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Original Article

An Application of Galois Fields to Beam Alignment and Synchronization Issues in Reconfigurable Intelligent Surfaces for Future Wireless Communication Systems

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Abstract *- Discrete mathematical structures in number theoretic methods have been successfully applied to various electronic and communication engineering research issues. As a result, an effort has been made here to extend a number-theoretic-based model for a phased array antenna to the reflection mechanism of a Reconfigurable Intelligent Surface (RIS). The model is based on a discrete mathematical structure/finite field approach (Galois field) with the so-called Zech logarithm at the heart of the integerization/discretization process. The aim of the exercise being to propose and validate an alternative approach to handle beam alignment and synchronization issues in the application and implementation of the state of the art intelligent surfaces for Beyond Fifth Generation (B5G) and Sixth Generation (6G) wireless communications with the ultimate objective of reducing hardware complexity and cost. Several new technological paradigms are highlighted and discussed, which show promise in optimizing resources to control channel dynamics for future wireless communications. Finally, numerical analysis concerning the proposed model is carried out, and subsequent simulations verify the method's applicability.*

Keywords *- Electromagnetic waves/beams, Finite fields, Galois fields, Reconfigurable intelligent surfaces, Zech logarithm.*

1. Introduction

The dawn of 6G wireless communications is expected to primarily usher in an era characterized by features such as holographic teleportation, higher immersion rates for augmented and virtual reality, the internet of nano-things, the internet of bio-nano-things, and the internet of space things [1]. This includes denser and higher computer or communication connectivity involving satellites, deep space probes, and unmanned aerial vehicles. Additionally, it promises enhanced human-machine interactions and automated network management. This new generation of wireless technology aims to integrate mainstream technologies like sensing, imaging, localization, and communications onto a single platform, with key performance indicators focused on ultra-high throughput, ultra-dense connectivity, and ultra-low latency. The key enabling technologies to serve as the backbone for 6G wireless communications have been identified to be the Ultra-Massive, Multiple-Input, Multiple-Output (UM-MIMO) antenna systems and smart communication environments powered by Reconfigurable Intelligent Surfaces (RIS) [2-4]. Furthermore, the Terahertz (THz) portion of the electromagnetic spectrum (0.1-10 THz) [5] has been identified as the band of frequencies that can supply close to unlimited bandwidth for 6G communications. However, the THz frequencies are subject to very high path-loss and molecular absorption in addition to scattering and multipath issues that may result from Non-Line Of Sight (NLOS) scenarios – consequently, once again, the channel sets the limit for the quantity and quality of communication possible, thereby, prompting the wireless communication community to ask the question, "Can the channel be controlled?" [2], fortunately, the answer came in affirmative in the form of smart communication environments such as smart walls powered by reconfigurable intelligent surfaces of which contemporary research has already provided two promising solutions in the form of the MIT R-Focus RIS wall paper and NTT Docomo RIS sliding glass [2] as prototypes for the proffered smart communication environments.

In line with [3], however, accurate alignment and synchronization issues still affect the reflection of electromagnetic beams to RIS. As a result, the approach proposed here applies the mathematics of finite fields, particularly Galois fields [6, 7], to enhance directional accuracy for the reflected RIS beams. Following the discussions in [8], a phased antenna array can be designed to radiate Electromagnetic (EM) waves of equal strength in a definite amount of directions subject to choice, if the elements of the array are driven with equal amplitudes and phase-angles having values satisfying a specific structure of Galois fields involving the so called Zech logarithm (a discrete logarithm) [9]. In the discussions to be pursued here, we will give an outline of the mathematical structure of Galois fields of the form GF (p^n) and GF (p^n*) [10]. [11], GF stands for Galois field, and 'P' is any prime number, while 'n' stands for an integer value. Most importantly, the work here intends to extend the idea of utilizing finite fields for phased array antenna design to an array of reflecting units in RIS. The advantage of such an innovation lies in offering an alternative route to beam alignment and synchronization in RIS operation, and the electronic configuration network structure needed to steer the RIS beam. Thus, the scheme developed here can serve as an alternative effective solution to RIS system design compared to the works in [12, 13].

2. Potential Motivating Applications

The main leading idea of 6G communications and beyond is the integration of main stream communication technologies on a single platform, thus, as finite field mathematics is applied extensively in coding and cryptography, applying it to the design of beam alignment issues for RIS inspires the idea of both coding and beam steering utilizing the same scheme. In particular, the advantages of this approach to the design of RIS is made manifest when the highly line of sight (LOS) nature of the THz waves are considered - a cascade of THz beams can be achieved with such a scheme, and an idea of multiplexing the beams without complicated electronics sharply comes into focus. In a discrete mathematical approach, such a cascade can progress in the form of the famed Fibonacci rabbit sequence [14] as presented in Figure 1 below: this approach can lead to beam alignment and coding of the information-bearing beams on a single platform. Operationally in terms of deployment, this approach can be applied within in-door scenarios where the RIS can be made to be reflecting waves of a chosen amplitude in a definite amount of directions and in outdoor scenarios where RIS designed with this scheme can be placed intermittently as repeaters on smart walls to propagate THz waves in a desired direction through longer distances. The approach can also be applied where installation of RIS and attendant equipment maybe difficult, for instance in the design and deployment of smart undergrounds [2] where the complex topology of the radio environment negatively affects the coverage of wireless signals or in the construction of smart train stations where waiting passenger platforms can be illuminated to enhance signal coverage for individual users and cluster of users. The idea of cost and complexity reduction in using the finite field approach for designing RIS becomes evident to the case of Large Intelligent Surfaces (LIS) [2], which are seen as the next frontier after UM-MIMO; these type of RIS involve quite a complicated level of automation/ intelligence and RF chains; thus the benefits that can be accrued from interfacing LIS as shown in (Figure 2) with RIS designed with the finite field approach is obvious, since the interface can be seen as a kind of extended cognition for LIS, and in general, highly intelligent surfaces of the future, offloading the burden of cognition to a lower level intelligence environment in-order to increase system efficiency and reduce cost [15] with the added advantage of the improved green nature of such RIS technology also reducing human exposure to EM waves. In addition, the emerging field of quantum computers/quantum computing is based on a fundamentally discrete nature i.e. 'energy quanta' [16], thus, it will be advantageous to exert more research efforts in this direction which can result in a 'quantum RIS' for quantum communications to the integer/quantum nature of discrete mathematical structures. Furthermore, in line with [17] one can also envisage an RIS with reflection mechanism based on Julia/Mandelbrot sets/fractal geometry, and an RIS whose radiating/reflecting elements are arranged based on the distribution of prime numbers amongst a given range of positive integers; such self similar patterns may apply to beam alignment processes and coding schemes for future intelligent surfaces.

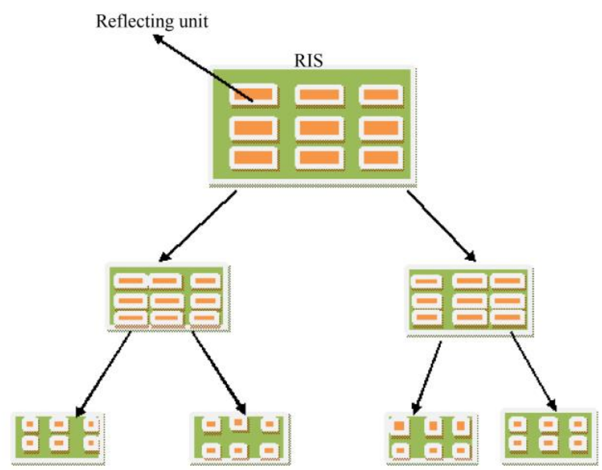

Fig. 1 Fibonacci sequence multiplexing for RIS

Thus, the ideas presented herein are hinged on the philosophy that as the mathematics of calculus was fundamental to the science and technology of the industrial revolution, the field of discrete mathematics with finite fields as a major player will be the backbone of the computer/ information age, thereby escorting the field of engineering gradually from the world of applied mathematics to that of pure mathematics with the discrete/disconnected nature of the objects laying modeling assumptions bare and rendering the entire engineering process unambiguous. In other words, the contribution of the work presented here is centered on identifying, demonstrating and verifying the benefits that can be realized by applying a discrete mathematical structure such as Galois fields to RIS system design and operation (Figure 3). Subsequently, previous works concerned with RIS system design in connection with its beam alignment and synchronization is considered.

2.1. Related Works

The concepts of discrete mathematical structures have been applied extensively to various engineered communication systems such as spread spectrum communications, coding and cryptographic communications [18-20], the ambiguity problem in radar communications [21] and shift register dynamics and design [22]. Recently, beam management, as indicated by [23, 24], in the presence of meta-surface RIS has become an open problem in contemporary metamaterial surface research.

The discussions carried out in [3] pointed out that mechanical actuation using mechanical rotation and translation, functional materials in the form of liquid crystal displays and grapheme and electronic devices such as the Positive-Intrinsic-Negative (PIN) diodes, Field Effect Transistors (FET), Field Programmable Gate Arrays (FPGA), Application Specific Integrated Circuits(ASIC) and Micro-Electromechanical Switches (MEMS) were the main approaches in contemporary technology for reconfiguring RIS elements for highly controllable reflections/dynamic range of reflection.

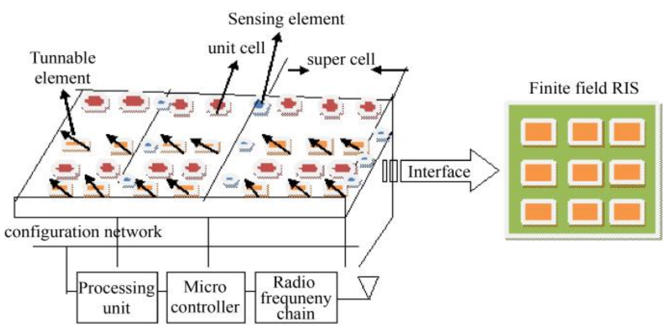

Fig. 2 Large intelligent surface interfaced to finite field RIS for cost efficiency

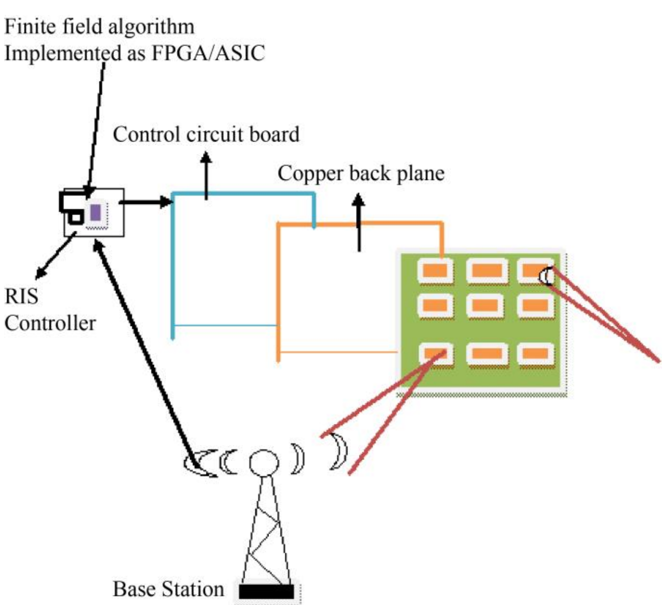

Fig. 3 RIS system architecture

Discussions in [2] went further to highlight the attendant electronics (dedicated radio frequency chains, power amplifiers and signal processing electronics) that form the intrinsic structure of more complex RIS, such as Large Intelligent Surfaces (LIS), which enables the surface to be smart with on-board sensing capabilities. Further discussions in [3] noted that in practice, it is desirable to have independent control of the amplitude and phase shift of each RIS element for optimizing the reflection design, and also that phase shift control was of higher cost to implement compared to amplitude control. As a result, a two-level discrete control was proposed in [12, 13] for amplitude and phase shift. In particular, [25] proposes a feedback-based iterative beam steering with an embedded network of controllers to enhance intelligence and autonomous operation in wireless networks based on the revolutionary enabling technology for RIS known as Hyper Surfaces (HSF) [26].

Machine learning approaches for manipulating EM beams in connection with RIS have been extensively addressed in [27-31]. The work in [27] handles in-depth machine learning approaches for beam forming and beam alignment in mm-Wave networks by designing novel beam forming technologies for low-latency and cost-effective networks with the attendant analysis of their performance. [32-34] all pointed out and demonstrated that beam steering for RIS in general was centered around designing super-strates above the meta surface antennas, thus [35] studied 2-D beam-steering by a reconfigurable Partial Reflecting Surface (PRS) by manipulating the states of PIN diodes, while [36] analyzed phase manipulation by capacitive loading in a leaky wave micro-strip antenna for beam steering in a desired direction. In [37], a fluidically loaded dielectric lens was reported to have been deployed for single element beam steering in a modified antipodal Vivaldi antenna element and in [38] it was shown that beam steering in both the azimuth and elevation planes can be implemented by rotating two metal transmit-array lenses placed above a meta-surface antenna.

Furthermore, [39] demonstrated that azimuthally beam-steering can be accomplished by arranging the antenna elements in MIMO in a daisy-chain structure, while [40, 41] showed that electronic steering of radiation pattern in MIMO can be obtained via application of varactors. Very recently, in [42], beam steering of THz MIMO antenna using grapheme-based intelligent reflective surface was reported. Moreover, [43] an RIS-based steerable beam forming antenna with near field eigenmode feeder was discussed, by presenting a novel, power and hardware efficient antenna system leveraging the eigenmodes of over the air propagation matrix from an active multi antenna feeder to a large Reflective Intelligent Surface (RIS). Moreover, the 6G communication era is expected to perfect more sophisticated functions for the RIS, such as RIS-based modulation [44-47], RIS-based multi-stream transmitters and meta-surface based transmitters [48-50], and RIS-based encoding [51].

2.2. Contributions of the Work

In all the above-mentioned approaches, it is evident that an extensive amount of electronics and material resources in the form of electronic/optoelectronic/photonic devices and circuits, in addition to formulated machine learning approaches with the attendant hardware, are needed to configure the operation of the RIS. Thus, in the scheme presented here, the aim would be to develop theoretically an alternative approach that will only require a simple electronic network within the RIS controller that can be broken down to ordinary modular / modulo 2 arithmetic due to the discrete nature of the Galois field domain. It is hoped that the results of this approach can be used in tandem with the more sophisticated electronic approaches highlighted above, at least to reduce cost or complexity once the basic configuration network has obtained the channel state information. Motivations for this contribution involves taking a closer look at the works in [12, 13] and the discussions in [3] which reveal that the central idea of continuously varying amplitude or phase in terms of discrete phase shift control of RIS is focused on applying diodes/PIN diodes, varactors and MEMS on each reflecting element or meta- material particle in order to optimize the communication performance as presented in (Figure 4.) below [12], however as pointed out in [3], this approach is practically difficult to implement for high resolution reflecting elements when applying PIN diodes considering that for an 8 level phase shift for example, $log_2 g8$ = 3 (base 2), PIN diodes are required; as such, this renders the reflecting element design more challenging and requires more controlling pins at the RIS controller to control the required PIN diodes. Further, [3] indicates that although a single varactor diode can achieve multi-level phase shifts, it requires a wide range of biasing voltages and thus is more costly to implement. In addition, [3] also emphasizes that phase shift control is more costly to implement than amplitude control. Furthermore, [3] also highlighted that resonant circuit models for the reflecting elements revealed a non-linear relationship between the amplitude and phase shifts, implying that both cannot be changed independently, further undermining discrete individual phase control for the reflecting elements. Consequently, [12] asserts that for RIS with a large number of reflecting elements, it is more cost effective to implement only discrete and finite amplitude/phase shift levels that require only a small number of control bits for each reflecting element, however, as a trade-off, this may impose some limitation on the quality of the communication; these inherent limitations in RIS system design can be surmounted considerably by subscribing to the discrete mathematical generalized framework offered by the Galois field approach. The Galois field scheme based on the Zech discrete logarithm can achieve discrete phase shifts without requiring individual electronic switches for the reflecting elements, because [8] points out that a phased antenna array can be designed to radiate EM waves of equal strength in a definite amount of directions subject to choice, if the elements of the array are driven with equal amplitudes and phase-angles having values satisfying a specific structure of Galois fields involving the so called Zech logarithm. In other words, with a fixed amplitude/voltage, a single varactor can be used in this scheme to achieve multi-level discrete phase shifts without requiring a wide range of biasing voltages, but rather, relies on one fixed amplitude/voltage for biasing all the reflecting elements. Thus, the finite field algorithm [8] can be implemented as an FPGA or ASIC within the RIS controller to control a varactor for multi-level phase shifts. In addition, such a controller can generally be made adaptive by incorporating machine/deep learning paradigms in its algorithm to cover mobile users' continuous motion. Finally, the work to be carried out presently shows how this scheme for a phased array antenna can be extended to a reflect array where an RIS can be implemented.

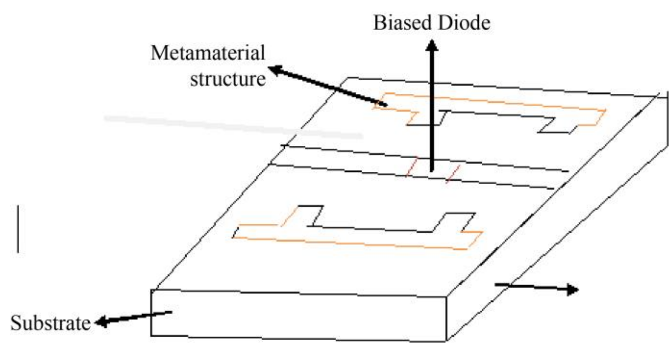

Fig. 4 Metamaterial particle for realizing a digital metasurface

2.3. Challenges Encountered in the Work

Implementing a metanaterial reflect array based on the Galois field approach will involve designing and fabricating a simple digital integrated circuit/digital device in the form of an FPGA or ASIC, which will implement the finite field algorithm based on the Zech logarithm within the RIS controller. Due to financial and technical constraints, mathematical modeling and computer simulation have been utilized in the work presented here as laboratory tools. The validation of the model at this level is solely based on establishing that a reflect array-based RIS can be implemented theoretically (rather than experimentally for now) to reflect electromagnetic beams of equal amplitude in any discrete desired number of directions, subject to a discrete mathematical technique.

3. Materials and Methods

3.1. The Electromagnetic View

The work in [8] deals with a phased array antenna, which functionally is a radiating structure or an array of radiating antennas/elements. To extend the case to an RIS, the RIS structure to be used has to be considered. Functionally, the different flavors in which an RIS can be implemented according to [2] are, as reflect array antennas and meta material surfaces such as large intelligent surfaces (active surfaces contributing to amplification), intelligent reflecting surfaces (optimized for beam steering and focusing with unit elements not having amplification effect on the impinging radio waves but only on the phase response), digitally controllable scatterers (operation based on mutual coupling among elements comprising the surface) and soft-ware controllable surfaces(operation based on software defined networking technologies). The reflect array antenna-based meta-surfaces will be used here, as the substrate for the model, since structurally it is conceptually the most aligned with a phased array antenna [52]. The only marked difference between the two antenna arrays is portrayed in the fact that for a classic planar array such as the phased array, beam focusing in specific directions are accomplished by assigning a progressive phase distribution to the elements, while for a reflect array one needs to account for the feed position, In addition to assigning a progressive phase distribution to the elements (Figure 5) [53].

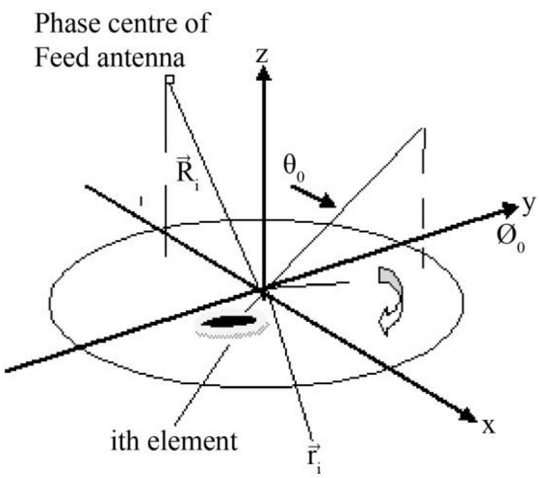

Fig. 5 Co-ordinate system for reflect array and elements

In-terms of design, the reflect array as a high gain antenna commands a large number of elements on its aperture which makes a full-wave simulation quite challenging [53], thus as indicated in [54], two basic methods for the analysis of the radiation performance of reflect array antennas are usually utilized- the array theory method and the aperture method. For our purposes in the present work, the array theory method suffices. Structurally, reflect arrays have a low mass, a low profile and a flat surface, thus combining the best qualities of a phased array antenna and a parabolic reflector [54-56]. Similar to a phased array, many reflecting elements on the reflect array are approximated as identical elements, enabling an array summation or far field transformation of currents to calculate the radiation pattern [57-59]. In line with [54] and (Figure 6), and to the so-called Q-model, the radiation pattern of a two-dimensional planar array with M × N elements can be calculated as;

$$E(\theta,\emptyset) = \sum_{m=1}^{M}\sum_{n=1}^{N} cos^{q_e}\,\theta\,\frac{cos^{q_f}\theta_f(m,n)}{|\vec{r}_{mn}-\vec{r}_f|}\; e^{-jk(|\vec{r}_{mn}-\vec{r}_f|-\vec{r}_{mn}\bullet\,\hat{u})}cos^{q_e}\theta_e(m,n)e^{j\varphi_{mn}} \quad (1)$$

The above radiation pattern equation requires the evaluation of a double summation; therefore, to enhance computational speed-up and efficiency, the double summation in (1) is replaced with an inverse discrete Fourier transform defined as [54].

$$f(p,q) = \frac{1}{N_xN_y}\sum_{m=0}^{N_x-1}\sum_{n=0}^{N_y-1} F(m,n)\, e^{\frac{j2mp\pi}{N_x}}\; e^{\frac{j2nq\pi}{N_y}} \quad (2)$$

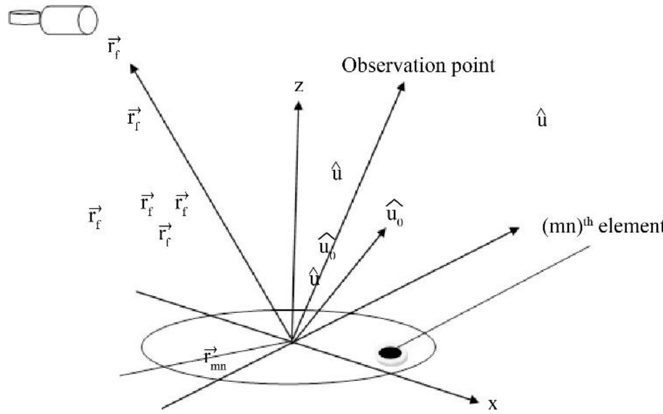

Fig. 6 Co-ordinate system for reflect array based on Q-model

This Fourier representation of the radiation pattern in (1) is necessary and sufficient for our purposes here because it matches exactly the discretization process for the reflections of the reflect array, which our model proposes. After all, the spectral functions will be obtained in a discrete number of angular coordinates. As indicated in [54], these points in the (u, v) plane are defined by the Fourier transform as;

$$u = \frac{2\pi}{N_xd_xk_0}p,\ p = 0,1,2,\dots N_x-1;$$
$$v = \frac{2\pi}{N_yd_yk_0}q,\ q = 0,1,2,\dots\ N_y-1. \quad (3)$$

3.2. The Finite Field Approach

Finite fields are fundamentally mathematical structures built on the concept of numbers, the theory of numbers and the mathematics of arithmetic. The mathematical objects involved in the study of numbers include the set of real numbers (comprising the set of rational ‘ℚ’ and irrational numbers ‘ℙ’) denoted as ‘ℝ’ (i.e. the real line, ℝ = {-∞,..., 0,…, +∞}) and in particular the subset of real numbers ‘ℤ’ known as the integers which are positive or negative whole numbers including 0 (ℚ = { a / b: a, b ϵ ℤ ∀ b ≠ 0}, and where the symbol ℤ has its roots in the German word for number ‘Zahlen’) [60, 61]. The field of study, which is number theory, is mainly concerned with logical statements and assertions (Theorems in connection with the integers) which can be verified via a system of proofs (logical deductions made based on valid arguments in connection with the so-called axioms). At the same time, arithmetic generally involves manipulations and computations in connection with ℝ and mostly ℤ. As noted in [11], the most powerful and important ideas of coding theory in communication engineering are based on arithmetic systems of finite fields, particularly Galois fields. To appreciate the meaning of a finite field, one needs to understand several basic concepts in the subject of number theory, starting with basic notions in arithmetic such as divisibility, the Euclidean algorithm, modular arithmetic, groups, Abelian groups, Rings, Commutative Rings, Integral Domains and fields. The set of rational numbers $\mathbb{Q}$ and real numbers $\mathbb{R}$ are algebraic systems that are examples of a field. Generally, a field is a set $\mathscr{F}$ which is an algebraic structure in which all the axioms of groups, rings and integral domains are satisfied. Consequently, the set $\mathscr{F}$ with the addition operation and multiplication form an Abelian group, i.e. the zero element 0, the unity element 1, the negative (-a), and the reciprocal (1/a), a ≠ 0 are all unique. A field with finite elements is known as a finite field [11]. Galois proved mathematically that the number of elements should be p^n, where p is a prime and n is a positive integer. A Galois field denoted as GF (p^n) is a finite field with p^n elements. By the notation, when the value of n is one, we have the field GF (p), [62, 63]. The field can be expressed as a set. Z_p,{0,1, …, $p^n - 1$} In this set, each element has an additive inverse, and non-zero elements have a multiplicative inverse. On the other hand, the field GF (p^{n*}) gives all the non-zero primitive roots of p, yielding the set {1, g^1, g^2,…, g^{p-2}}, where g is a primitive root of p; thus, the so-called Zech logarithm generates all such 'g' prime powers of p^n excluding the zero element. Analytically, for a phased array operating at some frequency 'f' with wavelength λ and element spacing of λ/2, having a periodic sequence with period $p^n - 1$, [8] gives the number of discretely directed radiations as;

$$a_0 = 0 \quad (4)$$

$$a_n = \exp\,[2\pi j Z(n) / p^m - 1] \quad (5)$$

For n= 1,2,… $p^m - 2$,.

m= 0, I, 2, 3, 4,…∞.

Where Z(n) is defined to be a Zech logarithm in GF(p^m) of the form;

$$\alpha^{Z(n)} = 1 - \alpha^n \quad (6)$$

With all a_n for n ≠ 0 mod ($p^m - 1$), having a magnitude of 1, and the periodic correlation sequence c_n given by;

$$c_o = p^m - 2 \quad (7)$$

$$c_n = -1 - \exp\,[2\pi j n / p^m - 1] \quad (8)$$

With a corresponding power spectra expressed as;

$$|A_k|^2 = p^m, \quad k = 2, 3…, \; p^m - 2, \quad (9)$$

Note: for any two numbers a and b, a ≡ b (mod n) defines a congruence between the integers a and b, meaning that the difference a-b is divisible by n, i.e. mathematically, a – b ǀn.

Table 1. Table showing polynomials for $2^4 - 1$ terms

index(k)	Polynomial terms ($2^4 - 1$)	polynomial	N bit word 2^4
K=0	$0x^3$+$0x^2$+$0x^1$+$0x^0$	0	0000
K =1	$0x^3$+$0x^2$+$0x^1$+$1x^0$	1	0001
K=2	$0x^3$+$0x^2$+$1x^1$+$0x^0$	x	0010
K=3	$0x^3$+$0x^2$+$1x^1$+$1x^0$	$x + 1$	0011
K=4	$0x^3$+$1x^2$+$0x^1$+$0x^0$	x^2	0100
K=5	$0x^3$+$1x^2$+$0x^1$+$1x^0$	$x^2 + 1$	0101
K=6	$0x^3$+$1x^2$+$1x^1$+$0x^0$	$x^2 + x$	0110
K=7	$0x^3$+$1x^2$+$1x^1$+$1x^0$	$x^2 + x + 1$	0111
K=8	$1x^3$+$0x^2$+$0x^1$+$0x^0$	x^3	1000
K=9	$1x^3$+$0x^2$+$0x^1$+$1x^0$	$x^3 + 1$	1001
K=10	$1x^3$+$0x^2$+$1x^1$+$0x^0$	$x^3 + x$	1010
K=11	$1x^3$+$0x^2$+$1x^1$+$1x^0$	$x^3 + x + 1$	1011
K=12	$1x^3$+$1x^2$+$0x^1$+$0x^0$	$x^3 + x^2$	1100
K=13	$1x^3$+$1x^2$+$0x^1$+$1x^0$	$x^3 + x^2 + 1$	1101
K=14	$1x^3$+$1x^2$+$1x^1$+$1x^0$	$x^3 + x^2 + x + 1$	1111

Thus, a phased array operated at a wavelength λ, having element spacing of λ/2 and driven with equal amplitude and phase angles based on (5), will reflect energies into $\boldsymbol{p^m - 3}$ distinct directions to the index k=2 to k= $\boldsymbol{p^m - 2}$ (i.e. 13 directions for a $\boldsymbol{2^4 - 1}$ Galois field - Table 1).

3.3. Integration of the Electromagnetic View and the Finite Field Approach

Finally, combining (2) and (5), our model for the reflect array/RIS will be;

$$f(p,q) = \frac{1}{N_x N_y} \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} \exp\,[2\pi j Z(n) / p^M - 1] \times e^{\frac{j2mp\pi}{N_x}} e^{\frac{j2nq\pi}{N_y}} \quad (10)$$

Where the power of the prime 'p' has been chosen to be 'M' to differentiate from 'm' of the discrete Fourier transform.

4. Numerical Analysis and Results

The inverse discrete Fourier transform expression (10) of our model involves a double summation; therefore, at the heart of the algorithm for our simulations is a double FOR loop structure which implements (10) accordingly with our prime number p=2, and the prime power M=4, and N_x= N_y= 15 i.e. $2^4 - 1$ possible elements (Table 1) which will reflect in 14 possible directions (i.e. excluding the 0 element as was the case for the set of primitive roots of the prime). Also, to further illustrate, N_x= N_y= 14 is also utilized, which gives a required reflection in 13 possible directions. The plot of the radiated field as a function of three dimensions and the azimuth angle is shown below. Figure 7 shows the reflected electromagnetic energy with the field components at right angles to each other and pointed in 14 directions as illustrated by Table 1. Figure 8 shows the reflected electromagnetic beam as a function of the azimuth angle, and as can be seen, the discrete number of directions is maintained (quasi-omni-directional) to the circular function. Further, the number of points in (31) is reduced to N_x= N_y= 14 and this gives 13 distinct directions of reflection as in Figure 9, taking note of the fact that the vertical axis for the plot has been zoomed in to take a closer observation of the plot, which reveals the electromagnetic wave undulations.

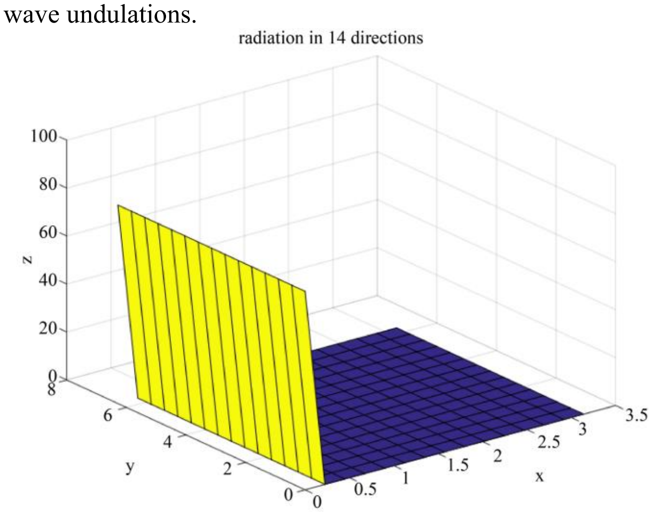

Fig. 7 Three-dimensional plot of radiated field for 14 directions

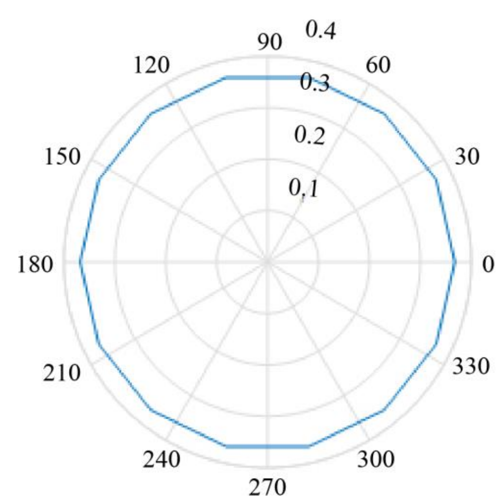

Fig. 8 Radiated field vs Azimuth angle for 14 directions

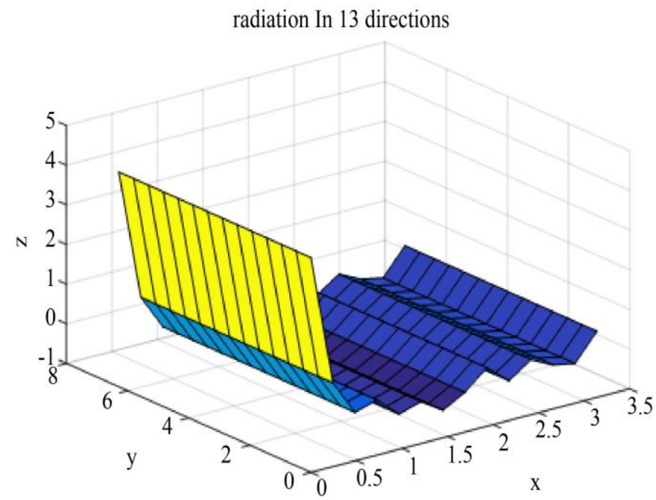

Fig. 9 Three-dimensional plot of radiated field for 13 directions.

To further illustrate the flexibility and applicability of the model, we can extend the range of radiation/reflection directions by choosing the prime p=2 and the prime power M=5, and N_x= N_y= 31 i.e. $2^5 - 1$ prime elements will reflect in 30 possible directions, excluding the 0 element. The three-dimensional plot for 30 directions is shown in Figure 10, while in Figure 11, the radiation to the azimuthal angle is shown for 30 radiation directions.

Within this set of prime elements, we can choose a lesser number of radiation directions by, say, letting N_x= N_y= 27, which gives radiation in 26 directions as shown in Figure 12 regarding the azimuth angle.

In addition, Figure 13 incorporates the elevation angle into the three-dimensional surface plot, thereby giving a clearer view of the electromagnetic fluctuation/radiation structure from the antenna surface. In contrast, Figure 14 shows 30 radiation directions in three dimensions regarding a spherical function.

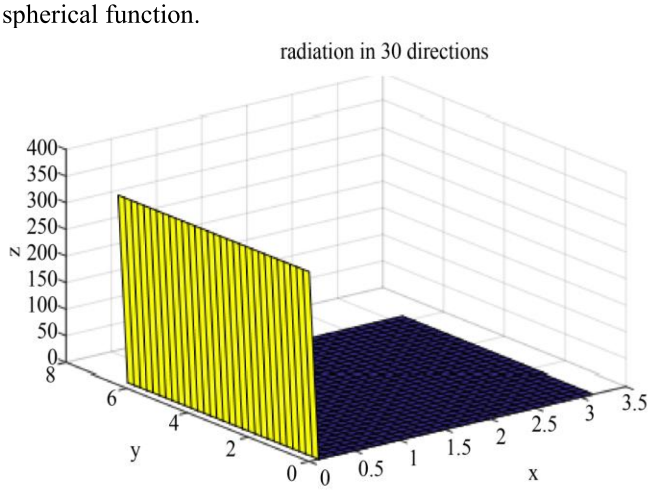

Fig. 10 Three-dimensional plot of radiated field for 30 directions

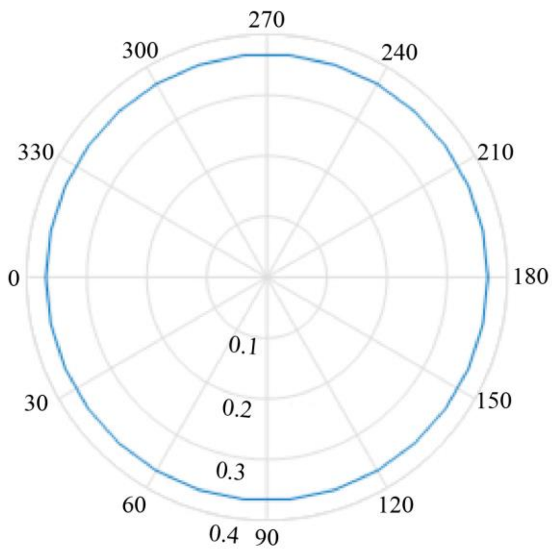

Fig. 11 Radiated field vs Azimuth angle for 30 directions

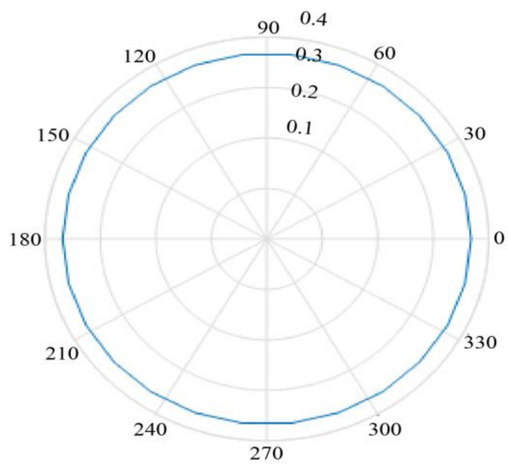

Fig. 12 Radiated field vs Azimuth angle for 26 directions

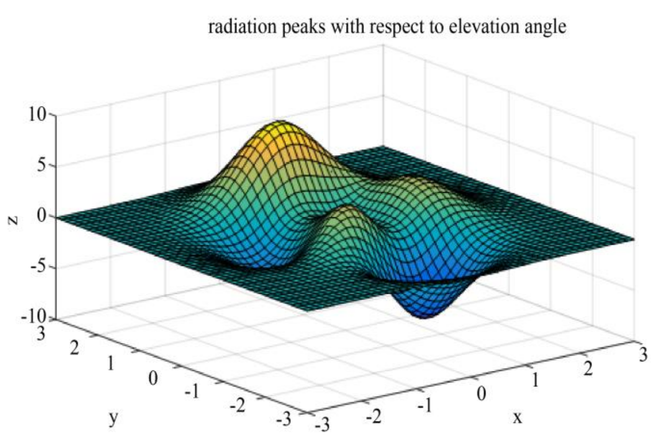

Fig. 13 Three-dimensional plot of radiated field vs elevation angle for 26 directions of radiation

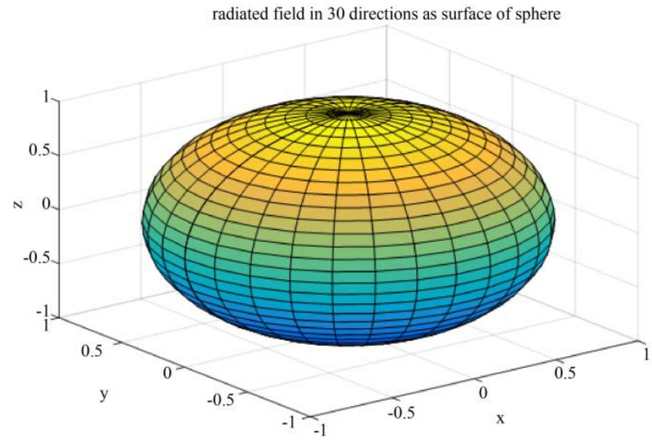

Fig. 14 Three-dimensional radiated field in 30 directions in terms of a spherical function

5. Conclusion

The work presented here has endeavored to extend a number-theoretic approach applied to the radiation mechanism of a phased array antenna to reconfigurable intelligent surfaces comprising the fundamental channel element for 6G communications and beyond. The effort was carried out to control the channel dynamics, hardware complexity and cost via the alignment and synchronization of beams emanating from the RIS by utilizing a model based on Galois fields. Other discrete mathematical structures that can be applied to improve the control of channel characteristics for 6G and beyond were also highlighted, along with some hopefully advantageous technological schemes also discussed. Finally, it can be inferred that RIS in general and in particular with this design scheme could result in a mobile communications channel that will be more deterministic and hence more predictable and will thus yield itself, or be more amenable to the methods of linear system analysis. The work can be taken further by experimentally validating the Galois field approach proposed here which was validated solely by simulations, and investigations may also be carried out to the possible effect of other number theoretic schemes such as the extended Euclidean algorithm, the Chinese remainder theorem and the distribution of primes in a given range of positive integers to the reflection mechanism of intelligent surfaces.

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