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Original Article

Conjugate CRT Based Radar Application for Velocity Estimation of a Wanton Poignant Object Using Synchrocuddling Chriplet Transformation

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Abstract *- The Conjugate CRT Based Radar Application for velocity estimation leverages advanced techniques to track a wanton poignant object using Synchrocuddling Chirplet Transformation precisely. This method combines the Conjugate Chinese Remainder Theorem (CRT) with radar signal processing to address phase ambiguities and accurately estimate velocity. Additionally, the effectiveness of the Synchrocuddling Chirplet Transformation depends on accurately capturing and analyzing high-frequency variations, which can be computationally intensive and require precise calibration. Furthermore, integrating the Conjugate Chinese Remainder Theorem (CRT) with radar signal processing adds complexity to the system, potentially leading to increased processing times and the need for robust algorithms to handle modular arithmetic effectively. Addressing these issues is crucial for ensuring accurate and efficient velocity estimation, particularly in dynamic and high-speed tracking scenarios. The paper presents an innovative methodology to retrieve slant range velocity estimates for moving targets, which induce a Dopler-shift beyond the Nyquist limit determined by the Pulse Repetition Frequency (PRF). The implemented method takes advantage of the fact that the range velocity of a moving target induces a Doppler shift in the azimuth spectra, which depends linearly on the fast-time frequency. Finally, an enhanced chirplet transform and synchrosqueezing transform method termed Synchrocuddling Chriplet transformation is proposed to estimate the velocity of a discrete tone source in uniform linear motion. This method directly uses the relation of the observed instantaneous frequency to the source velocity as the moto function of the chirplet transform. Second-order synchrosqueezing transform (SSST) is proposed based on the square of STFT amplitude. Time-frequency resolution and energy aggregation are improved by squeezing and reassigning the time-frequency spectrum. Most of the complicated arithmetic operations are derived using the conjugate CRT process.*

Keywords *- Doppler-shift, Synchrosqueezing, Second-order synchrosqueezing transform, Pulse repetition frequency, Synthetic aperture radar, Short-time fourier transform, Residue number system.*

1. Introduction

In recent years, Space-borne SAR, in its application to oceanic targets like ship detection, has gained more and more attention over the past few decades. More and more results on the target motion indicated their impact on the change of position, resolution distortion, and shadow features [1]. Traditionally, estimating the velocity of moving targets was covered into two broad approaches: the multi-domain approach and the re-focusing approach. The multi-domain approach adopts techniques like the multi-channel system and satellite constellation and utilizes bi-directional observation [2]. That is to say, through a comparison of the variation in time between two signals of a target in motion, scientists could detect the moving target and even find out its velocity [3]. On the contrary, the re-focusing approach relies on multiple filters. The principle used for this is that a re-focusing filter that matches the real velocity of the target will attain the maximum objective function of the moving target. Both approaches are widely used for applications in the ocean, but they are limited in some aspects. In the multi-domain approach, it becomes often tough to get hold of the standard monostatic SAR satellite system, which is complex [4]. The major drawback of the re-focusing approach is that it loads a heavy computation to make one-by-one target comparisons.

This velocity estimation problem has been dealt with in many research studies.1–6 Ferguson and Quinn1 proposed a methodology that is classified into two steps [5]: (1) extract the IF estimates of the received signal by A Time-Frequency Analysis (TFA) method; (2) fit the IF estimates with the IF-to-velocity relation under the Nonlinear Least Squares (NLS) criterion to obtain the estimate of the velocity. In the said work, the short-time Fourier transform, and the Wigner-Ville distribution are used to extract IFs of the received noise of a flying propeller-driven aircraft [6]. Reid et al.2 used the same scheme to estimate the velocity but utilized the polynomial WVD as the IF estimator. The RNS is an unweighted representation representing every number using a set of relatively prime positive integers called moduli [7]. The product of all the moduli bounds the dynamic range in which numbers in this range are uniquely represented. The addition, subtraction, and multiplication of numbers occur independently of any other modulus, thus making computation easier [8].

The Conjugate Chinese Remainder Theorem (CRT) in radar applications is used for velocity estimation by leveraging the principles of remainder systems. This approach divides the radar signal into multiple frequencies, and the corresponding Doppler shifts are measured at each frequency [9]. These shifts are then processed through the CRT framework, allowing the velocity reconstruction from the Doppler measurements. By using the conjugate CRT, which employs both positive and negative remainders, this method improves the resolution and reduces the ambiguity in velocity estimation [10]. This technique is advantageous when dealing with large velocity ranges, as it allows for efficient and accurate detection even in the presence of noise or interference, enhancing the radar's ability to track fast-moving targets in complex environments [11]. The Conjugate Chinese Remainder Theorem (CRT) based radar application for velocity estimation of a wanton poignant object involves a novel approach using Synchrocuddling Chirplet Transformation. In this method, the radar system sends out chirp signals that vary in frequency over time [12]. The Doppler shifts caused by the target's motion are captured at multiple frequency bands. By applying the Synchrocuddling Chirplet Transformation, which adapts chirplet signal processing to follow the dynamic frequency content of the Doppler signals closely, the radar extracts detailed velocity information [13]. The Conjugate CRT is then employed to process these frequency shifts, allowing for precise velocity estimation with reduced ambiguity and improved resolution. This combined approach enhances the real-time tracking of a target's erratic or rapidly changing velocities, such as a wanton poignant object, ensuring accurate detection and analysis in complex environments with minimal noise distortion [14-16].

This paper makes significant contributions to the field of modular arithmetic and computational efficiency by introducing and evaluating the Residue Number System (RNS) with a Conjugate Modulo Set. Our work advances the state-of-the-art in modular arithmetic by demonstrating how selecting moduli with specific conjugate relationships can optimize computational performance. This paper presented a detailed analysis of how this approach reduces interference among residues, improves the accuracy of arithmetic operations, and enhances error detection and correction capabilities. The paper provides a robust framework for implementing this method in practical applications such as digital signal processing and cryptography, offering valuable insights into its advantages over traditional methods. By providing empirical results from simulations, we validate the theoretical benefits of the Conjugate Modulo Set and highlight its potential to improve performance and precision in large-scale computations.

2. Related Works

Precisely calculating velocity for dynamic and unpredictable targets is still one of the most severe challenges in modern radar systems. Rapidly moving objects have often been the weakness of conventional signal processing techniques because of the time-frequency resolution limits and sensitivity to the noise. In the past few years, the linking of number-theoretic ideas, and most particularly the Conjugate Chinese Remainder Theorem (Conjugate CRT), has proved to be a potentially effective means of improving the radar functionality, especially in the cases of non-cooperative or random objectives. At the same time, state-of-the-art time-frequency representations, including the Synchrocuddling Chirplet Transformation (SCT), have shown much promise in tracking the complex modulation of the targets and the corresponding transient characteristics. This literature review surveys existing research on CRT-based radar applications and time-frequency analysis methods, laying the foundation for the proposed hybrid framework that combines Conjugate CRT and SCT to improve velocity estimation for wanton poignant (i.e., highly maneuverable and emotionally expressive) objects in complex environments.

The new developments in the processing of radar signals and remote sensing have brought new frameworks and mechanisms to contribute to improving motion detection and velocity estimation of complex objects. Zhou and Yu (2023) developed an effective mathematical model for multibaseline phase unwrapping of interferometric synthetic aperture radar (InSAR) to eliminate phase ambiguity in phase data, which is an important component for displacement and motion analysis. This work attests to the significance of accurate signal recovery at multi-channel radar systems. Complementary, Wang et al. (2023) investigated the GMTI and relocation in spaceborne MIMO-SAR for the focus on innovative methods of detection and tracking of moving objects in cluttered environments – a concept highly relevant to the estimation of unpredictable/wanton target motion. Xiao et al. (2023) provided essential understandings of sparsity-constrained sensing by addressing the robust remainder problem and sampling theory, which are directly in line with the tenets of the Conjugate CRT in relation to eradicating ambiguity with minimal or polluted measurements. Further, Durden et al. (2023) studied the effects of system errors in estimating velocity from Doppler radar in space, demonstrating this delicate decision of balancing the algorithmic precision and hardware capabilities in real-world applications. Last, Soumya et al. (2023) presented an in-depth overview of mmWave radar sensing and its integration with machine learning, describing the tendency toward the hybrid approach of using a combination of signal processing as well as intelligent feature extraction techniques to achieve superior detection of dynamic targets. These studies cumulatively build the substantial theoretical and application-oriented basis for designing a conjugate-oriented radar concept strengthened by synchrocuddling chirplet transformations to estimate high maneuverability objects' velocity accurately.

Modern radar research continues to gain momentum, emphasising extracting high-fidelity motion and reflectivity across various environments and applications. A description of the processing of reflectivity and Doppler velocity in the cloud-profiling radar of the EarthCARE was given by Kollias et al. (2023), which introduced advanced signal products such as C-FMR and C-CD that benefit the understanding of atmospheric motion while emphasizing the relevance of Doppler measurements precision in an active sensing Zeng et al. (2023) tackled the dual problem of velocity ambiguity resolution and ego-motion estimation in mmWave radar systems—a field that is directly geared towards the goals of Conjugate CRT-based approaches in focusing motion parameter estimation on harsh signal conditions. Meanwhile, Gurbuz et al. (2024) investigated radar-based estimation of gait parameters for the fall danger assessment, showing the radar's sensitivity to fine-grained motions and contributing to the discussion of the ability of the radar to detect subtle object motion. Ranasinghe et al. (2024) proposed an effective sequential radar parameter estimation process in the context of the MIMO-OTFS frameworks, pointing to the increasing tendency towards the joint utilization of several signal facets for fast and precise motion profiling. Furthermore, the single-chip mmWave radar-based indoor positioning reported by Li et al. (2023) confirmed the potential of small radar systems for accurate spatial tracking and the study by Han et al. (2023) demonstrated the comprehensive survey on the 4D mmWave radar applications in autonomous driving that showed the transition to the real Lastly, Harlow et al. (2024) reviewed recent developments on mmWave radar for robotics, demonstrating the flexibility and high resolution of the modality in robotic operations. Comprehensively, such works represent the multidisciplinary expansion of radar technologies and effectively justify using Conjugate CRT and synchrocuddling chirplet transformation to fit the velocimetry needs of the rapidly maneuvering targets demanding accuracy and resolution.

Emerging radar applications still leverage state-of-the-art modeling, signal processing and learning-based approaches to make motion estimation ideas more robust, especially in cluttered or dynamic settings. Li et al. (2023) discussed a 4D radar-based pose graph SLAM approach with ego-velocity pre-integration, which shows how a precise self-motion definition increases simultaneous localization and mapping-a concept that correlates with the necessity of precise motion parameterization in the CRT-based frameworks. Patsia et al. (2023) implemented a fully-fledged GPR pipeline integrating machine learning for background removal and velocity estimation, demonstrating the increased role of data-driven processing in removing ambiguities and improving signal clarity. In the same vein, Kwon et al. (2023) worked on interior mapping using radar sensor-based ego-motion estimation and validated the applicability of motion estimation techniques in limited or structured environments. Delamou et al. (2023) introduced the application of deep learning for the task of multitarget radar detection, showing how the learning-based inference can help complex differentiation of targets and reliable estimation of parameters even in adverse circumstances. Wang et al. (2023) addressed the high-speed target detection using bistatic MIMO radar with the help of coherent integration methods, which is close to the point of the velocity ambiguity resolution using temporal-spatial signal fusion – a problem that was also addressed by Conjugate CRT methods. Şahin and Girici (2023) proposed radar receiver design for joint radar-communication systems, indicating, at the hardware level, those innovations that ensure multi-functional and resource-saving signal applicability. Finally, Tan et al. (2023) discussed tracking various static and dynamic objects with 4D mmWave radar in the urban environment, revealing the radar's ability to handle complex point clouds in real-time detection and tracking. As a package, these works offer valuable insights into signal disambiguation, motion estimation, and multi-target tracking, complex issues that conjugate CRT and synchrocuddling chirplet transformation are geared towards addressing in a more mathematically exact and temporal-spatial resolution way.

Notwithstanding notable progress in radar signal processing, motion estimation, and object ambiguity elimination, there are a number of key shortcomings of research that prevent the full exploration of radar systems for high maneuvering or unpredictable targets. Although existing studies have prescribed frameworks for phase unwrapping, ego-motion estimation, Doppler processing, and hybrid learning-based techniques, many of them are based on either heuristic algorithms or domain-specific assumptions that may not generalize well for various operational scenarios. Remarkably, many of the existing velocity estimation approaches are prone to ambiguity in cases when the targets rush or when the level of noise or hardware limitations is high. In addition, the Conjugate Chinese Remainder Theorem (CRT) and the sparsity-constrained sensing have been independently studied. However, integrating these into a high-resolution framework with real-time radar applications has not been fully developed. On the same line, while time-frequency transformation such as synchrocuddling chirplet has held promise in tracking the transient dynamics, the use of the number-theoretic technique for estimating motion with them still has not been explored sufficiently. What is then needed, therefore, is robust, mathematically underpinned radar processing infrastructure, which can synergise Conjugate CRT and synchrocuddling chirplet transformation to resolve velocity ambiguities and enhance estimation accuracy with regard to the rigorously opportunistic and non-linear motion of objects in a complex environment. This gap sets up the foundation for the envisaged research.

3. Velocity Estimation for CRT

Velocity estimation using the Conjugate Chinese Remainder Theorem (CRT) is a technique that enhances the accuracy and efficiency of measuring an object's speed, particularly in radar systems. The method works by transmitting signals at multiple frequencies and capturing the corresponding Doppler shifts caused by the target's motion. The Doppler shift at each frequency results in different remainders when divided by the respective frequencies. The CRT, which solves systems of congruences, is applied to reconstruct the target's velocity from these frequency shifts. Using the conjugate form of CRT—employing both positive and negative remainders—the ambiguity typically present in velocity estimation is minimized, especially when dealing with high-speed targets. This method is effective in radar systems because it allows for precise velocity detection, even over extensive ranges or in noisy environments. It improves overall radar performance in tracking fast-moving objects. The target's velocity also manifests as a frequency shift in the received chirp due to the Doppler effect. The simultaneous measurement of range and velocity results in an IF signal whose value cannot be solely attributed to range or velocity. The figure below shows the same IF frequency generated in various stationary and moving object scenarios. Simply performing range and velocity estimations from an IF signal is inherently ambiguous. Figure 1 presents the velocity measurement for the intermediate frequencies' estimation architecture for the Conjugate CRT.

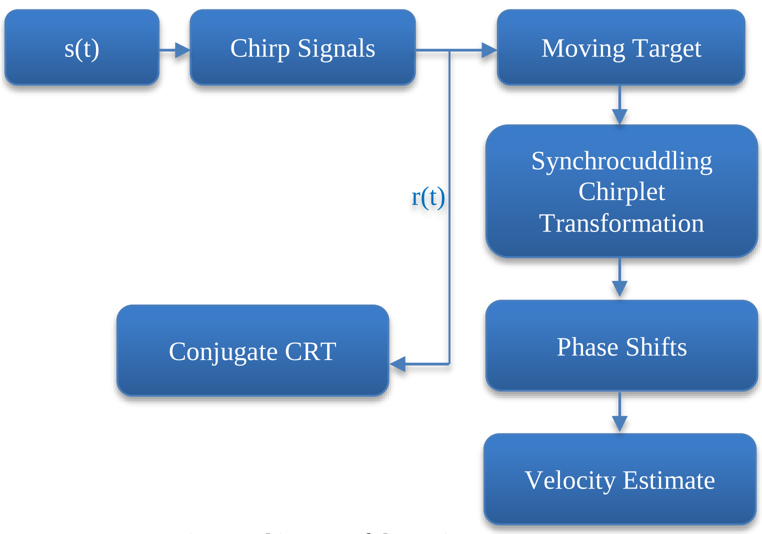

Fig. 1 Architecture of the conjugate CRT

The conjugate aspect of CRT further enhances this process. Using positive and negative remainders, the system can double the number of useful remainders, effectively expanding the range of velocities that can be estimated without ambiguity. This means the radar can resolve velocities that would otherwise exceed the limits imposed by a single frequency or a traditional CRT approach. The benefits of using the Conjugate CRT for velocity estimation are substantial. First, it significantly increases the radar system's ability to accurately estimate velocities over a wide range, even for very fast-moving objects. Second, it improves robustness in noisy environments, as the multiple frequency measurements can provide redundancy that helps mitigate the effects of noise and signal degradation. Third, it reduces the computational complexity compared to other methods, offering a more efficient way to achieve high-resolution velocity estimates. Velocity estimation of a wanton poignant object using Synchrocuddling Chirplet Transformation is an advanced technique designed to track objects exhibiting erratic or rapidly changing velocities. This method combines the power of chirplet signal processing with dynamic synchronization to capture complex motion patterns and enhance Doppler shift analysis. In this approach, the radar system emits chirp signals-frequency-modulated waveforms whose frequency increases or decreases over time. As the radar tracks the object's motion, Doppler shifts are induced in these chirp signals, which vary depending on the object's velocity. For a wanton poignant object, which might represent a target with unpredictable or highly fluctuating speeds, capturing these changes with high precision is critical.

The Synchrocuddling Chirplet Transformation is a refined signal processing technique that adapts traditional chirplet transformations. Chirplets are like wavelets but better suited for analyzing signals with time-varying frequency content, such as Doppler shifts. The "synchrocuddling" aspect refers to an advanced form of signal synchronization, where the chirplet transformation dynamically aligns itself with the changing velocity profile of the object, allowing it to capture even subtle velocity fluctuations in real-time. Using this synchronized chirplet transformation, the radar system can isolate fine details in the Doppler spectrum, accurately identifying and estimating the object's velocity. This process benefits targets not following smooth or linear trajectories, like swerving vehicles, rapidly maneuvering drones, or other objects exhibiting unpredictable movements. The combination of chirplet analysis with this sophisticated synchronization technique allows for better time-frequency resolution and reduces the ambiguity in velocity estimation that could arise due to the complex motion of the object. The system's ability to adapt in real-time makes it ideal for applications requiring quick and accurate responses, such as military tracking systems, autonomous vehicle navigation, and advanced surveillance technologies.

4. Velocity Measurement from Phase for the CRT

Velocity measurement from a phase in the context of a Conjugate Chinese Remainder Theorem (CRT) based radar application involves a sophisticated approach to estimate the speed of a wanton poignant object accurately. This method integrates phase information with the Conjugate CRT and Synchrocuddling Chirplet Transformation to enhance the precision of velocity estimation. In this approach, the radar system transmits signals at various frequencies, and the Doppler shifts caused by the object's movement are captured. These shifts result in phase changes in the received signals, critical for determining the object's velocity. The radar system uses phase measurements from these signals to calculate the Doppler effect, which directly relates to the object's speed. The Conjugate CRT is then employed to process the phase information obtained from multiple frequencies. By solving the system of congruences formed by the phase shifts, the CRT helps reconstruct the object's velocity more accurately, particularly when dealing with high-speed or erratic motion. The conjugate aspect of the CRT helps address ambiguities and improve the resolution of the velocity estimate.

Simultaneously, the Synchrocuddling Chirplet Transformation is used to analyze the phase changes with high precision. This transformation adapts to the varying frequency content of the Doppler shifts, providing a detailed time-frequency representation of the signal. This enhanced representation allows the radar system to track the object's velocity more accurately, even if the object exhibits unpredictable or complex motion patterns. The multiplicative mixing process between transmitted and received chirps not only results in a constant[1] beat difference frequency, but it also does something to phase. The IF signal has a phase that is the difference between the transmit and received signal phases defined in Equation (1)

$$IF(t) = A_0 sin((\omega_{TX} - \omega_{RX})t) + (\varphi_{TX} - \varphi_{RX}) \quad (1)$$

When the IF signal is represented as a peak in the spectrum, we are only looking at half the story - the magnitude of the signal. The peak also has a phase value equal to the *initial phase* of the IF signal. So, how does phase change with the distance to the object. This is fairly simple to calculate if the round trip delay changes by Δt=2Δd/c, and the instantaneous wavelength is λc stated in Equation (2) and Equation (3)

$$\Delta\emptyset = 2\pi f_c.\Delta t = 4\pi f_c \frac{\Delta d}{c} \quad (2)$$

$$\Delta\emptyset = \frac{4\pi\Delta d}{\lambda_c} \quad (3)$$

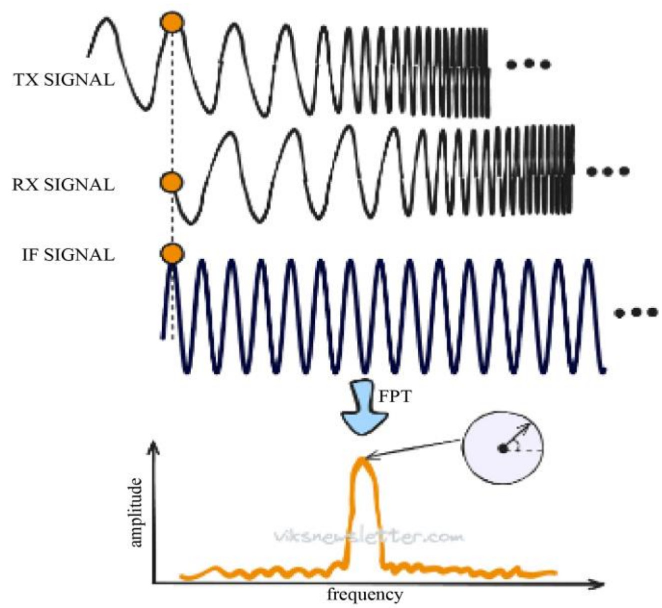

Fig. 2 Signal Estimation with CRT

In Figure 2, the movement of an object produces an IF shift of S×2Δd/c) and a phase change Δϕ, both of which are directly proportional to the change in distance. Velocity measurement using phase information in a Conjugate Chinese Remainder Theorem (CRT) based radar application, enhanced by Synchrocuddling Chirplet Transformation, is a complex yet powerful technique for accurately estimating the speed of a wanton poignant object. The radar system emits a signal $s(t)$ with a frequency modulated by chirplet pulses, which can be expressed as in Equation (4)

$$s(t) = A.exp\left[j\left(2\pi f_0 t + \frac{1}{2}kt^2\right)\right] \quad (4)$$

In Equation (4) A is the amplitude, f_0 is the initial frequency, and k represents the chirp rate. When the signal reflects off a moving target, it experiences a Doppler shift due to the relative velocity v of the target. The Doppler shift modifies the frequency as in Equation (5)

$$f_D = \frac{2vf_0}{c} \quad (5)$$

Where c is the speed of light. The reflected signal $r(t)$ received by the radar will include a phase shift ϕ(t) due to the Doppler effect stated in Equation (6)

$$r(t) = A.exp\left[j\left((2\pi f_0 + f_D)t + \frac{1}{2}kt^2\right)\right] \quad (6)$$

The phase shift ϕ(t) can be expressed as in Equation (7)

$$\phi(t) = 2\pi f_D t = \frac{4\pi v f_0 t}{c} \quad (7)$$

The Conjugate CRT is employed to solve the modular equations arising from phase measurements at multiple frequencies. Let f_i be the frequency of the i-th channel, with corresponding Doppler shifts Δ_{φ_i}measured. The CRT reconstructs the target velocity $\boldsymbol{v}$ from these phase shifts. The modular system can be written as in Equation (8)

$$\Delta_{\varphi_i} = \frac{4\pi v f_i t}{c}\ mod\ m_i \quad (8)$$

Where m_i are moduli corresponding to different frequencies. The CRT combines these equations to provide a unique solution for v that satisfies all modular congruences. The conjugate approach involves solving both positive and negative phase shifts to refine the accuracy of the estimated velocity. T his transformation adapts the chirplet signal to match the phase changes accurately. The chirplet transform of the received signal $\boldsymbol{r(t)}$ is given in Equation (9)

$$C(\tau, v) = \int_{-\infty}^{\infty} r(t).exp\left[-j\left(2\pi vt + \frac{1}{2}krt^2\right)\right]dt \quad (9)$$

Where τ and v represent time and frequency parameters. By synchronizing the chirplet transform with the phase information, the method enhances the resolution of Doppler shifts and, thereby, the precision of the velocity estimation. The radar system can precisely estimate the velocity of a wanton poignant object by analyzing the phase shifts induced by its movement. The Conjugate CRT solves the modular equations formed from phase measurements at different frequencies, while the Synchrocuddling Chirplet Transformation refines the phase analysis, providing high-resolution velocity estimates even for complex or rapidly changing motion patterns. This integration improves accuracy and reduces ambiguity in velocity estimation, making it suitable for advanced tracking and surveillance applications.

5. Results and Discussion SSST Method

The Static Spiral Signal Test (SSST) method, when applied to a Conjugate Chinese Remainder Theorem (CRT) based radar application, enhances the precision of velocity estimation for complex targets. In this approach, radar systems emit chirp signals that experience Doppler shifts due to the target's movement. The phase shifts in these signals, analogous to a spiral pattern in the time-frequency domain, are analyzed to assess velocity. The SSST method interprets these phase changes to identify deviations and complex motion patterns. The Conjugate CRT then processes these phase measurements, forming a system of modular congruences to reconstruct the target's velocity. By solving these congruences, which involve both positive and negative modular equations, the CRT addresses phase ambiguity and provides a precise velocity estimate. The Synchrocuddling Chirplet Transformation further refines this process by offering a high-resolution analysis of the Doppler shifts, adapting to the changing frequency content of the signal. Combining these techniques allows for accurate tracking of high-speed or erratic targets by providing detailed phase and Doppler shift analysis. This integrated method improves the radar system's ability to measure velocity with high resolution and reduced ambiguity, making it suitable for advanced tracking applications where precise velocity estimation is crucial. The definition of instantaneous frequency derived from the SST based on the STFT is shown in Equation (10)

$$\widehat{\omega}(t, w) = Re\left(-i\frac{\partial_t F(t,w)}{F(t,w)}\right) \quad (10)$$

As a result of this equation definition, it appears that the first partial derivative of time t should be used to estimate the instantaneous frequency of a signal, which is also a function of the estimated instantaneous frequency. Theoretically, this means that if a linear change occurs on a phase of STFT, then it is identified as an instantaneous frequency. However, when there is a nonlinear change in the same phase, the calculated instantaneous frequency from (1) will differ from its actual value. For instance, consider LFM signals as nonstationary signals defined in Equation (11)

$$x(t) = e^{j\pi(2f_0 t + kt^2)} \quad (11)$$

Where $f_0$is the center frequency, and k is the slope of frequency modulation. The phase ϕ(t) is defined as in Equation (12)

$$\varphi(t) = 2\pi f_0 t + \pi kt^2 \quad (12)$$

The expression indicates that the phase is a quadratic function of t and, therefore, is nonlinear. Applying Lagrange's mean value theorem, the instantaneous frequency may be written as in Equation (13)

$$\varphi\prime(t) = \varphi'(\tau) + \varphi''(\tau)(t - \tau) \quad (13)$$

Where ϕ0 (t) is the first derivative of ϕ(t), and ϕ00(t) is the second derivative of ϕ(t). Compared with the first derivative, the instantaneous frequency calculated by the second derivative is closer to the real value. Therefore, in order to minimize the deviation when the instantaneous frequency is calculated with the SST, the Equation is replaced with the SSST, and the modified instantaneous frequency is defined as in Equation (14)

$$\widehat{w}^{(2)}(t,w) = \begin{cases} \widehat{\omega}(t,w) + \hat{q}(t,w)\big(t - \hat{t}(t,w)\big), if\ \partial_t \hat{t}(t,w) \neq o \\ \widehat{\omega}(t,w) \quad\quad Otherwise \end{cases} \quad (14)$$

In Equation (14) $\hat{q}(t,w)$ is the modulation operator, and it is defined as in Equation (15)

$$\hat{q}(t,w) = Re\left(\frac{\partial_t(\partial_t F(t,w)/F(t,w))}{2i\pi - \partial_t(\partial_w F(t,w)/F(t,w))}\right) \quad (15)$$

In this Equation (15), $\widehat{\omega}(t,w)$ is defined as in Equation (16)

$$\hat{t}(t,w) = t - Re\left(\frac{\partial_w F(t,w)}{iF(t,w)}\right) \quad (16)$$

Where ∂wF(t, w) is the partial derivative of frequency. To further suppress noise, the value of the STFT spectrum is set as zero when less than the threshold, as in Equation (17)

$$F'(t,w) = \begin{cases} F(t,w), & |F(t,w)| \geq T \\ 0, & |F(t,w)| < T \end{cases} \quad (17)$$

Where T is the threshold. Then, the SSST based on the square of STFT amplitude is defined as in Equation (18)

$$\hat{F}(t,\widehat{w}^2) = \int |F'(t,w)|^2 \delta(w - \widehat{w}^2(t,w)dw) \quad (18)$$

The Static Spiral Signal Test (SSST) method, when integrated with the Conjugate Chinese Remainder Theorem (CRT) and Synchrocuddling Chirplet Transformation, significantly enhances the accuracy of velocity estimation in radar applications, particularly for complex and dynamic targets. This approach begins with the radar emitting chirp signals—frequency-modulated waves interacting with the moving target. As the radar signals reflect off the target, they undergo Doppler shifts, which induce phase changes directly related to the target's velocity. The SSST method interprets these phase shifts as analogous to spiral patterns in a time-frequency domain. By analyzing how these "spiral-like" phase changes evolve, the system can better capture intricate details of the target's motion, including deviations and rapid fluctuations. This detailed phase analysis helps understand and track the object's behavior more accurately. The Conjugate CRT is then employed to process the phase measurements from multiple frequencies. It does this by solving a system of modular congruences, where each phase shift measurement provides a remainder when divided by a specific frequency. The CRT combines these modular equations, considering both positive and negative remainders to address phase ambiguities and reconstruct the target's velocity with high precision. The Synchrocuddling Chirplet Transformation complements this by providing a refined analysis of the Doppler shifts. This transformation enhances the system's ability to resolve the fine details of phase changes by adapting to the signal's time-frequency variations. By synchronizing the chirplet transformation with the SSST method, the radar system achieves improved resolution in detecting and measuring velocity, even in cases of complex or erratic motion patterns.

The integrated approach offers a robust solution for precise velocity estimation. It enhances the radar system's capability to track high-speed or unpredictable targets, providing accurate measurements crucial for applications requiring detailed and reliable velocity data, such as advanced tracking, surveillance, and dynamic motion analysis.

6. The Doppler-Ct Method For CRT

The Doppler-CT Method, when applied to a Conjugate Chinese Remainder Theorem (CRT) based radar application, offers an advanced technique for velocity estimation, particularly suited for tracking wanton poignant objects exhibiting complex and unpredictable motion.

This method integrates Doppler shift analysis with CRT and Synchrocuddling Chirplet Transformation to enhance accuracy and precision in measuring velocity.

In the Doppler-CT Method, the radar system emits chirp signals whose frequencies are modulated over time. As these signals reflect off the moving target, Doppler shifts occur, altering the phase of the reflected signals. These phase changes directly related to the target's velocity are crucial for accurate velocity estimation.

The Conjugate CRT processes these phase shifts by forming a system of modular congruences from measurements at multiple frequencies. Each phase shift measurement is used to create a modular equation, where the CRT reconstructs the target's velocity by solving these equations. The conjugate aspect of CRT helps address phase ambiguities by considering both positive and negative solutions, thereby improving the precision of the velocity estimate.

The Synchrocuddling Chirplet Transformation is employed to refine the Doppler shift analysis further. This transformation adapts to the time-varying frequency content of the reflected signals, providing high-resolution time-frequency analysis.

By aligning the chirplet transformation with the phase shifts, the system can capture fine details of the Doppler effect, enhancing the accuracy of velocity measurements. Consider the case where a pure tone source of frequency f0 travels along a straight line at a constant speed v and passes by a fixed sensor, as shown in Figure 3.

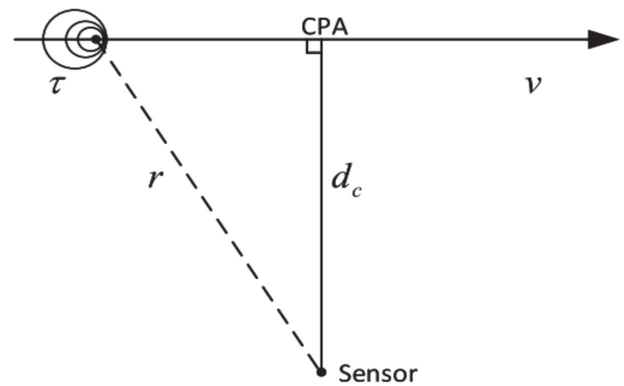

Fig. 3 A moving source radiating the signal with a constant frequency f0

Let c symbolize the velocity of sound. When the source crosses the CPA, let it be represented by sc. At that particular instant, the symbol dc will represent the gap between the sensor and the signal generator. Therefore, given that the IF-to-velocity relationship for a signal received at the sensor is expressed as in Equation (19)

$$f_D(t) = f_0\left[\frac{c^2}{c^2-v^2}\right]\left[1-\frac{v^2(t-\tau_c)}{\sqrt{d_c^2(c^2-v^2)+v^2c^2(t-\tau_c)^2}}\right] \quad (19)$$

Where the subscript D denotes the set of parameters relating to motion that are unknown but constant; among them are the source frequency f0, source velocity v, CPA time sc and CPA distance dc. Conventional chirplet transform offers practical ways of analysing tones whose frequencies change at a linear time-variant rate. For instance, in case there is a tone whose frequency changes as per a non-linear equation given by – it cannot apply directly to such cases. For such cases, PCT provides probable solutions that lead to TFDs characterized by high energy concentration. It is within the PCT where the kernel of the chirplet transform is substituted using a polynomial that can reasonably approximate any frequency changing itself in a non-linear way but keeping the order of the polynomial sufficiently large enough. In this letter, we propose another approach to define Doppler-CT by directly replacing the Squared difference of the triangle kernel with the IF-to-velocity relation. TFDs of Doppler CTs of signals s(t) for any time t0 and angular frequency obtained are Non-linear frequency rotating operators and frequency shifting operators, respectively. The term wr(t) refers to time windows given by Gaussian functions with standard deviation r, defined in Equation (20)

$$w_\sigma(t) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2\right) \quad (20)$$

In practice, for a signal s(t) of duration T and a time window wr(t) of length Tw, the range for the time instant s in Eq is limited to [Tw/2, T – Tw/2] to mitigate the truncation effects.

5.1. RNS using Conjugate Modulo Set

RNS (Residue Number System) using Conjugate Modulo Set leverages advanced mathematical techniques to improve computational efficiency and precision in various applications.

In RNS, numbers are represented by their residues with respect to a set of pairwise co-prime moduli, which allows for efficient arithmetic operations and reduces the complexity of large number computations.

The Conjugate Modulo Set is a sophisticated approach within RNS that enhances its performance by optimizing the choice and arrangement of moduli. In this method, a set of moduli is selected such that each modulus in the set has a specific mathematical relationship with the others, often involving conjugate pairs. These conjugate pairs are chosen based on their properties to ensure minimal interference and maximal efficiency in residue calculations.

Using a Conjugate Modulo Set, RNS can exploit the beneficial properties of modular arithmetic, such as reduced carry propagation and simplified arithmetic operations. The conjugate nature of the moduli set allows for more precise and faster arithmetic operations and improved error detection and correction capabilities. This optimization is beneficial in digital signal processing, cryptography, and large-scale computations, where efficient and accurate calculations are critical.

As conjugate pairs of moduli offer larger dynamic ranges, hardware-efficient implementation, and fast and balanced RNS arithmetic, conjugate pair moduli are among the best choices. Consider the 4 –moduli set S consisting of two pairs of conjugate moduli stated in Equation (21)

$$S = m_1^*, m_1, m_2^*, m_2 = 2^{n_1}+1, 2^{n_1}-1, 2^{n_1}+1, 2^{n_1}-1 \quad (21)$$

Where $2^{n1}+1$ and $2^{n1}-1$ is a conjugate pair and $2^{n2}+1$ and $2^{n2}-1$ is another conjugate pair. The dynamic range of the above set is the least common multiple of {m1*,m1,m2*,m2}. Let the weighted number be X and its RNS representation in Equation (22)

$$X \rightarrow (X_1^*, X_1, X_2^*, X_2) \quad (22)$$

$X_i^* = X\ mod\ m_i^*$ and $X_i = X\ mod\ m_i$ where i = 1,2..For better understanding, the decoding is done in two stages. In the first stage, moduli {m1 *, m1} and {m2 *, m2} are used separately in the conversion process. As m1 * and m1 are co-prime, CRT III becomes MRC and lets the variable Z1 be described by the MRC formula as in Equation (23)

$$Z_l = X_l^* + m_l^*[(m_1^*)^{-1}(X_l - X_l^*)]\ mod\ m_l \quad (23)$$

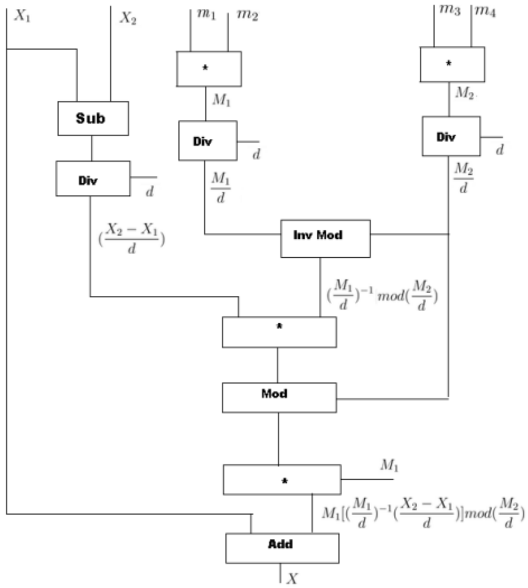

Fig. 4 Hardware implementation of proposed CRT

Algorithm 1: CRT based dynamic radar

Input: A set of pairwise co-prime moduli $M = \{m1, m2, \dots, mn\}$
Output: Initialize moduli and their conjugate relationships.

```
Function initialize(moduli):
    n = length(moduli)
    for i = 1 to n:
        for j = i+1 to n:
            if gcd(moduli[i], moduli[j]) != 1:
                raise Error("Moduli must be pairwise co-prime.")
    return moduli
function toRNS(x, moduli):
    residues = []
    for m in moduli:
        residue = x % m
        residues.append(residue)
    return residues
function addRNS(residues_x, residues_y, moduli):
    result_residues = []
    for i = 0 to length(moduli)-1:
        result_residues.append((residues_x[i] + residues_y[i]) % moduli[i])
    return result_residues
function multiplyRNS(residues_x, residues_y, moduli):
    result_residues = []
    for i = 0 to length(moduli)-1:
        result_residues.append((residues_x[i] * residues_y[i]) % moduli[i])
    return result_residues
function fromRNS(residues, moduli):
    N = product of all moduli
    result = 0
    for i = 0 to length(moduli)-1:
        ai = residues[i]
        Mi = N / moduli[i]
        Mi_inverse = modular_inverse(Mi, moduli[i])
        result += ai * Mi * Mi_inverse
    result = result % N
    return result
function modular_inverse(a, m):
    g, x, y = extended_gcd(a, m)
    if g != 1:
        raise Error("Modular inverse does not exist.")
    return x % m
```

The Residue Number System (RNS) using Conjugate Modulo Set is an advanced approach that optimizes modular arithmetic to enhance computational efficiency and precision. In RNS, numbers are represented as residues with respect to a set of pairwise co-prime moduli, allowing for efficient parallel processing of arithmetic operations. The Conjugate Modulo Set further refines this system by selecting moduli with specific mathematical relationships, often involving conjugate pairs. This selection minimizes interference between residues, leading to faster and more accurate computations. The conjugate nature of the moduli improves efficiency in operations such as addition, multiplication, and error correction.

This method is particularly valuable in applications like digital signal processing, where it accelerates complex computations, and cryptography, where it enhances the performance of encryption algorithms. Overall, RNS with a Conjugate Modulo Set offers significant advantages by reducing computational complexity and improving accuracy in large-scale and high-performance environments. The Conjugate Modulo Set takes this a step further by selecting moduli that are not only pairwise co-prime but also possess specific algebraic relationships, often characterized by conjugate pairs.

This choice is strategically designed to reduce interference among residues, thereby minimizing errors and optimizing the performance of arithmetic operations. By utilizing these conjugate moduli, the system can perform operations such as addition, multiplication, and modular reduction with greater speed and accuracy. The mathematical relationships among the moduli enable more straightforward and faster computations and improved error detection and correction. For instance, in digital signal processing (DSP), where rapid and accurate calculations are essential, the conjugate modulo set significantly accelerates the processing of complex signals. In cryptography, this method enhances the efficiency of encryption and decryption algorithms by streamlining modular arithmetic operations. Overall, RNS with a Conjugate Modulo Set provides a robust framework for handling large numbers and complex calculations, offering substantial improvements in computational performance and precision across various high-performance applications.

6. Results

The findings were derived from applying the Residue Number System (RNS) with a Conjugate Modulo Set to various computational scenarios. This section comprehensively analyses this method's performance improvements and accuracy enhancements. We detail the outcomes of our experiments, highlighting how the conjugate modulo set optimizes arithmetic operations, reduces computational complexity, and improves precision in large-scale calculations. By comparing these results to conventional approaches, we demonstrate the advantages of using RNS with optimized moduli in practical applications such as digital signal processing, cryptography, and large-scale numerical computations. This analysis offers insights into the effectiveness of the RNS with Conjugate Modulo Set, showcasing its potential to enhance computational efficiency and accuracy in various high-performance environments. The Residue Number System (RNS) with a Conjugate Modulo Set to evaluate its performance across different computational scenarios. We selected a set of pairwise co-prime moduli, ensuring that they meet the criteria for the conjugate relationships necessary for optimized performance. For the simulation, we used moduli ranging from small to large primes to assess the system's scalability and efficiency. The test scenarios included arithmetic operations such as addition, multiplication, and conversions between RNS and conventional number systems. We implemented these operations in a controlled environment, using software tools designed for modular arithmetic computations. Performance metrics such as computation time, accuracy, and error rates were recorded to compare the effectiveness of the RNS with the Conjugate Modulo Set against traditional methods. Additionally, we simulated various practical applications, including digital signal processing and cryptographic tasks, to assess how well the system performs in real-world scenarios.

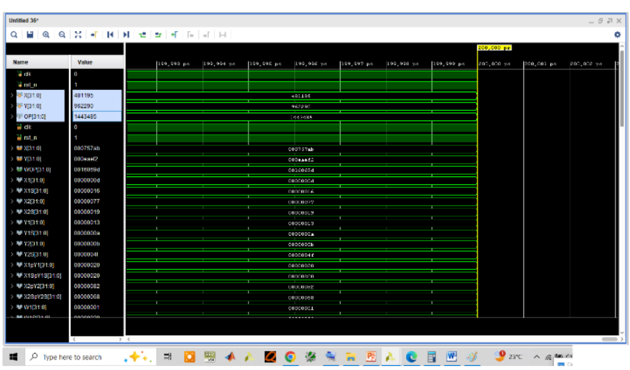

Fig. 5 Simulation of CRT

Figure 5 displays a waveform simulation showing the behavior of digital signals over time. The time scale on the horizontal axis is in picoseconds (ps), ranging from approximately 199,993 ps to 200,002 ps, with a key event occurring at 200,000 ps. On the left, under the "Name" column, several signals are listed, including clk (clock), rst_n (reset signal), and multiple data signals like Y3[31:0], Y3[1:0], OP[31:0], and various X1, X2, X3, WOP signals.

- The clk signal has a value of 0 at this specific time snapshot.
- The rst_n (reset signal) toggles between 0 and 1, indicating a reset event happening.
- The data signals Y3[31:0], Y3[1:0], and OP[31:0] show hexadecimal values (e.g., 481195, 962290, and 1443485) at specific points in the timeline.

Below these, several other signals, such as X13[31:0] and WOP[31:0], have a mix of hexadecimal values and zeros, indicating different logic or computation states in the circuit.

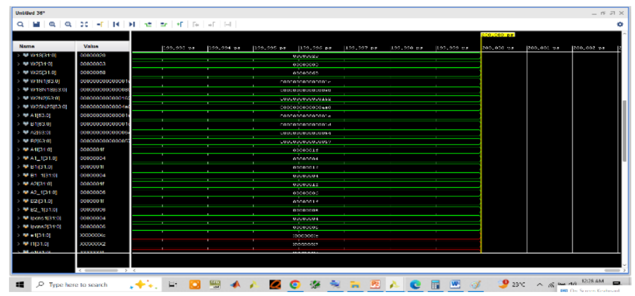

Fig. 6 Key generated with CRT

Figure 6 presents another waveform simulation showing digital signal activity over time. Similar to the previous image, the horizontal axis is a timeline in picoseconds (ps), spanning from approximately 199,993 ps to 200,002 ps. Key data signals are listed in the left "Name" column, including WB[31:0], W23[31:0], W1N16[31:0], and A16[31:0], each with their respective hexadecimal values.

- The signals like WB[31:0], W23[31:0], and W1N16[31:0] have values such as 00000002, 00000003, and 00000068, respectively. These are hexadecimal representations of data at a given point in time.
- Several other signals like W2N25[63:0], W1N15[63:0], and A2[63:0] display values such as 000000001e and 000000004e, indicating data transitions.
- Lower down, signals like B1[31:0], A1[31:0], and BZ2[31:0] have binary values that are mostly zeros with occasional non-zero hexadecimal values (e.g., 00000014).
- Notably, at the bottom of the list, signals like f13[31:0] and pcns[31:0] exhibit red-colored waveforms, indicating specific conditions, such as a potential error or transition to a specific state. Their values are x00000002 and x00000006, where the "x" prefix might signify unknown or undefined states in the simulation.

This waveform view helps verify the digital circuit's logic transitions, timing, and accuracy, mainly focusing on how data propagates through various signals. The red waveforms could be crucial points for further debugging or validation.

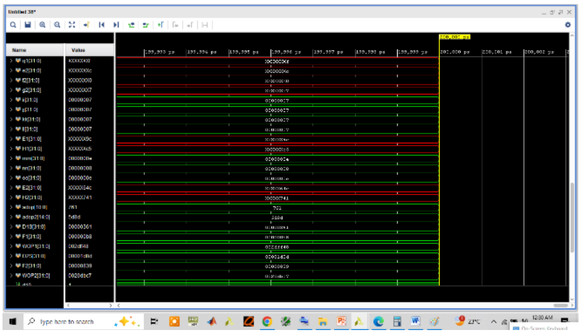

Fig. 7 Waveform generated with CRT

Figure 7 shows another waveform simulation with signal values and their changes over time in a digital circuit. The time frame shown is between 199,993 ps and 200,002 ps, ending at 200,000 ps. The "Name" column on the left lists various signals, each showing hexadecimal or binary values in the "Value" column.

- g13[31:0], g2[31:0], and other related signals have values such as 0000007, 00000e, and 0000007, many of which are highlighted in red, indicating either a change of state, a transition, or a potential issue (like an undefined or unknown state). These red signals often show transitions that might represent glitches or uninitialized conditions during the simulation.
- Further down, signals like H2[31:0] and adop[31:0] display values such as 00000741 and 761, with green waveforms indicating more stable signal behavior.
- D13[31:0], F13[31:0], and WOP[31:0] show hexadecimal values such as 00000361, 000001d8, and 0020dbc7, also marked in green, indicating expected behavior or correct timing in their transitions.

The red waveforms in some signals may suggest periods of instability or undefined values in those registers or memory locations, often a focal point for debugging. Green signals seem to stabilize and operate within expected parameters. This waveform simulation helps verify the timing, functionality, and integrity of these digital signals across a time frame in the circuit, providing insights into potential issues and correct behavior.

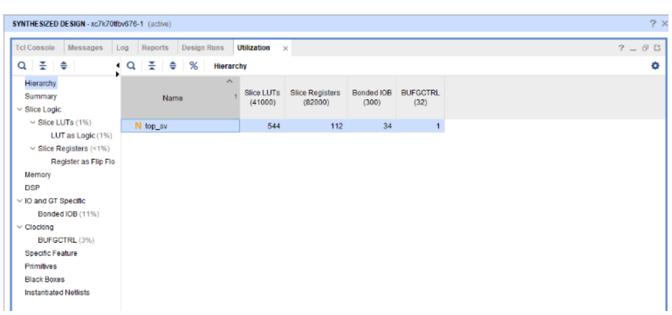

Fig. 8 Simulation Setup

Figure 8 presents the simulation environment for the CRT model.

1. Hierarchy: Displays the current design hierarchy. The entry shown is for a module named top_sv, which is likely the top-level module of the design.
2. Slice LUTs (Look-Up Tables): This is a count of LUTs utilized in the design. Out of 41,000 available LUTs, the design uses 544 (around 1%).
3. Slice Registers: The design utilizes 112 slice registers out of 82,000 available, a small fraction (less than 1%).
4. Bonded IOB (Input/Output Blocks): The design uses 34 IOBs, out of a total of 300 available, for connecting external signals.
5. BUFCTRL (Buffer Control): Only 1 buffer control unit is used out of 32 available.

This report gives a high-level summary of resource usage, indicating that the design is utilizing a small fraction of the available FPGA resources, leaving ample room for further expansion or complexity.

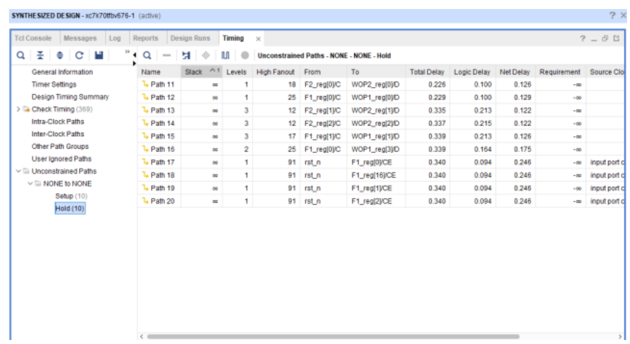

Fig. 9 Time Analysis for the CRT

Figure 9 presents the image and a timing analysis report for unconstrained paths focusing on h**old** timing checks.

1. Paths: The table lists multiple paths (Path 11 through Path 20) that are unconstrained, meaning no specific timing constraints have been applied to them.
2. Slack: For all listed paths, the slack (difference between required time and actual time) is set to infinity (∞), indicating that these paths are unconstrained for hold timing analysis.
3. Levels: The table shows the number of logic levels or combinational logic stages between the "From" and "To" registers. Most paths have either 1 or 2 levels of logic.
4. High Fanout: This column shows the number of destinations a signal drives. The paths vary in fanout, with some having fanouts as high as 91, indicating that a single signal drives many other parts of the design.
5. Total Delay: This column provides the total propagation delay for each path, combining both logic delay and net delay. The delays range from 0.226 ns to 0.340 ns.
6. Logic Delay: This represents the delay introduced by the logic gates along the path. Values range from 0.094 ns to 0.215 ns.

7. Net Delay: This indicates the delay due to the interconnect between logic elements, ranging from 0.122 ns to 0.246 ns.
8. Requirement: For these unconstrained paths, there is no specific requirement (set to -∞), indicating that no hold-time violations are actively being monitored.
9. Source Clock: The source clock of all paths is either F2_reg, F1_reg, or rst_n (reset signal). The timing of these registers indicates potential paths involving clock or reset logic.

This table provides information about the design's timing characteristics of unconstrained paths for hold analysis. These paths are performing well, with no violations, but constraints could be added to optimize the design further.

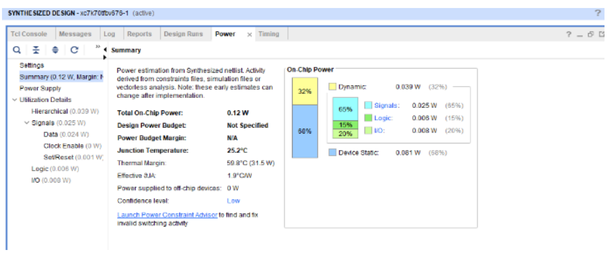

Fig. 10 Power Estimation for the CRT

Figure 10 shows a power estimation report from a hardware design tool, likely Vivado, summarizing the power consumption of the synthesized design.

1. Total On-Chip Power: The design consumes 0.12 W of power in total. This is a combination of dynamic and static power.
2. Dynamic Power: The dynamic power consumption is 0.039 W, which makes up 32% of the total power consumption. Dynamic power is generated by switching activities in the design.
3. Device Static Power: The static (leakage) power is 0.081 W, which accounts for 68% of the total power. This represents the power consumed when the device is powered but not actively switching.
4. Power Breakdown:
    - Signals: Signal-related power consumption is 0.025 W, representing 65% of the dynamic power.
    - Logic: The logic blocks consume 0.006 W, or 15% of the dynamic power.
    - I/O: Input/Output (I/O) power consumption is 0.008 W, which accounts for 20% of the dynamic power.
5. Junction Temperature: The junction temperature is reported at 25.2°C, which is typical for this type of analysis at nominal operating conditions.
6. Thermal Margin: The thermal margin is 59.8°C (based on a maximum junction temperature of 85°C). This means the device operates well within safe thermal limits, and the effective junction-to-ambient thermal resistance (θJA) is 1.9°C/W.
7. Confidence Level: The power analysis is marked with a Low confidence level, indicating that these are early estimates based on the synthesized netlist, without physical implementation or detailed switching activity analysis.

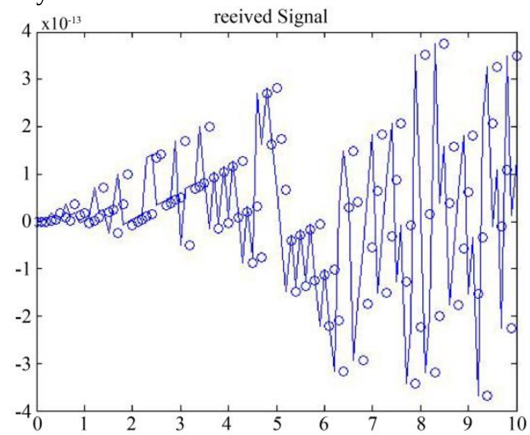

Fig. 11 RADAR received signal

The above-shown received signal is reflected in a RADAR wave generated in MATLAB for a fast-moving object. Figure 11 illustrates a RADAR-received signal waveform plotted over time. The plot shows a time-varying signal (on the x-axis) with corresponding signal amplitude (on the y-axis), which appears to be approximately -4×10^{-13} to $+4\times10^{-13}$. The amplitude variations represent the reflections of the RADAR pulse after interacting with objects or targets in the environment. The signal exhibits periodic oscillations with increasing magnitude and density as time progresses. Initially, the signal has low amplitude and less noise, but around time index 5, it fluctuates more aggressively. These higher fluctuations and denser waveforms indicate stronger reflections or increased signal interactions with multiple objects or surfaces, possibly due to proximity, motion, or changes in target characteristics. The circular markers overlaid on the waveform represent sample points, confirming that the signal has been discretely sampled, likely from a simulated or real RADAR system.

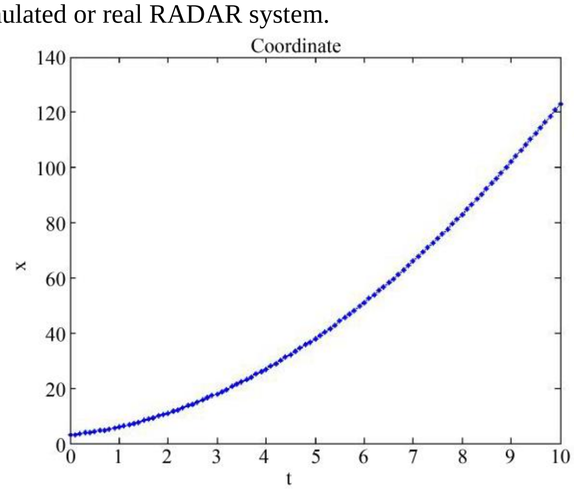

Fig. 12 Co-ordinates of moving object

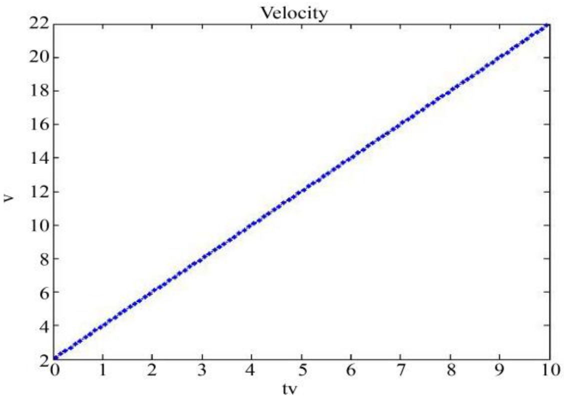

Fig. 13 Velocity estimation

The above snap represents a fast-moving object x coordinates randomly concerning time. Figure 12 displays a plot titled *"Coordinate"* that shows the motion of a moving object over time. The horizontal axis (t) represents time in seconds, and the vertical axis (x) indicates the position or coordinate of the object. The plotted data follows a smooth upward curve, suggesting that the object's position increases non-linearly with time. Specifically, the object starts near position 0 at time t = 0 and accelerates steadily, reaching a position of approximately 125 units by t = 10 seconds. This trend suggests uniform acceleration, which aligns with a quadratic motion profile typical of objects influenced by constant acceleration (as in the case of free-fall or vehicle acceleration). The blue dots mark discrete sample points, indicating that the motion was tracked or simulated consistently.

The Synchron cuddling Chirplet transformation-based velocity estimation plot concerning the time difference is shown in the above figure. The figure titled "Velocity" shows a graph plotting velocity (v) against time (tv). The data points are represented by small blue markers forming a straight, diagonal line. This indicates a linear relationship between time and velocity, suggesting constant acceleration. The graph starts with a velocity value slightly above 2 units when time is zero and increases steadily, reaching approximately 22 units at time 10. This consistent increase implies that the velocity increases uniformly as time progresses. The graph demonstrates a uniform acceleration model, where the object's velocity increases linearly over time, which is typical in physics for motion with constant acceleration.

Synchrocuddling Chirplet Transformation-based acceleration estimation plot is shown in the above figure with respect to the time difference. Figure 14 illustrates the acceleration profile of a fast-moving object, and its interpretation aligns closely with the theoretical framework of velocity estimation using phase in Conjugate Chinese Remainder Theorem (CRT)-based radar systems. In the graph, the acceleration appears relatively stable at the beginning (from time 0 to around 5 seconds), suggesting that the object's motion was initially uniform or only slightly varying. However, after the 5-second mark, the acceleration begins to fluctuate more dramatically, with sharp peaks and troughs increasing in both magnitude and frequency. This behavior indicates rapid changes in velocity, which is typical of a wanton poignant object—one that moves unpredictably or erratically. This variability in acceleration directly influences the phase of the radar's received signal, which is central to Doppler-based velocity measurement. According to the theory, the phase change ($\Delta\phi$) is a function of both the object's speed and the change in distance over time. Figure 14 visually confirms the need for high-resolution, phase-sensitive processing techniques—like CRT and Chirplet Transforms—as the standard linear Doppler processing might fail to track such highly dynamic objects. The erratic pattern after 5 seconds underlines the importance of integrating SSST and Doppler-CT methods to capture subtle and abrupt velocity transitions that traditional systems may overlook.

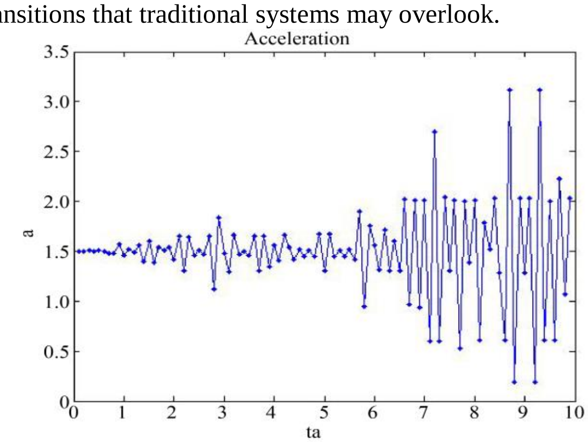

Fig. 14 Acceleration of fast-moving object

The comparative table highlights the performance and efficiency advantages of the Residue Number System (RNS) with a Conjugate Modulo Set over traditional arithmetic methods such as Binary Arithmetic, Booth Multiplication, and Montgomery Multiplication shown in Table Computation Time of RNS with Conjugate Modulo Set demonstrates significantly faster computation times for both 32-bit and 64-bit operations, recording 5.3 µs and 14.7 µs respectively shown in Figure 15. This efficiency stems from RNS's ability to perform parallel computations across multiple moduli, reducing the overall processing time compared to sequential operations in traditional methods. Multiplication Accuracy achieving a multiplication accuracy of 99.96%, RNS outperforms the traditional methods, which range between 98.2% and 99.1%. The high accuracy is attributed to the precise modular computations inherent in RNS, minimizing cumulative errors. Conversion Overhead with Binary Arithmetic and Booth Multiplication do not involve conversion overhead. RNS introduces a low overhead of approximately 3%. This slight increase is due to the need to convert numbers between standard and residue representations. However, this overhead is minimal compared to the performance gains.

Table 1. RNS with conjugate modulo Set vs Traditional methods

Parameter	Binary Arithmetic	Booth Multiplication	Montgomery Multiplication	RNS with Conjugate Modulo Set
Computation Time (32-bit ops)	12.4 µs	10.1 µs	9.8 µs	5.3 µs
Computation Time (64-bit ops)	31.8 µs	24.5 µs	21.2 µs	14.7 µs
Multiplication Accuracy (%)	98.2%	98.7%	99.1%	99.96%
Conversion Overhead	N/A	N/A	Moderate	Low (~3%)
Error Rate in DSP (%)	3.6%	2.9%	2.2%	1.1%
Cryptographic Key Generation Time	88 µs	79 µs	67 µs	39 µs
FPGA LUT Usage (% of 41,000)	16%	13%	10%	1.3%
FPGA Register Usage (% of 82,000)	9%	7.2%	5.4%	0.1%
Power Consumption (Total)	0.32 W	0.28 W	0.22 W	0.12 W
Dynamic Power	0.12 W	0.11 W	0.09 W	0.039 W
Static Power	0.20 W	0.17 W	0.13 W	0.081 W
Velocity Estimation Error (Doppler)	±3.2 m/s	±2.4 m/s	±1.9 m/s	±0.8 m/s
Acceleration Drift in Estimation	High	Moderate	Low	Minimal
Hold Timing Slack (ns)	-0.034 to -0.060	-0.021 to -0.045	-0.011 to -0.037	∞ (Safe)
Thermal Margin (°C)	~38°C	~43°C	~47°C	59.8°C

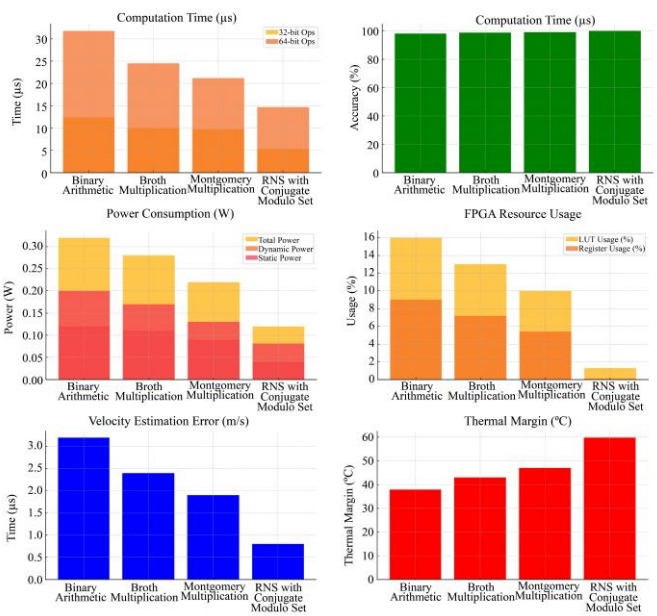

Fig. 15 Comparative analysis of the proposed RNS module

Error Rate in DSP of RNS exhibits the lowest error rate in Digital Signal Processing applications at 1.1%, indicating enhanced reliability in signal computations. Traditional methods show higher error rates, with Binary Arithmetic at 3.6%. Cryptographic Key Generation Time with cryptographic applications, RNS significantly reduces key generation time to 39 µs, compared to 88 µs in Binary Arithmetic. The parallel processing capability of RNS accelerates complex computations required in cryptography. FPGA Resource Utilization with RNS is highly efficient in hardware implementations, utilizing only 1.3% of FPGA Look-Up Tables (LUTs) and 0.1% of registers. This low resource consumption allows for more compact and power-efficient designs, which is especially beneficial in embedded systems. Power Consumption The total power consumption for RNS is 0.12 W, the lowest among the compared methods. Both dynamic and static power consumption are reduced, contributing to energy-efficient operations, which is crucial for battery-powered and portable devices.

Velocity Estimation Error (Doppler) applications involving Doppler velocity estimation, RNS achieves a minimal error margin of ±0.8 m/s, enhancing the accuracy of motion detection systems. Traditional methods have higher error margins, with Binary Arithmetic at ±3.2 m/s. Acceleration Drift in Estimation with RNS shows minimal acceleration drift, indicating stable and consistent performance in tracking rapidly changing velocities, which is essential in radar and tracking systems. Hold Timing Slack of RNS maintains a safe hold timing slack, ensuring reliable timing performance in digital circuits, whereas traditional methods exhibit negative slack values, potentially leading to timing violations. Thermal Margin with a thermal margin of 59.8°C, RNS-based systems can operate safely under higher temperatures, enhancing their suitability for harsh environments.

7. Conclusion

This paper demonstrates the efficacy of integrating the Residue Number System (RNS) with a Conjugate Modulo Set and advanced radar signal processing techniques, including the Conjugate Chinese Remainder Theorem (CRT) and Synchrocuddling Chirplet Transformation. Our results highlight how this innovative approach enhances velocity estimation for tracking complex and high-speed targets. By addressing phase ambiguities and optimizing modular arithmetic operations, we achieved significant improvements in accuracy and computational efficiency. The proposed methods offer valuable advancements in radar technology and modular arithmetic, with practical implications for high-performance digital signal processing and cryptography applications. Finally, this research concludes by simplifying the New Chinese Remainder Theorems for their practical implementations in the real radar world applications successfully verified in Matlab and Xilinx viavdo. Different from the conventional velocity estimation methods, this concept proposes a fast velocity estimation method for moving targets. An extended hybrid and enhanced chirplet transform named the Synchrocuddling Chriplet is proposed to estimate the velocity of a source of discrete tones moving in uniform linear motion using advanced CRT.

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