Adaptive Robust Control for Perturbed Coupled Nonlinear Twin Rotor Multiple Input-Multiple Output System

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Abstract

The paper proposes an approach to design adaptive robust control for tracking control a TRMS (Twin Rotor MIMO System), which is a perturbed coupled nonlinear system with two degrees of freedom. The here proposed controller uses an infinite horizon and continuous time nonlinear model. Hence it always guarantees the adaptive robust tracking stability of obtained closed loop systems in real time, without using an additional penalty function in objective function as usual. The obtained simulation result by using this controller has confirmed its promising applicability in practice.

Keywords — *TRMS, tracking control, Adaptive robust control, LQR*

I. INTRODUCTION

Some decades ago, the research of the control of Twin Rotor MIMO Systems (TRMS), which is depicted symbolically in Fig.1, has been considered as a benchmark of controlling the flight of air vehicle such as helicopter or UAV (unmanned air vehicle). Therefore several control methods and techniques for TRMS have been proposed and implemented regularly, in which the conventional control method namely PID and the modern one such as MPC are all employed. For example: in 2002 Ahmad et al. provided the open loop control along longitudinal axis [1]; in 2007 Lu at al. proposed the time optimal control based on LQR [2], in 2010 Pratap et al. introduced a sliding mode state observer controller [3]; in 2014 Pandey et al. presented a PID controller, and in 2012 proposed Ramalakshmi et al. a nonlinear control approach based on Lyapunov [4], or an optimal LQR for the stabilization around an equilibrium had been introduced by Pandey et al. in 2015 [5] etc.,. Moreover, if in control problem of TRMS, there are some required constraints which are not ignorable, then the methods introduced by Akbar Rahideh in 2009 based on MPC seem to be good alternative solutions to overcome [6]. However, all these methods are restricted if the TRMS is additionally disturbed and if the trajectory to be tracking is a complicatedly desired hover [12].

This approach can be considered as an extension of the method, which is already proposed in [7] for bilinear discrete time systems. The extension here means that this approach is established for nonlinear continuous time systems without time discretizing them as well as without implementation of any constrained optimization algorithm as usual by applying MPC techniques.

Moreover, since the discrete model obtained by discretizing could not reflect all inter-sample behaviors of the real system, which may be cause a number of critical event in practical applications, this proposed sample data controller with its avoidance of model discretization improves therefore indirectly the internal control performance of closed loop systems.

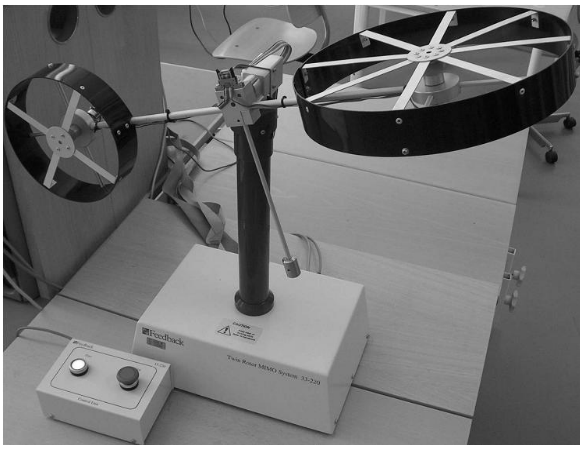

Fig 1: Twin rotor multiple input-multiple output system

II. MAIN CONTENT

A. *Nonlinear continuous time model of TRMS*

A various number of TRMS model has been proposed in [8, 9, 10]. Under which this paper uses the TRMS model given in [10], where the pivot length is not negligible. Simulation results obtained in [10] show that this model is much precise than the other introduced earlier. This model was established by using the Euler-Lagrange equations and has an equivalent continuous time state equation as follows:

$$\begin{cases} \dot{x} = f(x,u) \\ y = Cx \end{cases} \quad (1)$$

where

$\underline{x} = (\alpha_v, \alpha_h, \dot{\alpha}_v, \dot{\alpha}_h, \omega_m, \omega_t)$ is the vector of pitch angle, yaw angle, derivative of pitch angle, derivative of yaw angle, velocity of main rotor and velocity of tail rotor respectively.

$$f(x,u)=\left(\dot{\alpha}_v,\dot{\alpha}_h,f_1(x,u),\ \dots\ ,f_4(x,u)\right)^T \text{ with}$$
$$\left(f_1(x,u),\ \dots\ ,f_4(x,u)\right)^T=M(q)^{-1}\left[I(u)-N(q,\dot{q})\right]$$

where $q=(\alpha_v,\alpha_h,\alpha_m,\alpha_t)$ is the vector of pitch angle, yaw angle, angle of main rotor and angle of tail rotor respectively, and

$$M(q)=\begin{pmatrix} a_1 & i_{12} & 0 & a_6 \\ i_{21} & i_{22} & a_7\cos\alpha_v & 0 \\ 0 & a_7\cos\alpha_v & a_7 & 0 \\ a_6 & 0 & 0 & a_6 \end{pmatrix}$$

$$i_{12}=a_2\sin\alpha_v-a_3\cos\alpha_v,\ i_{21}=a_2\sin\alpha_v-a_3\cos\alpha_v$$
$$i_{22}=a_5+a_4\cos^2\alpha_v$$

$$N(q,\dot{q})=\begin{pmatrix} 0 & c_{12} & 0 & 0 \\ c_{21} & 0 & 0 & 0 \\ c_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\dot{q}+\begin{pmatrix} b_2\sin\alpha_v+b_1\cos\alpha_v \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$c_{12}=\sin\alpha_v\left(a_4\dot{\alpha}_h\cos\alpha_v+a_7\omega_m\right)$$
$$c_{21}=-2a_4\sin\alpha_v\cos\alpha_v\dot{\alpha}_h-a_7\omega_m\sin\alpha_v$$
$$+\left(a_2\cos\alpha_v+a_3\sin\alpha_v\right)\dot{\alpha}_v$$
$$c_{31}=-a_7\dot{\alpha}_h\sin\alpha_v,\ I(\underline{u})=\left(I_v,I_h,I_m,I_t\right)^T$$
$$I_v=l_mk_{fv}\omega_m\left|\omega_m\right|\gamma-k_{tr}\omega_r\left|\omega_r\right|-B_v\dot{\alpha}_v-F_v\,\mathrm{sgn}(\dot{\alpha}_v)$$
$$-k_gl_mk_{fv}\omega_m\left|\omega_m\right|\gamma\dot{\alpha}_h\cos\alpha_v$$
$$\gamma=\frac{1}{1-\left(r_{mr}/4\left(H+l_m\sin\alpha_v\right)\right)^2}$$
$$I_h=l_tk_{fh}\omega_r\left|\omega_r\right|\cos\alpha_v-k_{tm}\omega_m\left|\omega_m\right|\cos\alpha_v$$
$$-B_h\dot{\alpha}_h-F_h\,\mathrm{sgn}(\dot{\alpha}_h)-C_c\left(\alpha_h-\alpha_{h0}\right)$$
$$I_m=\tau_m-\mathrm{sgn}(\omega_m)k_{tv}\omega_m^2-B_{mr}\omega_m$$
$$I_t=\tau_t-\mathrm{sgn}(\omega_t)k_{th}\omega_t^2-B_{tr}\omega_t$$
$$u=\left(u_1,u_2\right)^T=\left(\tau_m,\tau_t\right)^T$$

$$y=\left(\alpha_v,\alpha_h\right)^T=Cx$$
$$=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}x$$

B. Receding Horizon Control model

It is obviously that the function $f(x,u)$ of TRMS model given in (1) is continuously differentiable. Therefore, at the current time instant t_k and during a short time interval $[t_k,t_{k+1})$ with $t_{k+1}=t_k+\delta_k,\ 0<\delta_k<<1$ afterward it can be approximated by:

$$f(x,u)\approx f(x_k,u_{k-1})+A_k\left(x-x_k\right)+B_k\left(u-u_{k-1}\right)$$
$$=A_kx+B_ku+d_k$$

where

$$A_k=\left.\frac{\partial f}{\partial x}\right|_{x_k,u_{k-1}},\quad B_k=\left.\frac{\partial f}{\partial u}\right|_{x_k,u_{k-1}} \qquad (2)$$
$$d_k=f(x_k,u_{k-1})-A_kx_k-B_ku_{k-1}$$

Therefore, the original nonlinear state equation in (1) can be now replaced accordingly during the same time interval $[t_k,t_{k+1})$ by a linear model:

$$\dot{x}=f(x,u)=A_kx+B_ku+d_k$$

It is clearly that all matrices A_k, B_k and vectors d_k is determined, since $x_k=x(t_k)$ at the current time instant t_k are measureable and $u_{k-1}=u(t_{k-1})$ at the previous time instant is already known.

Hence, the original nonlinear model (1) of TRMS can be now replaced accordingly during the current time interval $[t_k,t_{k+1})$ by the following determined LTI model:

$$\begin{cases}\dot{x}=A_kx+B_ku+d_k \\ y=Cx\end{cases} \qquad (3)$$

Each model (3) can be replaced the original model (1) only during the appropriate time interval $[t_k,t_{k+1})$ and all of them together with $k=0,1,\dots$ will be called hereafter the receding horizon LTI models as depicted in Fig.2.

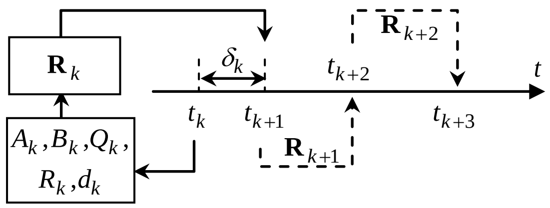

Fig 2: Receding horizon control with optimization

C. Adaptive robust controller design

In the following, the obtained LTI model (3) will be used to design the state feedback controller $u(x)$ based on linear quadratic variation technique to control TRMS (1) during an appropriate time interval $[t_k,t_{k+1})$. The obtained optimal controller, which is obviously also valid only during the next time interval $[t_{k+1},t_{k+2})$, will be denoted by $\mathbf{R}_k$, $k=0,1,\dots$ as illustrated in Fig.2. The merged controller from them:

$$\mathbf{R}=\mathbf{R}_k,\ t_{k+1}\le t<t_{k+2},\ k=0,1,\dots \qquad (4)$$

for all time domain t, will be called the receding horizon controller. Consistently, the purpose of this receding horizon controller $\mathbf{R}$ is the asymptotical convergence to zero of tracking error $e_k=w(t_k)-y(t_k)$ of closed loop system for all k, where $w(t)$ is the desired output.

With (4) the designing of $\mathbf{R}$ can be now replaced by determining of all instant controllers

$\mathbf{R}_k$, $k=0,1,\ldots$. In order to avoid tracking errors $w(t_{k-1})-y(t_{k-1})$ by designing of $\mathbf{R}_k$, which could be remaining from previous control time instant t_{k-1}, an alternative desired value r_k for the current time instant t_k given below will be used instead of the original $w(t_k)$:

$$r_k = w(t_k)+\left[w(t_{k-1})-y(t_{k-1})\right]. \qquad (5)$$

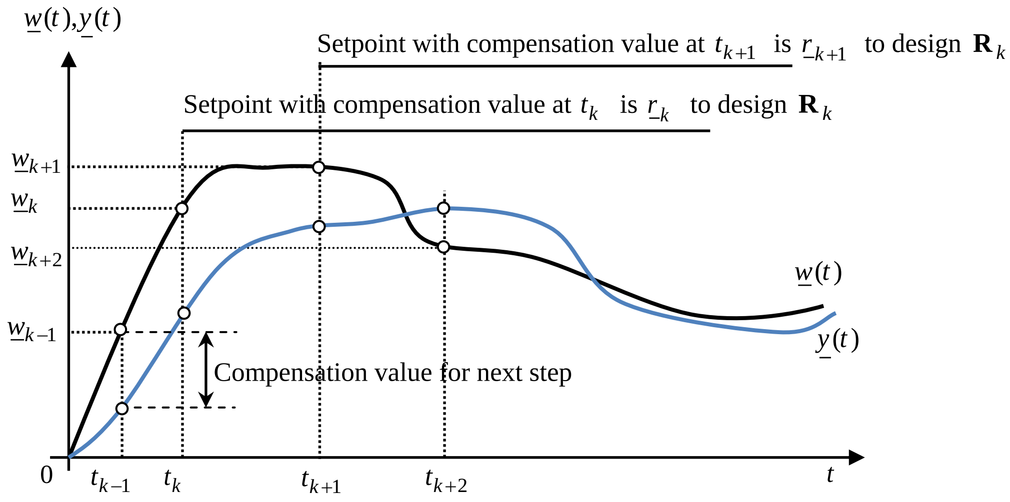

Fig 3: Set Point with Compensation Value to Design Control

Now, for a possible usage of optimal variation technique to design the controller $\mathbf{R}_k$, such that the outputs y of linear time invariant system (3) converge asymptotically to desired output r_k, it is required firstly this tracking problem to be converted correspondingly in a stabilizing control problem.

Signify the steady state of closed loop system of (3) after tracking phase with $x_s[k]$ and $u_s[k]$, then this steady state must be satisfied:

$$\begin{cases} 0 = A_k x_s[k]+B_k u_s[k]+d_k \\ r_k = C x_s[k] \end{cases}$$

which implies immediately:

$$\begin{pmatrix} x_s[k] \\ u_s[k] \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} -d_k \\ r_k \end{pmatrix} \qquad (6)$$

if the matrix

$$F_k = \begin{pmatrix} A_k & B_k \\ C & \mathbf{0} \end{pmatrix} \qquad (7)$$

is invertible. Then, with the new symbols:

$$z = x - x_s[k],\; v = u - u_s[k]$$

the tracking control problem of (3) to an alternative desired value r_k will be converted correspondingly in the stabilization problem of following nominal system:

$$\dot{z} = A_k z + B_k v\,. \qquad (8)$$

It is easy to recognize that for the invertibility of matrix F_k the number of inputs u and of output y must be coincided.

For optimal stabilizing this above obtained nominal system the following cost-function could be used:

$$J_k = \frac{1}{2}\int_0^\infty \left[z^T Q_k z + v^T R_k v\right] dt \to \min \qquad (9)$$

where Q_k, R_k are two arbitrarily chosen symmetric positive definite matrices. Thence, based on the continuous time variation technique the optimal input v it is deduced:

$$v = -R_k^{-1} B_k^T L_k z = -\mathbf{R}_k z \qquad (10)$$

where the symmetric positive definite matrix L_k is obtained by solving the algebraic Riccati equation:

$$L_k B_k R_k^{-1} B_k^T L_k - A_k^T L_k - L_k A_k = Q_k \qquad (11)$$

and it is equivalent to:

$$\begin{aligned} u(x) &= v + u_s[k] \\ &= -\mathbf{R}_k z + u_s[k] \\ &= -R_k^{-1} B_k^T L_k \left(x - x_s[k]\right) + u_s[k] \end{aligned} \qquad (12)$$

The obtained value $u(x)$ above is sent subsequently to the system (1) as control signal for a while of $t_{k+1} \le t < t_{k+2}$. To receive the next control value for the next time interval $t_{k+2} \le t < t_{k+3}$ all calculation steps above have to be repeated.

For a convenient implementation of proposed approach, the following algorithm has been established, which summarizes completely all calculation steps (2)-(13) given above.

1) Choose arbitrarily two symmetric positive matrices Q, R and their update factors $0<\mu<1,\ \gamma>1$. Select

a sufficiently small moving distance $\delta > 0$ along time axis for the control horizon. Set $\underline{\hat{x}} = \underline{0}$, $\underline{\hat{u}} = \underline{0}$, $\underline{\hat{y}} = \underline{0}$ and $t = 0$.

2) Measure the current state and output vector $\underline{x}, \underline{y}$ from system and then determine $A, B, \underline{d}, \underline{\hat{\varsigma}}, \underline{\hat{\upsilon}}, \underline{r}$ as follows:

$$A = \left.\frac{\partial f}{\partial x}\right|_{\underline{x},\underline{\hat{u}}}, \quad B = \left.\frac{\partial f}{\partial u}\right|_{\underline{x},\underline{\hat{u}}}, \quad \underline{d} = \underline{f}(\underline{x}, \underline{\hat{u}}) - A\underline{\hat{x}} - B\underline{\hat{u}}$$

$$\underline{\hat{\varsigma}} = (\underline{x} - \underline{\hat{x}})/\delta - \underline{f}(\underline{x}, \underline{\hat{u}}), \quad \underline{\hat{\upsilon}} = \underline{y} - C\underline{\hat{x}}$$

$$\underline{r} = \underline{w}(t) + \left[\underline{w}(t-\delta) - \underline{\hat{y}}\right]$$

3) Determine: $F = \begin{pmatrix} A & B \\ C & \mathbf{0} \end{pmatrix}$ If F is singular, then go back to the step 2).

4) Calculate: $\begin{pmatrix} \underline{x}_s \\ \underline{u}_s \end{pmatrix} = F^{-1} \begin{pmatrix} -\underline{d} - \underline{\hat{\varsigma}} \\ \underline{r} - \underline{\hat{\upsilon}} \end{pmatrix}$

5) Calculate $L, \underline{u}$ respectively as follows:

$$LBR^{-1}B^T L - A^T L - LA = Q,$$

$$\underline{u} = -R^{-1}B^T L(\underline{x} - \underline{x}_s) + \underline{u}_s$$

6) If $\underline{u} \notin U$ then set $R := \gamma R$ and go back to the step 5).

7) Send $\underline{u}$ to the controlled object for a while of δ.

8)Set $\underline{\hat{x}} = \underline{x}$, $\underline{\hat{u}} = \underline{u}$, $\underline{\hat{y}} = \underline{y}$, $R := \mu R$ and $t := t + \delta$. Then go back to the step 2).

Note that within the algorithm, not any solution $\underline{v}$ obtained from (11) to stabilizing the nominal system (9) could guarantee definitely the satisfaction of the required constraint $\underline{u} \in U$. However, based on the obviousness:

$$\lim_{\|R_k\| \to \infty} \underline{u} = \lim_{\|R_k\| \to \infty} \left(-R_k^{-1} B_k^T L_k \left(\underline{x} - \underline{x}_s[k]\right) + \underline{u}_s[k]\right)$$
$$= \underline{u}_s[k]$$

it could be needed for satisfying this unavoidable constraint $\underline{u} \in U$ an assumption, that $\underline{u}_s[k] \in U$ satisfies for all k.

III. SIMULATION RESULTS

The following simulation was carried out with particular parameter values of TRMS given in Table I.

Table I. Physical parameters of the trms

Symbol	Definition	Value	Unit
g	Gravity acceleration	9.81	m/s^2
m_t	Mass of the tail part of the beam	0.015	kg
m_{tr}	Mass of the tail rotor	0.221	kg
m_{ts}	Mass of the tail shield	0.119	kg
m_m	Mass of the main part of the beam	0.014	kg
m_{mr}	Mass of the main rotor	0.236	kg
m_{ms}	Mass of the main shield	0.219	kg
l_t	Length of the tail part of the beam	0.282	m
l_m	Length of the main part of the beam	0.254	m
r_{ms}	Radius of the main shield	0.155	m
r_{ts}	Radius of the tail shield	0.1	m
r_{mm}	Radius of the main rotor	0.007	m
r_{mt}	Radius of the tail rotor	0.007	m
m_b	Mass of the counterbalance beam	0.022	kg
m_{cb}	Mass of the counter-weight	0.068	kg
l_b	Length of the counterbalance beam	0.265	m
l_{cb}	Distance from the counter-weight to the pivot	0.25	m
r_{cb}	Radius of the counterbalance	1e-2	m
L_{cb}	Length of the counterbalance	3e-2	m
m_b	Mass of the pivot	0.09	kg
m_{b1}	Mass of the rear part of the pivot	0.05	kg
h	Length of the main part of the pivot	6e-2	m
h_1	Length of the tail part of the pivot	0.02	m
J_{mr}	Moment of Inertia of main rotor	21.624e-5	kgm^2
J_{tr}	Moment of Inertia of tail rotor	3.1432e-5	kgm^2
H	High from the base to the pivot	0.5	m
B_{mr}	Viscous friction constant of main motor	4.5e-5	kgm^2/s
B_{tr}	Viscous friction constant of tail motor	2.3e-5	kgm^2/s
B_v	Viscous friction constant of the pivot in vertical plane	0.6e-2	kgm^2/s
B_h	Viscous friction constant of the pivot in horizontal plane	0.1	kgm^2/s
k_{fv}	Coefficient of thrust due to main rotor	1.13e-5	kgm
k_{fh}	Coefficient of thrust due to tail rotor	2.23e-6	kgm
k_{tv}, k_{tm}	Main rotor drag coefficient	3.646e-7	kgm^2
k_{th}, k_{tt}	Tail rotor drag coefficient	2.436e-8	kgm^2
C_c	Cable spring constant	0.016	Nm/rad
α_{b0}	Steady yaw angle	-0.4602	rad
F_v	Sliding friction of the pivot in vertical plane	0.1e-2	Nm
F_h	Sliding friction of the pivot in horizontal plane	0.01	Nm

as well as with:

$a_1 = 0.0347, a_2 = 0.0013, a_3 = 2.497e-4, a_4 = 0.029,$
$a_5 = 0.0047, a_6 = 1.24e-5, a_7 = 6.36e-5, b_1 = 0.0408,$
$b_2 = 0.2154$

Obtained simulation results, which are obtained by applying proposed control algorithm above to adaptively tracking control the TRMS, are exhibited in Fig.4 and Fig.5. The simulation result of tracking behavior of system outputs $y(t)$ to the desired references $w(t)$ illustrated in this figure showed ones the tracking performance as desire.

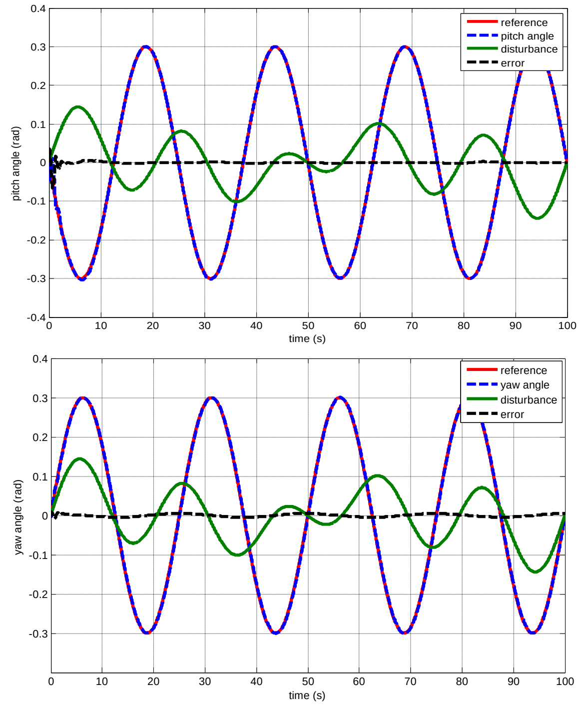

Fig 4: Sinus References and Time Dependent Disturbances

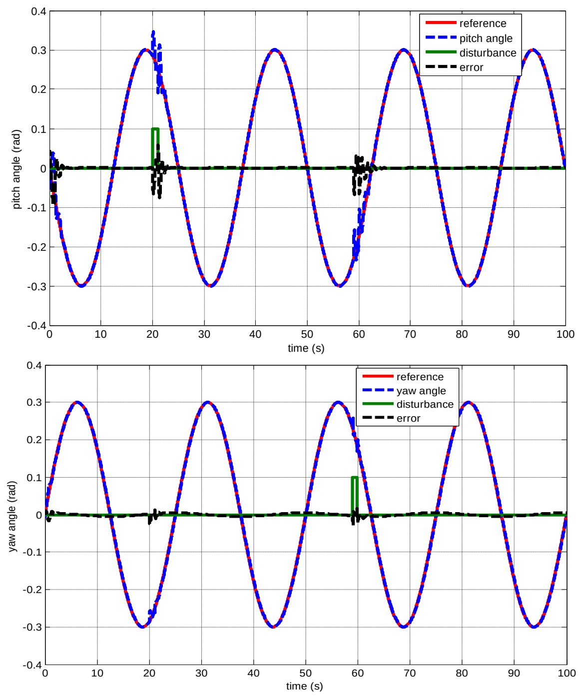

Fig 5: Sinus References and Pulse Disturbances

IV. CONCLUSIONS

The paper has presented an approach for asymptotically tracking control to any desired ability of closed loop system to desired sinus hovers $w(t)$ is exhibited. Again, the obtained convergence trajectory of a nonlinear smooth continuous time system subjected to unavoidable constraints of control signals $u(t) \in U$. This approach is created based on receding horizon technique with the movement of flexibly adjustable LQR along time axis. Thus, this proposed approach acts essentially as an adaptive constrained optimal controller in real time sample data systems.

To verify the desired control performance of proposed approach, this method had been also in the paper implemented to simulate the tracking control of a TRMS. The simulation result has definitely confirmed that the adaptive tracking performance has met the desired expectation and therefore the proposed method could be now completely applicable in practice.

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