Exact Linearization Control for Twin Rotor MIMO System

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Abstract:

Twin Rotor MIMO System (TRMS) is a non-linear object of which the dynamic behavior resembles that of a helicopter. The TRMS has been significantly used to verify control algorithms' performance and it attracts attention of scholars in the area of modeling and control. In this paper, an exact linearization controller is designed for the TRMS according to the Euler Lagrange model. The outputs of the controller are compared to the reference inputs. All related formulas are analyzed and experimental results are shows that TRMS tracks the desired trajectory accurately.

Keywords: *Exact linearization, Twin Rotor MIMO System*

I. INTRODUCTION

Twin Rotor multiple input - multiple output (MIMO) System (TRMS), as shown in Figure 1-3, is an experimental system developed by Feedback Instrument Ltd (Feedback Co., 1998) for MIMO experiments. TRMS consists of two rotors placed on a beam together with a counterbalance. The whole unit is attached to a tower allowing for safe helicopter control experiments. The movements in the vertical plane and in the horizontal plane are implemented by a vertical rotor (main rotor) and a horizontal rotor (tail rotor) respectively. The above rotors are driven by DC motors.

TRMS's dynamic characteristics are similar to that of a helicopter and they are nonlinear systems and have a significant cross coupling between two rotors. It is challenging for scientists to design a controller with acceptable outcome. Domestically and internationally, there have been a large number of publications regarding controllers for TRMS. Nevertheless, according to their experimental results, the outputs have mostly tracked desired trajectories but significant errors have been recognized. Therefore, in this paper, a design method based on the Euler-Lagrange model of TRMS will be proposed.

Figure 1. Twin Rotor MIMO System

The rest of the paper is organized as follows. The object's model and the controller's design are addressed in Section II. Simulation and evaluation will be demonstrated in Section III. Finally, conclusions and also future work will be mentioned in Section IV. The list of key notations used in this paper are provided in Table1

Table I Key Notations

Symbol	Unit	Feature Exaction Full name
$\alpha_{v/h}$	rad	Angular position of the beam in vertical/horizontal plane
$\alpha_{m/t}$	rad	Angular position of the main/tail propeller
g	m/s^2	Gravity acceleration
m	kg	Mass
J_1	kgm^2	Moment of Inertia of the beam
m_{T1}	kg	Total mass of the beam
l_{T1}	m	Centre of gravity of the beam
m_t	kg	Mass of the tail part of the beam
m_{tr}	kg	Mass of the horizontal rotor (tail rotor)
m_{ts}	kg	Mass of the tail shield
m_m	kg	Mass of the main part of the beam
m_{mr}	kg	Mass of the vertical rotor (the main rotor)
m_{ms}	kg	Mass of the main shield
l_t	m	Length of the tail part of the beam
l_m	m	Length of the main part of the beam
$r_{m/ts}$	m	Radius of the main/tail shield
$r_{mm/t}$	m	Radius of the main/tail rotor
J_2	kgm^2	Moment of Inertia of the counterbalance beam

m_b	kg	Mass of the counterbalance beam
m_{T2}	kg	Total mass of the counterbalance beam
m_{cb}	kg	Mass of the counter-weight
l_{T2}	m	Centre of gravity of the counterbalance beam
l_b	m	Length of the counterbalance beam
l_{cb}	m	Distance from the counter-weight to the pivot
r_{cb}	m	Radius of the counterbalance
L_{cb}	m	Length of the counterbalance
J_3	kgm^2	Moment of Inertia of the pivot
J_4	kgm^2	Moment of Inertia of the rear part of the pivot
m_h	kg	Mass of the pivot
m_{h1}	kg	Mass of the rear part of the pivot
h	m	Length of the pivot
h_1	m	chiều dài phần sau của chốt quay
J_{mm}	kgm^2	Moment of Inertia of motors
$J_{m/tp}$	kgm^2	Moment of Inertia of main/tail propellers
$\omega_{m/t}$	rad/s	Rotational speed of main/tail rotor
$J_{m/tr}$	kgm^2	Moment of Inertia of mail/tail rotor
H	m	High from the base to the pivot
k_g		Gyroscopic momentum parameter
M_v	Nm	Total moment in vertical plane
M_h	Nm	Total moment in horizontal plane
$M_{m/t}$	Nm	Total moment exerted on main/tail rotor
$B_{m/tr}$	kgm^2/s	Viscous friction constant of main/tail motors.
$B_{v/h}$	kgm^2/s	Viscous friction constant of the pivot in vertical/horizontal plane.
$F_{v/h}$	Nm	Sliding friction of the pivot in vertical/horizontal plane.
$\tau_{m/t}$	Nm	Electromagnetic moment of motor for vertical rotor/ horizontal motor

II. CONTROL DESIGN

Euler-Lagrange Equation

Lagrange function of TRMS is concluded from the sum of potential and kinetic energies [2] as follows.

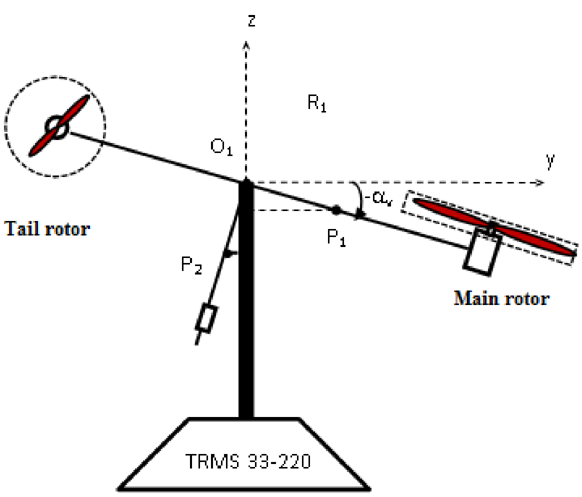

Figure 2. Vertical Position α_v of TRMS

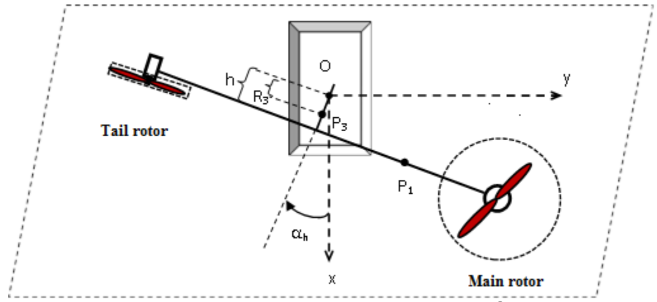

Figure 3. Horizontal Position α_h of TRMS

$$L = \frac{a_1}{2}\dot{\alpha}_v^2 + \frac{1}{2}\left(a_5 + a_4\cos^2\alpha_v\right)\dot{\alpha}_h^2 + a_6\omega_t\dot{\alpha}_v + a_7\omega_m\dot{\alpha}_h\cos\alpha_v + \frac{a_6}{2}\omega_t^2 + \frac{a_7}{2}\omega_m^2 - b_1\sin\alpha_v + \left(a_2\sin\alpha_v - a_3\cos\alpha_v\right)\dot{\alpha}_v\dot{\alpha}_h + b_2\cos\alpha_v \quad (1)$$

where

$a_1 = J_1 + J_2 + J_{tr}; a_2 = m_{T2}l_{T2}h; a_3 = m_{T1}l_{T1}h; a_4 = J_1 + J_{mr} - J_2$

$b_1 = m_{T1}l_{T1}g; b_2 = m_{T2}l_{T2}g; a_5 = J_3 + J_4 + m_{T2}h^2 + m_{T1}h^2 + J_2$

$a_6 = J_{tr}; a_7 = J_{mr}; \dot{\alpha}_t = \omega_t; \dot{\alpha}_m = \omega_m$

$$J_1 = \left(\frac{1}{3}m_t + m_{tr} + m_{ts}\right)l_t^2 + \left(\frac{1}{3}m_m + m_{mr} + m_{ms}\right)l_m^2 + \frac{1}{2}m_{ms}r_{ms}^2 + m_{ts}r_{ts}^2 + \frac{1}{2}m_{tr}r_{mt}^2 + \frac{1}{2}m_{mr}r_{mm}^2$$

$m_{T1} = m_t + m_{tr} + m_{ts} + m_m + m_{mr} + m_{ms}$

$$l_{T1} = \frac{(m_m/2 + m_{mt} + m_{ms})l_m - (m_t/2 + m_{tr} + m_{ts})l_t}{m_{T1}}$$

$$J_2 = \frac{1}{3}m_b l_b^2 + m_{cb}l_{cb}^2 + \frac{1}{4}m_{cb}r_{cb}^2 + \frac{1}{12}m_{cb}L_{cb}^2$$

$$m_{T2} = m_b + m_{cb},\ l_{T2} = \frac{m_b l_b/2 + m_{cb}l_{cb}}{m_{T2}}, J_4 = \frac{m_{h1}h_1^2}{3},\ J_4 = \frac{m_{h1}h_1^2}{3}$$

Apply Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = M_{ex}, \quad (2)$$

where $q = [\alpha_v \quad \alpha_h \quad \alpha_m \quad \alpha_t]^T$.

Then, the dynamic Euler-Lagrange of TRMS can be written as follows.

$$M\ddot{q} + C\dot{q} + G = M_{ex}, \quad (3)$$

where

$\ddot{q} = [\ddot{\alpha}_v \quad \ddot{\alpha}_h \quad \ddot{\alpha}_m \quad \ddot{\alpha}_t]^T$, $\dot{q} = [\dot{\alpha}_v \quad \dot{\alpha}_h \quad \dot{\alpha}_m \quad \dot{\alpha}_t]^T$

$G = [b_2\sin\alpha_v + b_1\cos\alpha_v \quad 0 \quad 0 \quad 0]^T$

$M_{ex} = [M_v \quad M_h \quad M_m \quad M_t]^T$

and

$$M = \begin{bmatrix} a_1 & z_1 & 0 & a_6 \\ z_2 & z_3 & a_7\cos\alpha_v & 0 \\ 0 & a_7\cos\alpha_v & a_7 & 0 \\ a_6 & 0 & 0 & a_6 \end{bmatrix}, \qquad (4)$$

where

$$z_1 = a_2\sin\alpha_v - a_3\cos\alpha_v, z_2 = a_2\sin\alpha_v - a_3\cos\alpha_v$$

$$z_3 = a_5 + a_4\cos^2\alpha_v$$

$$C = \begin{bmatrix} 0 & n_1 & 0 & 0 \\ n_2 & -a_4\sin\alpha_v\cos\alpha_v\dot{\alpha}_v & 0 & 0 \\ 0 & -a_7\dot{\alpha}_v\sin\alpha_v & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (5)$$

where

$$n_1 = \sin\alpha_v(a_4\dot{\alpha}_h\cos\alpha_v + a_7\omega_m)$$

$$n_2 = -a_4\sin\alpha_v\cos\alpha_v\dot{\alpha}_h - a_7\omega_m\sin\alpha_v + (a_2\cos\alpha_v + a_3\sin\alpha_v)\dot{\alpha}_v$$

$$M_v = l_m k_{fv}\dot{\alpha}_m|\dot{\alpha}_m|w - k_{tr}\dot{\alpha}_t|\dot{\alpha}_t| - B_v\dot{\alpha}_v - F_v sign(\dot{\alpha}_v) - k_g l_m k_{fv}\dot{\alpha}_m|\dot{\alpha}_m| w\dot{\alpha}_h\cos\alpha_v$$

(6)

where

$$w = \frac{1}{1 - \left(\frac{r_{mr}}{4(H + l_m\sin\alpha_v)}\right)^2}$$

$$M_h = l_t k_{fh}\dot{\alpha}_t|\dot{\alpha}_t|\cos\alpha_v - k_{tm}\dot{\alpha}_m|\dot{\alpha}_m|\cos\alpha_v - B_h\dot{\alpha}_h - F_h sign(\dot{\alpha}_h) - C_c(\alpha_h - \alpha_{h0})$$

(7)

$$M_m = \tau_m - sign(\dot{\alpha}_m)k_{tv}\dot{\alpha}_m^2 - B_{mr}\dot{\alpha}_m \qquad (8)$$

$$M_t = \tau_t - sign(\dot{\alpha}_t)k_{th}\dot{\alpha}_t^2 - B_{tr}\dot{\alpha}_t. \qquad (9)$$

Consequently, the above Euler-Lagrange model shows that TRMS lacks actuators with 2 inputs and 4 outputs.

A. *Control Design*

Firstly, equation (10) can be rewritten as follows:

$$M\ddot{q} + C\dot{q} + G = M_{ex} \qquad (11)$$

or

$$\begin{cases} M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1 = 0 \\ M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + \phi_2 = \begin{bmatrix}\tau_m \\ \tau_t\end{bmatrix} \end{cases} \qquad (12)$$

where

$$q_1 = [\alpha_v \quad \alpha_h]^T, \; q_2 = [\alpha_m \quad \alpha_t]^T$$

$$M_{11} = \begin{bmatrix} a_1 & a_2\sin\alpha_v - a_3\cos\alpha_v \\ a_2\sin\alpha_v - a_3\cos\alpha_v & a_5 + a_4\cos^2\alpha_v \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} 0 & a_6 \\ a_7\cos\alpha_v & 0 \end{bmatrix}, M_{21} = \begin{bmatrix} 0 & a_7\cos\alpha_v \\ a_6 & 0 \end{bmatrix}$$

$$M_{22} = \begin{bmatrix} a_7 & 0 \\ 0 & a_6 \end{bmatrix}$$

and

$$C_{11} = \begin{bmatrix} 0 & \sin\alpha_v(a_4\dot{\alpha}_h\cos\alpha_v + a_7\omega_m) \\ x & -a_4\sin\alpha_v\cos\alpha_v\dot{\alpha}_v \end{bmatrix}$$

$$x = -a_4\sin\alpha_v\cos\alpha_v\dot{\alpha}_h - a_7\omega_m\sin\alpha_v + (a_2\cos\alpha_v + a_3\sin\alpha_v)\dot{\alpha}_v$$

$$C_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C_{21} = \begin{bmatrix} 0 & -a_7\dot{\alpha}_v\sin\alpha_v \\ 0 & 0 \end{bmatrix}, C_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$\phi_1 = \begin{bmatrix} b_2\sin\alpha_v + b_1\cos\alpha_v - M_v \\ -M_h \end{bmatrix}$$

$$\phi_2 = \begin{bmatrix} sign(\omega_m)k_{tv}\omega_m^2 + B_{mr}\omega_m \\ sign(\omega_t)k_{th}\omega_t^2 + B_{tr}\omega_t \end{bmatrix}$$

From (4), we have

$$\begin{cases} M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1 = 0 \\ M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + \phi_2 = \begin{bmatrix}\tau_m \\ \tau_t\end{bmatrix} \end{cases} \qquad (13)$$

or

$$\begin{cases} \ddot{q}_2 = -M_{12}^{-1}(M_{11}\ddot{q}_1 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1) \\ M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + \phi_2 = \tau \end{cases} \qquad (14)$$

where $\tau = \begin{bmatrix}\tau_m \\ \tau_t\end{bmatrix}$.

A control problem for TRMS is that the first joint variable q_1 has to approach the reference trajectory q_{1r} regardless of the second joint parameter q_2. It can be seen that the first controller, namely inner loop, can be expressed in the following the equations as follows.

$$\begin{aligned} \tau &= M_{21}\ddot{q}_1 + C_{21}\dot{q}_1 + \phi_2 - M_{22}M_{12}^{-1}(M_{11}\ddot{q}_1 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1) \\ &= (M_{21} - M_{22}M_{12}^{-1}M_{11})\ddot{q}_1 + C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + \phi_2 - M_{22}M_{12}^{-1}(C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1) \end{aligned} \qquad (15)$$

or $\tau = D\ddot{q}_1 + \varphi(q,\dot{q})$ (16)

where

$$\begin{cases} \varphi(q,\dot{q}) = C_{21}\dot{q}_1 + C_{22}\dot{q}_2 + \phi_2 - M_{22}M_{12}^{-1}(C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + \phi_1) \\ D = M_{21} - M_{22}M_{12}^{-1}M_{11} \end{cases}$$

.

The closed loop system will have a joint variable set that is derived from the controller (17) and the model (18) as follows:

$$D\ddot{q}_1 = Dv \quad \Leftrightarrow \quad \ddot{q}_1 = v \,. \qquad (19)$$

In other words, the inner controller in Equation (20) has linearized q_1 in Equation (21) in the whole joint variable domain. Therefore, the inner controller can be rewritten as follows.

$$\tau = Dv + \varphi(q,\dot{q}) \,. \qquad (22)$$

The controller in Equation (9) is called the exact linearization controller. Obviously, despite the q_1 linearization, the system is unstable because of the second order integral in Equation (23).

After that, in order to apply the unstable q_1 to the desired values q_{1r}, the outer loop controller will be used.

$$v = \ddot{q}_{1r} + K_1 e_1 + K_2 \dot{e}_1 \,, \qquad (24)$$

where $e_1 = q_{1r} - q_1$. (25)

e_1 denotes the tracking error; K_1, K_2, are two arbitrary symmetric positive definite matrices. The outer loop controller in (26) probably makes the tracking error e_1 decrease to zero. From (27) and(28), we have:

$$\ddot{q}_1 = \ddot{q}_{1r} + K_1 e_1 + K_2 \dot{e}_1 \,, \qquad (29)$$

and

$$\underline{0} = \ddot{e}_1 + K_1 e_1 + K_2 \dot{e}_1 \,, \qquad (30)$$

and

$$\begin{pmatrix} \dot{e}_1 \\ \ddot{e}_1 \end{pmatrix} = \begin{pmatrix} 0 & I \\ -K_1 & -K_2 \end{pmatrix} \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix} = K \begin{pmatrix} e_1 \\ \dot{e}_1 \end{pmatrix} . \qquad (31)$$

where

$$K = \begin{pmatrix} 0 & I \\ -K_1 & -K_2 \end{pmatrix} . \qquad (32)$$

is the Hurwitz matrix.

Finally, the addition of the outer loop controller leads the system to having the total controller as shown in Equation(33).

$$\tau = D\left(\ddot{q}_{1r} + K_1 e_1 + K_2 \dot{e}_1\right) + \varphi(q,\dot{q}) \,. \qquad (34)$$

Figure 4 illustrates the diagram of the tracking system according to the exact linearization of q_1.

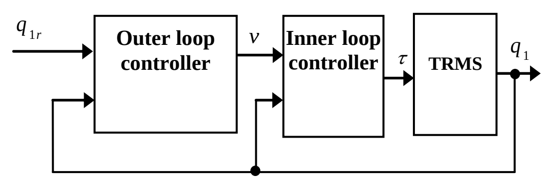

Figure 4. TRMS Control Diagram

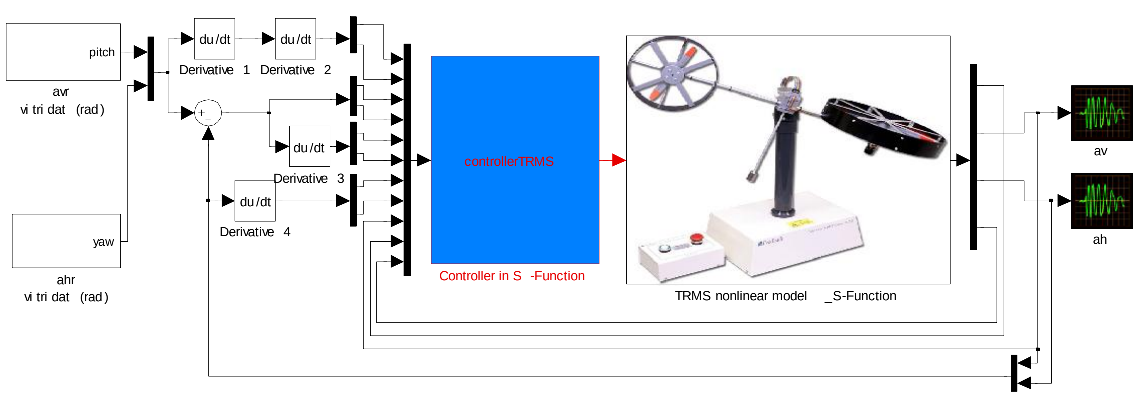

Figure 5. TRMS Control System Simulation

III. SIMULATION RESULTS

With the aim of verifying the performance of the proposed controller, MATLAB-SIMULINK has been used to conduct simulations according to parameters in Table II. The model of TRMS and controller are implemented via S-function. For the sake of simplicity, there will be one S-function for both the outer loop controller and the inner loop controller. The output position $\alpha_{v/h}$=av/h (α_v=pitch angle, α_h=yaw angle) and the reference angle $q_{1r} = [\text{avr ahr}]$. In order to observe effects of the cross reaction, the simulation with the 2 degree of freedom model and 2 reference inputs have been created. Let's choose K1=1.I and

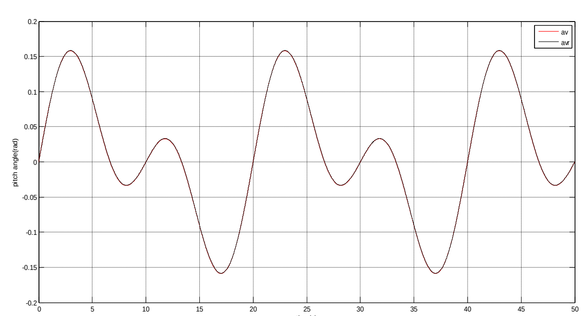

Figure 6. Vertical Position α_v with the Reference Input 0.09 sin(0.6283t)+ 0.09 sin(0.3142t)

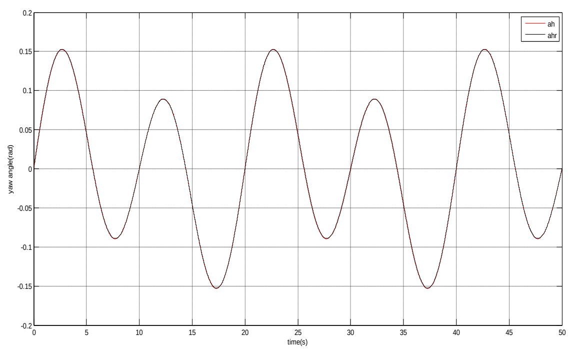

Figure 7. Horizontal Position α_h with the Reference Input: 0.12 sin(0.6283t)+ 0.045 sin(0.3142t)

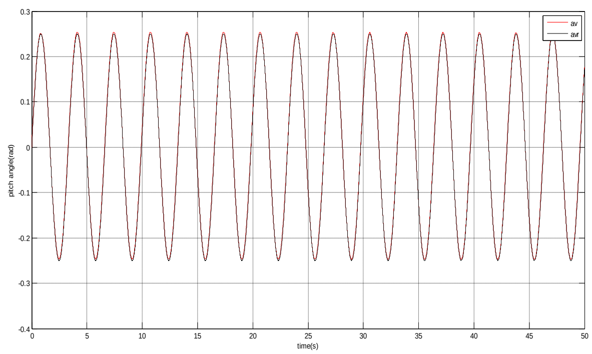

Figure 8. Vertical Position α_v with the Reference Input: 0.25 sin(1.9t)

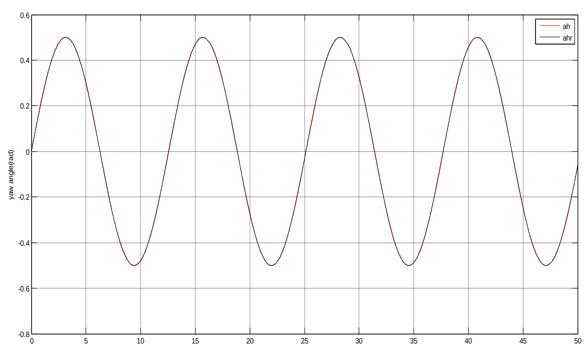

Figure 9. Horizontal Position α_h with the Reference Input: 0.5 sin(0.2t)

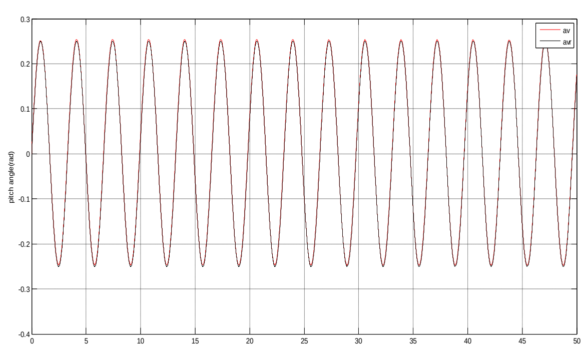

Figure 10. Vertical Position α_v with the Reference Input: 0.25 sin(1.9t)

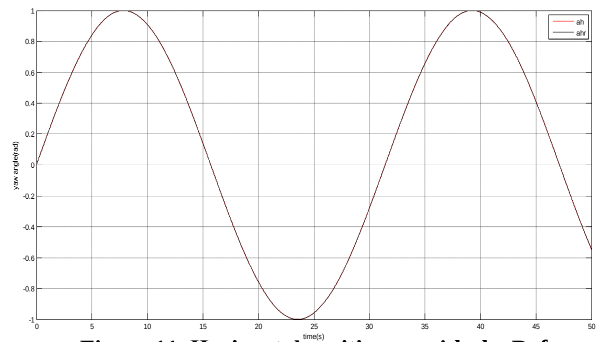

Figure 11. Horizontal position α_h with the Reference input: 1 sin(0.5t)

Table II TRMS Parameters

l_t	0.282 m	k_g	0.05
l_m	0.254 m	h	6e-2 m
l_b	0.265 m	h_1	0.02 m
l_{cb}	0.25 m	m_{h1}	0.05 kg
r_{ms}	0.155 m	m_h	0.09 kg
r_{ts}	0.1 m	g	9.81 m/s^2
m_{tr}	0.221 kg	L_c	3e-2 m
m_{mr}	0.236 kg	r_{cb}	1e-2 m
m_{cb}	0.068 kg	H	0.5 m
m_m	0.014 kg	r_{mt}	0.007 m
m_t	0.015 kg	r_{mm}	0.007 m
m_b	0.022 kg	m_{mrr}	0.042 kg
m_{ts}	0.119 kg	m_{trr}	0.016 kg
m_{ms}	0.219 kg	k_{chp}	0.00854
J_{mr}	21.624e-5 kgm^2	J_{tr}	3.1432e-5kgm^2
B_{mr}	4.5e-5 kgm^2/s	B_{tr}	2.3e-5 kgm^2/s
k_{tv}	23.03e-6	k_{th}	10e-6
B_v	0.6e-2 Nms/rad	F_v	0.1e-2 Nms/rad
B_h	0.1 Nms/rad	F_h	0.01 Nms/rad
C_c	0.016 Nm/rad	α_{h0}	-0.4602 rad

IV. CONCLUSIONS AND FUTURE WORK

In this paper, the 2-degree of freedom movement of TRMS has been discussed. The mathematical model of TRMS based on Euler Lagrange equations has been built by using MATLAB/SIMULINK. The controller is designed to be able to work in vertical and horizontal planes. Outputs of the controller have been compared to the reference inputs. It is clear that TRMS tracks the desired trajectory accurately. In our future work, the application of the controller for a real object would be implemented.

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