,1) \le CR\ \ or\ j = j_{rand} \\ x_{ij}^{(G)} & otherwise \end{cases} \tag{9}$$

$$x_{ij}^{(G)} = \begin{cases} u_i^{(G)} & if\ \ f\left(u_i^{(G)}\right) \le f\left(x_i^{(G)}\right) \\ x_i^{(G)} & otherwise \end{cases} \tag{10}$$

SDE assigns a fixed value to the crossover rate and mutation factor. It is commonly known that the F value determines the success and the rate of convergence quality. Here, a dynamic mutation factor is employed instead of a fixed F value for all members in each cycle. A unique random number is created for each dimension using the uniform distribution range of [0 1]. Fig 2 shows the illustration, where, in the process of mutation vector generation of the i$^{th}$ member, each differential vector component $dv_{i,j}$ is multiplied with a unique random number $rF_{i,j.}$ Exploration of a better solution is possible with the available diversity in the random number. As random number variation exists, D.E. with this process is called rDE in this work.[1]

This research explores Standard DE(SDE)and Dynamic DE(rDE) utilizing a range of binary mapping possibilities. Six unique ways of obtaining CCA thinning have been employed, as indicated in Table 1. The performance analysis of rDE is explored by comparing it with all the variants of D.E. algorithms.

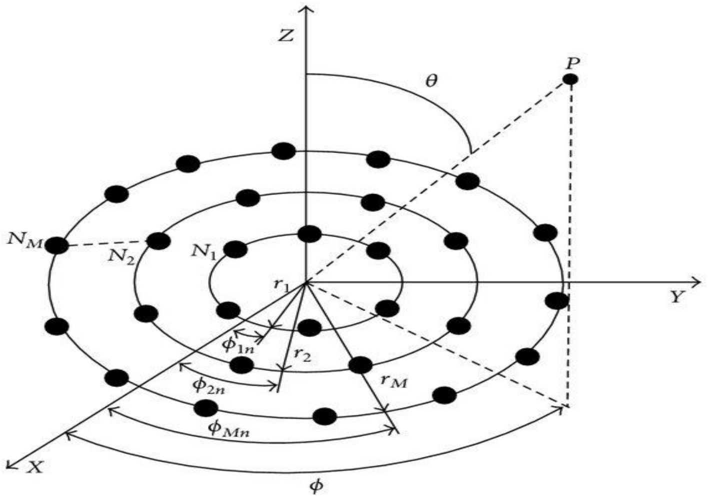

**Fig. 1 Planar circular concentric array antenna (PCCAA)**

## 5. Experimental Result

Uniform concentric two-ring arrays have been demonstrated through simulation experiments[27, 28]]. Table 2 displays the desired objectives and performance parameters for the scenario. A fixed λ/2 inter-element spacing has been assumed for simulation. The number of generations was 100, and the population was estimated at 20. All the considered algorithms underwent 20 independent trials to determine their statistical performances and allow for comparison. The MATLAB 2013a environment was used to create the entire simulation.

From the objective function value shown in Equation 4, the results have been achieved for the number of “on” array elements (NAE), PSLL, HPBW, FNBW, percentage of thinning, and point of convergence. No expected value of F and Cr exists for all the applications, as the performances of D.E. is highly influenced by F and Cr values. As a result, for two distinct sets of F=[0.5,0.9] and Cr=[0.8 0.5] values, the statistical results produced by different methods are displayed in Table 2.

Table 2 shows that the parameter values set [ F = 0.8; Cr =0.5] has demonstrated higher performance and hence has been considered for further experimentation on circular array thinning.

For the specified constraints of thinned radiation pattern for CCA as given in Table 3. The D.E. parameter Cr for rrDE has been examined and obtained mean performances over 20 independent trials are shown in Fig.3. It can be observed that a Cr value of 0.9 is the better choice in compare to 0.5, hence has been considered for the future part along with F value equal to 0.5. All six different algorithms as mention in the Table 1 are applied to get the solution for CCA thinning.

The mean objective function convergence characteristics have been shown in Fig. 4. Individual algorithm convergence over different trials is shown in Fig. 5. It can be observed that except rrDE, none of the algorithms has converged to the optimal solution in all trials.

At the same time, the performance of rrDE was appreciable. rrDE Performances were consistent and have shown significant reduction in array elements value in all trials while satisfying the desired value of SLL and FNB

The statistical performances obtained from all the algorithms show that with rrDE, the SSL values were -19.378dB along with the FNBW value of 11.368 $^0$ and obtained thinning is very close to 50%.

The convergence behaviour of different parameters in a trial with the rrDE algorithm has been presented in Fig. 6, where how fast rrDE can make the solution feasible can be observed.

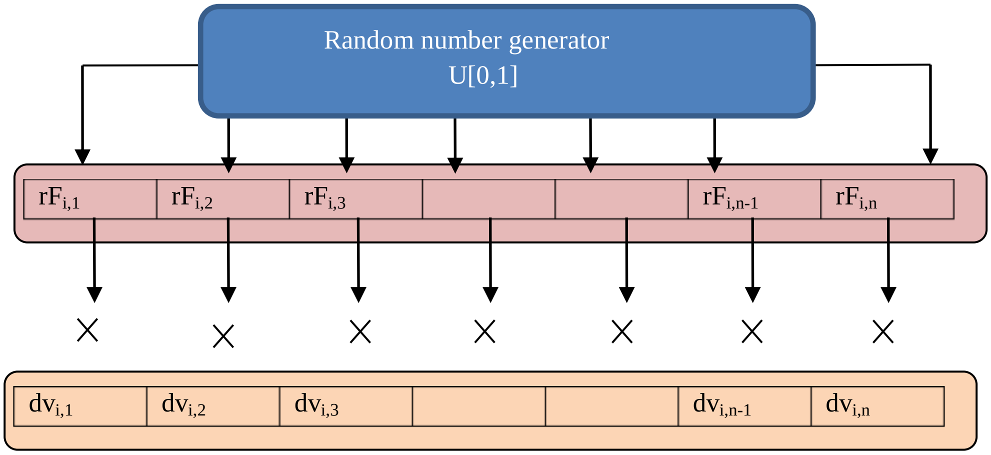

**Fig. 2 Dynamic mutation factor strategy in $rDE$**

**Table 1. Six different algorithms and their characteristics**

| Algorithm | rsDE | stSDE | spSDE | vpSDE | strDE | rrDE |
|---|---|---|---|---|---|---|
| Meta-heuristic | SDE | SDE | SDE | SDE | rDE | rDE |
| Transfer function | Rounding to discrete integer | Sigmoid | Sigmoid | Gaussian error function | Sigmoid | discrete integer rounding |
| Binarisation rule | Push to the binary boundary. | Threshold | Probabilistic | Probabilistic | Threshold | Push to the binary boundary |

**Table 2. Objectives and performances over 20 trials**

| Performance Parameter | | Objective Function | NAE | PSLL(dB) | η | HPBW | FNBW | Thinning (%) | NGC |
|---|---|---|---|---|---|---|---|---|---|
| stSDE | [F=0.5&Cr=0.9]<br>[F=0.8&Cr=0.5] | 916.95<br>40.626 | 41.55<br>39.95 | -19.128<br>**-20.131** | 0.46052<br>0.50416 | 1.1693<br>1.1922 | 2.6589<br>2.8709 | 16.9<br>20.1 | 25.6<br>58.8 |
| spSDE | | 2317<br>1238.1 | 38.45<br>40.15 | -17.721<br>**-18.802** | 0.4615<br>0.46881 | 1.175<br>1.1864 | 2.7334<br>2.7105 | 23.1<br>19.7 | 51.25<br>70.4 |
| eeDE | | 899.79<br>37.8 | 39.1<br>37.8 | -19.153<br>-20.16 | 0.49072<br>0.53391 | 1.2036<br>1.2896 | 2.785<br>3.0772 | 21.8<br>24.4 | 39.55<br>81.95 |

**Table 3. Desirable radiation pattern specifications**

| Antenna Array | NAE | PSLL (dB) | FNBW ($^0$) |
|---|---|---|---|
| CCA | 1st ring : 35 ; 2nd ring: 70 | -19 | 12 |

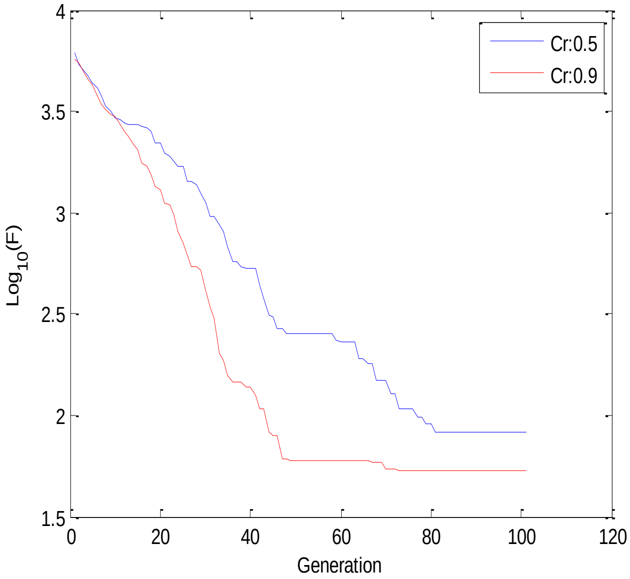

**Fig. 3 Comparative convergence characteristics of rrDE with different cr values in CCA**

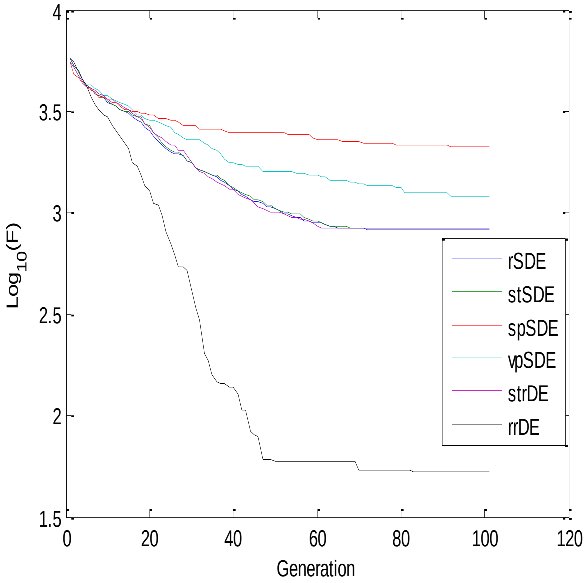

**Fig. 4 Mean convergence comparison for CCA thinning**

Fig.7 shows the obtained radiation pattern by the best solution of different algorithms, where the fulfilment of needed objectives in the pattern by rrDE appeared very easily. rrDE has also been compared with the All Element Array (AEA) and the existing Fire Fly Algorithm(FFA).

Table 4 gives the CCA-thinned array element status by the different algorithms, and the corresponding performances have presented in Table 5[29].

It is clear that with AEA, a very high value of SLL -12.453 appeared with 105 elements array while FFA has delivered-18.388dB with a thinning of 34.286 %. Of course, there is a little extra benefit in delivering the better FNBW value of 10.222 by AEA against rrDE, which has given the FNBW of 11.368, which still satisfies the requirement of specification very well.

At the same time, there is a massive reduction in array elements and just 48 elements to provide the the-19.438dB of SLL value. The obtained radiation patterns by different evolutionary algorithms are shown in Fig. 8, where the better performances of rrDE against others can observe quickly and clearly.

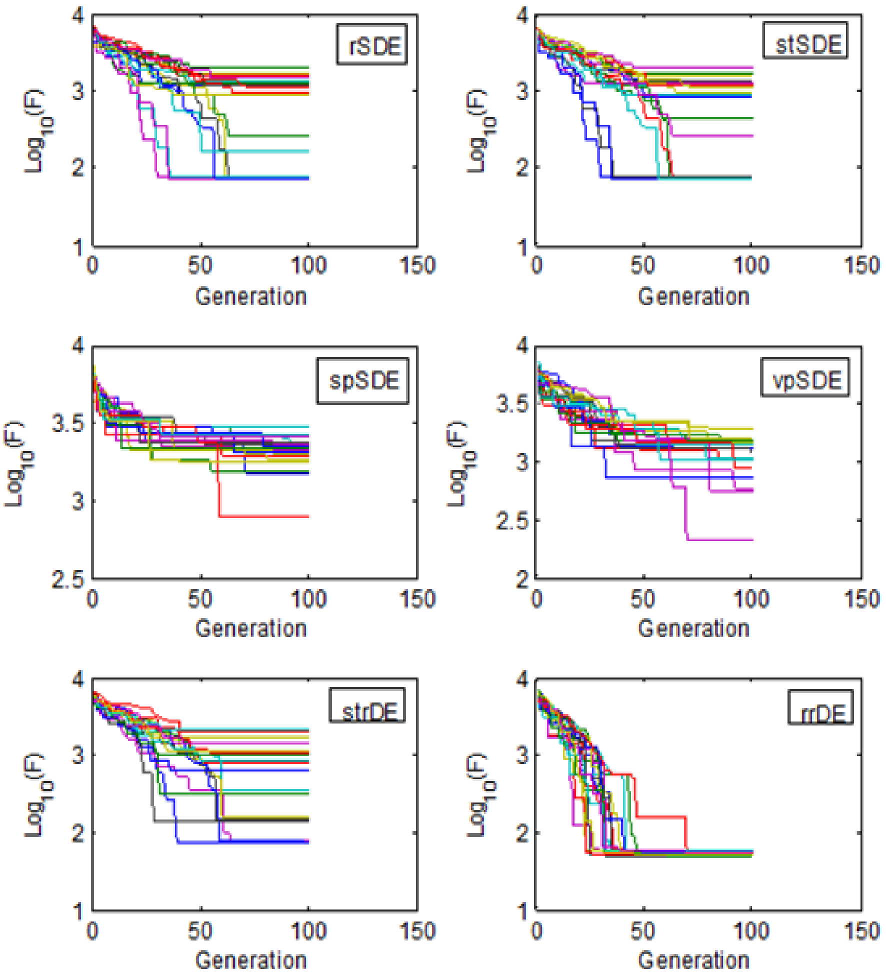

**Fig. 5 Individual algorithm convergence over different trials for thinning CCA**

**Table 4. CCA-thinned array element status of three different algorithms**

| Algorithms | Ring No. | Elements of CAA with Conditions 'on'=1 and 'off'= 0 |
|---|---|---|
| AEA | 1 | 11111111111111111111111111111111111 |
|  | 2 | 1111111111111111111111111111111111111111111111111111111111111111<br>1111111 |
| FFA | 1 | 01011111111010110000001101101111010 |
|  | 2 | 1101010111010011111011111101100101101001011111011111111011011111<br>0101001 |
| rrDE | 1 | 01011101101100010000101110001010000 |
|  | 2 | 0000100011010101011011001000000000001100111111110001111100111110<br>0010100 |

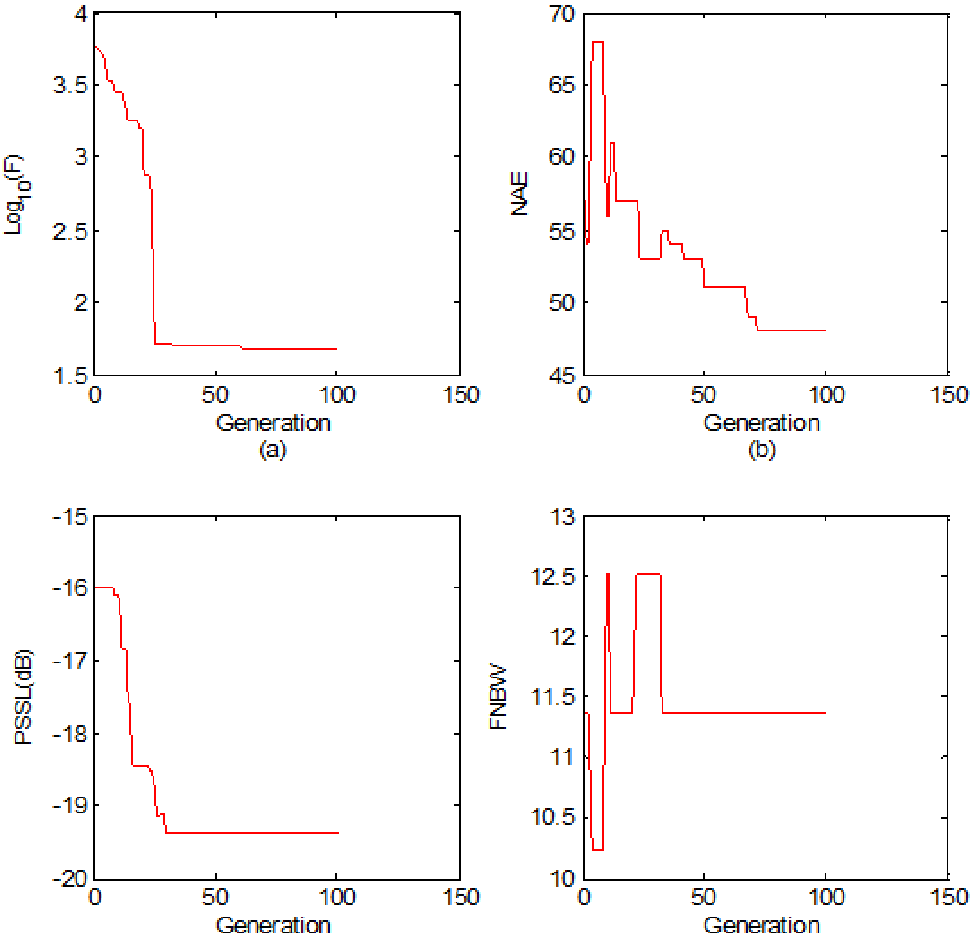

**Fig. 6 convergence behavior of different parameters with rrDE algorithm (a) Objective function (b) PSLL (c) NAE (d) FNBW**

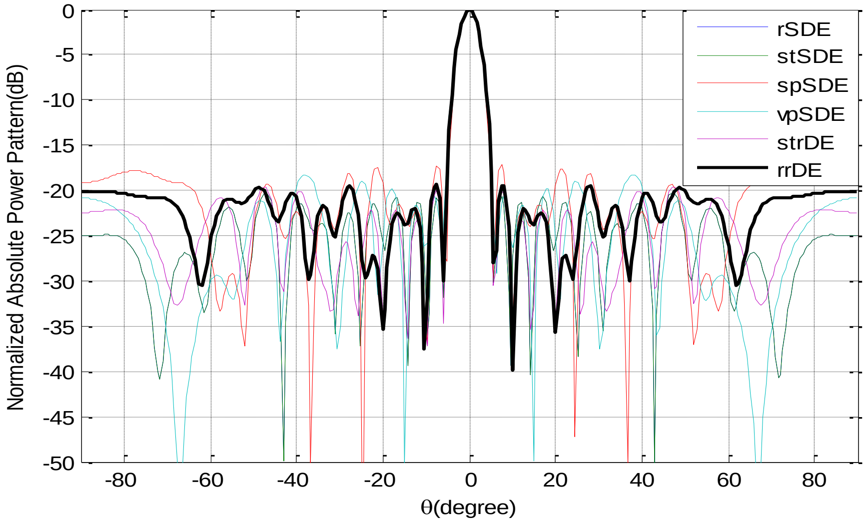

**Fig. 7 CCA radiation pattern using different D.E. variants**

**Table 5. Comparison of rrDE performances against AEA and FFA**

| Trial | NAE | PSLL(dB) | η | HPBW | FNBW | Thinning(%) |
|---|---|---|---|---|---|---|
| AEA | 105 | -12.453 | 0.1186 | 4.4924 | 10.222 | 0 |
| FFA | 69 | -18.388 | 0.26649 | 5.6383 | 11.368 | 34.286 |
| rrDE | 48 | -19.438 | 0.40495 | 5.6383 | 11.368 | 54.286 |

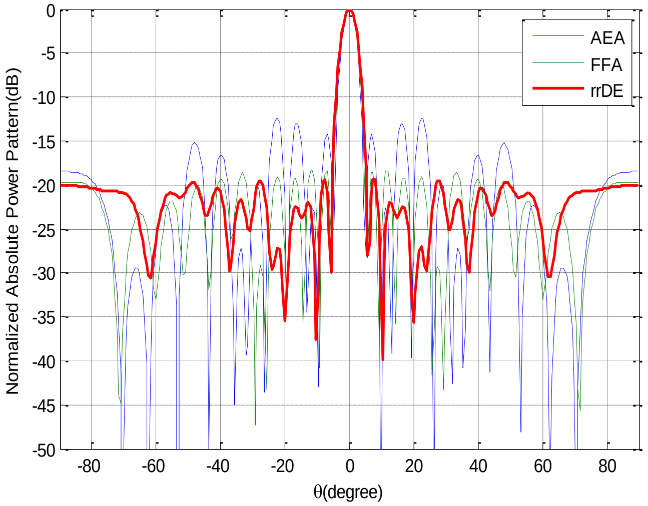

**Fig. 8 Comparison of CAA radiation pattern of rrDE with AEA and FFA**

## 6. Conclusion

Thinning of the array has always been the best approach towards achieving the desired radiation pattern with minimum side lobe level. Application of Differential Evolution on CCA to obtain thinning and detailed examination along with different variants of D.E. shows that the dynamic mutation factor technique used in D.E. has boosted its capability to generate a superior solution in achieving thinning of CAA compared to SDE and also shows that it has easily fulfilled the desired radiation pattern specifications for CCA with more % of thinning compared to that of AEA and FFA.

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