

# Tracking Control via ISS Stabilization and for Nonlinear System

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## Abstract

In this work, we introduce a new adaptive tracking controllers dealing with a MIMO nonlinear system in presence of input noise. The adaptive tracking controllers base on Input to State Stability (ISS) stabilization. An ISS stabilization is used to make the error tracking smoothly converges to an arbitrary sufficient small area around the neighborhood of the origin. The set of controller's parameter, which is a satisfy Hurwitz polynomial, is then updated by adaptive laws via a model reference system. Thanks to Lyapunov's theory, the stability of the closed loop system is demonstrated. Finally, simulation results corresponding to an Active Magnetic Bearing system (AMBs) illustrate the effectiveness of our proposed combination.

**Keywords** — ISS stabilization; MRAS; disturbance estimation; MIMO system; adaptive control; AMBs

## I. INTRODUCTION

In this paper, we consider a nonlinear MIMO system, which can be presented in Euler-Lagrange equation form. The Euler-Lagrange nonlinear system is a common model of many plants such as robot manipulators, mechanical Tora systems, Lavitat mechanical systems, etc. Consider the following dynamic model of Euler-Lagrange nonlinear system given by

$$\ddot{\mathbf{q}} + \mathbf{h}(t) = \mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) \quad (1)$$

where  $\mathbf{q} = (q_1, q_2, \dots, q_n)^T$ ;  $\mathbf{u} = (u_1, u_2, \dots, u_n)^T$  are output and input vectors respectively,  $\mathbf{h}(t) \in \mathbb{R}^n$  denotes the inputs noise vector,  $\mathbf{D}(\mathbf{q})$  is a positive definite symmetric function matrix depending on  $\mathbf{q}$ ,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is a coriolis matrix and vector of centrifugal which depends on  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$  and the  $\mathbf{G}(\mathbf{q})$  is vector of gravity terms. The control objective is making the system (1) is stable tracking a prior trajectory  $\mathbf{q}_m$  called reference trajectory, that means  $\lim_{t \rightarrow \infty} \mathbf{e}(t) = 0$ ,  $|\mathbf{e}(t)|$

is finite, where  $\mathbf{e}(t) = \mathbf{q} - \mathbf{q}_m$  is tracking error.

With the assumption that system has exactly known parameters and is unaffected by noise, there are many

successful control strategies in [1] including the state feedback controller. In other cases, when the system (1) has the presence of uncertainty and resistance to noise, most of adaptive control methodologies modulated by principles of certainty equivalence [4] are summarized in [2], [3]. These methods ensure error between output signal and the desired trajectory and its derivation which are always asymptotic. Nevertheless, in those papers, they do not focus on the performance of closed loop system, for instance, the settling time, overshoot and error between the output signal and its reference. Therefore, these mitigating problems are focused in this paper.

[5], [6] consider the problem in the case of presenting of input noise  $\mathbf{h}(t) \neq 0$ . The controller can derive the tracking errors to a neighborhood of the origin defined with a quite large radius. In [6], [7] sliding mode control is applied for this class of systems. However, a sliding mode control suffers from a well-known problem chattering due to the high gain and high-speed switching control. The undesirable chattering may excite previously unmodelled system dynamics and damage actuators, resulting in unpredictable stability. Therefore, the main contributions of this paper are summarized as follows: First, we develop an adaptive tracking control via ISS stabilization based on state feedback controller, and ensure the stability of the closed-loop system. Second, the set of controller's parameter, which is a satisfy Hurwitz polynomial, is then updated by adaptive laws via a model reference system.

The remainder of this paper is organized as follows. In section II, we introduce the new adaptive tracking control via ISS stabilization. Section III then provides the new adaptive tracking control combined with disturbance estimation. Simulation results are given in section IV and section V concludes this work.

## II. NEW ADAPTIVE TRACKING CONTROL VIA ISS STABILIZATION (NAC\_ISS)

Consider the following dynamic model of Euler-Lagrange with input noise (EULI) given as (1), with the assumption that all the state of the system are measurable and available for feedback. The state feedback controller introduced in [1], [2], and [5] without input noise

$$u = D(q) \left( \ddot{q}_m + K_1 \dot{e} + K_2 e \right) + C(q, \dot{q}) + G(q) \quad (2)$$

where  $K_1, K_2$  are two positive definite symmetric matrices. This method guarantees the closed loop system is asymptotically stable. In case,  $h(t) \rightarrow 0$  the new adaptive tracking controller in such as

$$u = D(q) \left( \ddot{q}_m + K_1 \dot{e}_p + K_2 e_p - \dot{v}(t) \right) + C(q, \dot{q}) + G(q) \quad (3)$$

where  $K_1, K_2$  are diagonal matrices,  $e_p(t) = q - q_m$  is tracking error and the disturbance compensation signal  $v(t)$  chosen as theorem 1.

**Theorem 1.** Consider the system – EULI (1) and the new state feedback controller (3) with  $K_1 = \text{diag}(k_{1i}); K_2 = \text{diag}(k_{2i})$  and  $k_{2i} > k_{1i}$   $i = 1, 2, \dots, n$  which are updated by the adaptive law:

$$\begin{aligned} \text{diag}(k_{1i}) &= -\alpha_1^{-1} \int_0^t (P_{21} e_1 + P_{22} e_2) e_p dt + \text{diag}(k_{1i}(0)) \\ \text{diag}(k_{2i}) &= -\alpha_2^{-1} \int_0^t (P_{21} e_1 + P_{22} e_2) \dot{e}_p dt + \text{diag}(k_{2i}(0)) \end{aligned} \quad (4)$$

where  $P$  is roots of Lyapunov equation  $A_m^T P + P A_m = -Q$ . If the disturbance compensation signal  $v(t)$  is chosen such as  $v(t) = \int_0^t m^T e dt$  then the error between output signal and desired trajectory and its derivation will have the “attractor”

$$W_1 = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \in R^{2n} \left\| e \right\| \leq \frac{l m^T}{d} \quad (5)$$

*Proof:*

The second derivative of the output error is expressed

$$\begin{aligned} \ddot{e} &= -\dot{v}(t) + \ddot{q}_m + K_1 \dot{e}_p + K_2 e_p + D^{-1}(q) h(t) \\ -\ddot{q}_m &= -K_1 \dot{e}_p - K_2 e_p + \ddot{q}_m + D^{-1}(q) h(t) \end{aligned}$$

$$\begin{bmatrix} \ddot{e}_p \\ \dot{e}_p \end{bmatrix} = \begin{bmatrix} Q & I \\ -K_1 & -K_2 \end{bmatrix} \begin{bmatrix} e_p \\ \dot{e}_p \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \ddot{q}_m + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \dot{q}_m + \begin{bmatrix} 0 \\ 0 \end{bmatrix} q_m \quad (6)$$

$$\dot{x}_p = A_p x_p + B_p p; x_p = \begin{bmatrix} e_p \\ \dot{e}_p \end{bmatrix}^T$$

Explicit output reference, and derivative output are created using the reference model for MIMO system as

$$\begin{bmatrix} \ddot{x}_{m1} \\ \ddot{x}_{m2} \\ \vdots \\ \ddot{x}_{mn} \end{bmatrix} = \begin{bmatrix} G_{m1}(s) & 0 & L & 0 \\ 0 & G_{m2}(s) & L & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & L & G_{mn}(s) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix}$$

where  $G_{mi}(s), i = 1, 2, \dots, n$  is described by the transfer function

$$G_{mi}(s) = \frac{x_{mi}}{r_i} = \frac{w_{mi}^2}{s^2 + 2z w_{mi} s + w_{mi}^2} \quad (7)$$

By the same way, the description of the reference model (8) is

$$\begin{bmatrix} \ddot{x}_{m1} \\ \dot{x}_{m1} \\ x_{m1} \end{bmatrix} = \begin{bmatrix} w_{m1}^2 e_{m1} - 2z w_{m1} \dot{x}_{m1} \\ e_{m1} \\ r_1 - x_{m1} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \ddot{x}_{mn} \\ \dot{x}_{mn} \\ x_{mn} \end{bmatrix} = \begin{bmatrix} w_{mn}^2 e_{mn} - 2z w_{mn} \dot{x}_{mn} \\ e_{mn} \\ r_n - x_{mn} \end{bmatrix}$$

$$\begin{aligned} x_m &= \begin{bmatrix} \ddot{x}_m \\ \dot{x}_m \\ x_m \end{bmatrix}; A_m = \begin{bmatrix} Q & I \\ w_m^2 I & -2z w_m I \end{bmatrix} \\ B_m &= \begin{bmatrix} Q \\ Q \\ Q \end{bmatrix} e_m = \begin{bmatrix} e_{m1} & L & e_{mn} \end{bmatrix}^T \end{aligned} \quad (9)$$

$$\dot{x}_m = A_m x_m + B_m u$$

Subtracting eq. (6) from eq. (10) yields:

$$\begin{aligned} e &= x_m - x_p \\ \dot{e} &= \dot{x}_m - \dot{x}_p = A_m e + A x_p + B p \end{aligned} \quad (10)$$

where:

$$\begin{aligned} A &= A_m - A_p = \begin{bmatrix} Q & I \\ w_m^2 I & -2z w_m I \end{bmatrix} - \begin{bmatrix} Q & I \\ w_p^2 I & -2z w_p I \end{bmatrix} = \begin{bmatrix} Q & Q \\ w_m^2 I + K_1 & -2z w_m I + K_2 \end{bmatrix} \\ B &= B_m - B_p = \begin{bmatrix} Q \\ Q \\ Q \end{bmatrix} e_1 = e_2; e^T = \begin{bmatrix} e_1 & e_2 \end{bmatrix} \\ e_1 &= x_{m1} - x_{p1}; e_2 = x_{m2} - x_{p2} \end{aligned} \quad (11)$$

The following Lyapunov function is used

$$V(e) = e^T P e + a^T a; P = P^T > 0, a > 0$$

where  $a$  is a vector and contains the non-zero elements of  $A$ . Take the derivative of  $V(e)$  with respect to time, we have

$$\begin{aligned} \dot{V}(e) &= e^T (A_m^T P + P A_m) e + 2e^T P A x_p + 2a^T a^T + 2p^T B^T P e \\ &= -e^T Q e + 2e^T P A x_p + 2a^T a^T + 2p^T B^T P e \end{aligned} \quad (12)$$

The chosen matrix  $Q$  in [1], [8]

$$Q = \begin{bmatrix} K_1^2 & Q \\ Q & 2(K_2^2 - K_1) \end{bmatrix}$$

where  $Q$  is a zero matrix with an appropriate dimensions. Obviously,  $Q$  is positive definite matrix when  $k_{2i}^2 > k_{1i}, i = 1, n$ ,  $P$  is the solution of the equation  $A_m^T P + P A_m = -Q$

$$P = \begin{bmatrix} 2K_1 K_2 & K_1 \ddot{\theta} \\ K_1 & K_2 \ddot{\theta} \end{bmatrix}$$

$$\text{set } 2e^T P A x_p + 2a a^T = Q \tag{13}$$

where

$$e^T = (e_1 \ e_2), \dot{a} = e_2; a = (a_1 \ a_2); a = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, A = \begin{bmatrix} 0 & 0 \\ a_1 & a_2 \end{bmatrix}, x_p = \begin{bmatrix} \ddot{\theta} \\ \dot{\theta} \\ \theta \end{bmatrix}$$

$$\dot{a}_1 = -a_1^{-1}(P_{21}e_1 + P_{22}e_2)e_p;$$

$$\dot{a}_2 = -a_2^{-1}(P_{21}e_1 + P_{22}e_2)\dot{a}_p$$

$$\dot{a}_1 = \text{diag}(k_{1i}); \dot{a}_2 = \text{diag}(k_{2i})$$

$$a_1 = \text{diag}(a_{1i}); a_2 = \text{diag}(a_{2i});$$

$$P_{21}; P_{22} \hat{=} R^{n \times n}; i = 1, n$$

The solution of eq. (13) is (4) and eq. (12) becomes

$$\begin{aligned} \mathcal{V}(\dot{e}) &= -e^T Q e + 2p^T B^T P e \\ &= -e^T \begin{bmatrix} 2K_1^2 & Q \\ Q & 2(K_2^2 - K_1) \end{bmatrix} e - 2p^T \begin{bmatrix} Q \\ K_1 & K_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \dot{\theta} \\ \theta \end{bmatrix} \\ &= -2 \sum_{i=1}^n k_{1i}^2 e_i^2 - 2 \sum_{i=1}^n (k_{2i}^2 - k_{1i}) e_{n+i}^2 - \\ &\quad - 2(k_{11}e_1, \dots, k_{1n}e_n, k_{21}e_{n+1}, \dots, k_{2n}e_{2n})p \\ l &= \max_i(k_{1i}, k_{2i}); d = \min_i(k_{1i}, k_{2i}^2 - k_{1i}); |p(t)| \leq m \\ \frac{dV}{dt} &\leq -d|e|^2 + l|p||e| \leq (lm - d|e|)|e| \end{aligned} \tag{14}$$

Eq. (14) shows that if  $|e| > \frac{lm}{d}$  then  $\mathcal{V} < 0$ . Therefore, we always have monotonically decreasing error  $e(t)$  when  $e(t) \hat{=} W_1$  in (5).

### III. ACTIVE MAGNETIC BEARING SYSTEM

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In this section, an active magnetic bearing system as Fig. 1, used to illustrate our proposed methods. According to [10], AMB system replaces mechanical bearings used in electric engine working

in special environments. Because it uses magnetic forces to support the movement of the spindle without mechanical contacting, the new technology bearings have a number of advantages compared to other types of conventional bearings. However, this system is the instable, so a feedback control loop is necessary to stabilize system. Formerly, the other projects [10], [11], and [12] have been modeled the AMB 2 DOF system in linear and nonlinear state space which is difficult to consider disturbances inputs. The modeling of the AMBs in Euler-Lagrange equation is presented in [13]. The control objective is that the position  $x$  and  $y$  of a suspended object follow the position reference  $x_m$  and  $y_m$  with parameters of AMBs as Table 1.

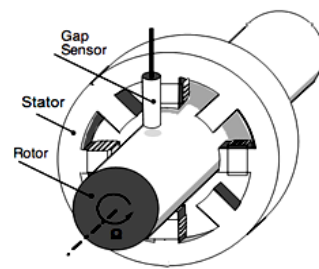


Fig. 1. Two degree of freedom AMB

#### A. AMBs in Euler-Lagrange equation form

Fig. 2a shows a shaft rotating at an angular speed of  $w_{rm}$  around the  $k$ -axis. The three perpendicular  $x$ - $y$ - and  $z$ -axes are in the rotational coordinate reference frames. The bottom of the shaft is fixed to the origin of these axes. At a length  $l_{rt}$  from the origin, a cylindrical magnetic bearing rotor is fixed to the shaft. Fig. 2b shows the difference. In Figure 2b1, a view along the  $y$ -axis is shown. The  $k$ -axis is inclined by an angular position  $q_y$  from the  $z$ -axis. A moment (or torque)  $N_y$  is applied around the  $y$ -axis. Note that the angular position and moment are defined around the  $y$ -axis using the right-hand rule. In Fig. 2b2, a view along the  $x$ -axis is shown. The angular position and the moment are also defined as  $q_x$  and  $N_x$  respectively, again, based on the right-hand rule.

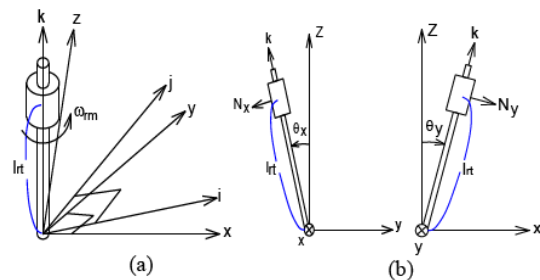


Fig. 2. Coordinates; (a) Coordinate system; (b) view 1; view 2

Displacements from the cylindrical rotor center alignment on the  $x$  - and  $y$  -axes can be obtained from Fig. 2b

$$\begin{aligned} x &= l_{rt} \cos q_y \dot{q}_x - l_{rt} \dot{q}_y \sin q_x \\ y &= -l_{rt} \cos q_x \dot{q}_y - l_{rt} \dot{q}_x \sin q_y \end{aligned} \quad (25)$$

In [13] we obtain the following dynamics equation

$$\begin{aligned} \frac{J_i}{K_{i,rt}^2} l_{rt} \ddot{q}_x &= -\frac{w_{rm} J_k}{K_{i,rt}^2} l_{rt} \dot{q}_y - \frac{mgh}{K_{i,rt}^2} y - i_y \\ \frac{J_j}{K_{j,rt}^2} l_{rt} \ddot{q}_y &= \frac{w_{rm} J_k}{K_{j,rt}^2} l_{rt} \dot{q}_x - \frac{mgh}{K_{j,rt}^2} x + i_x \end{aligned} \quad (26)$$

and substituting (25) into (26), we have

$$u + h(t) = D(q) \ddot{q} + C(q, \dot{q}) + G(q) \quad (27)$$

where:

$$\begin{aligned} D(q) &= \begin{bmatrix} \frac{2J_i}{K_{i,rt}^2 \cos^2 q_x} & 0 \\ 0 & \frac{2J_j}{K_{j,rt}^2 \cos^2 q_y} \end{bmatrix} & G(q) &= \begin{bmatrix} \frac{mgh}{K_{i,rt}^2} y \\ \frac{mgh}{K_{j,rt}^2} x \end{bmatrix} \\ C(q, \dot{q}) &= \begin{bmatrix} \frac{J_i \sin q_x}{K_{i,rt}^3 \cos^3 q_x} \dot{q}_x - \frac{w_{rm} J_k}{K_{i,rt}^2 \cos q_y} \dot{q}_y \\ \frac{w_{rm} J_k}{K_{j,rt}^2 \cos q_x} - \frac{J_j \sin q_y}{K_{j,rt}^3 \cos^3 q_y} \dot{q}_x \end{bmatrix} \end{aligned}$$

with  $q_x, q_y$  is very small, so we assume  $q_x \gg q_y$ ,

$$\begin{aligned} D(q) = D(q)^T > 0, \quad \frac{1}{\cos^2 q_x} &= \frac{\sin q_x}{\cos^2 q_x} \dot{q}_x = -\frac{\sin q_x}{l_{rt} \cos^3 q_x} \dot{q}_x \\ D(q) &= \begin{bmatrix} \frac{2J_i \sin q_x}{K_{i,rt}^3 \cos^3 q_x} \dot{q}_x & 0 \\ 0 & \frac{2J_j \sin q_y}{K_{j,rt}^3 \cos^3 q_y} \dot{q}_y \end{bmatrix} = C(q, \dot{q}) + C^T(q, \dot{q}) \end{aligned}$$

The modeling a system in Euler Lagrange equation has the following advantages: *First*, It is convenient to apply the modern control methods such as Li-Stoline, adaptive inverse dynamic [1]. *Second*, we do not consider to approximate  $\sin q_{x(y)} \gg q_{x(y)}$  as [10], [11]. Therefore, compare to the model built in [10], [11]

and [12], the model established in this paper is more accurate. *Third*, the AMBs has a nonlinearity. Consequently, the decoupling a time-invariant linear system using state variable feedback [11] do not apply. *Fourth*, the effects of disturbance on the system are not considered in [10], [11] and [12]. There are two kinds of disturbance be concerned: caused by exogenous signals and error model. In this case, they can be recast into the disturbance and will be compensated by disturbance estimation techniques.

TABLE I. THE PARAMETERS OF AMBS

Parameters	Values
Mass of rotor (kg)	$m = 12.4$
Displacements from the cylindrical rotor center alignment to fixed origin (m)	$l_{rt} = h = 0.21$
Moment of inertia in $k$ -axis (kg.m <sup>2</sup> )	$J_k = 6.88 * 10^{-3}$
Moment of inertia in $i$ and $j$ -axis (kg.m <sup>2</sup> )	$J_i = J_j = 2.22 * 10^{-1}$
Speed of rotor (rpm)	10000
The ratio electromagnetic-current (N/A)	$K_i = 102.325$
The ratio electromagnetic-displacement (N/A)	$K_s = 4.65 * 10^5$
The gravity acceleration (kg. m/s <sup>2</sup> )	$g = 9.81$

**B. The results simulation of our proposed controllers**

We will compare our proposed method NAC\_ISS to Adaptive Control based on ISS stabilization (AC\_ISS) in [2]. We choose the fixed matrices  $K_1, K_2$  of AC\_ISS and initial matrices  $K_1(0), K_2(0)$  of NAC\_DE such as:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix} \\ K_1(0) &= \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \quad K_2(0) = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix} \end{aligned}$$

and the disturbance compensation signal  $v(t)$  is chosen such as  $|p(t)| \leq 5, "t$ .

Comparison between the position  $x$  and  $y$  of a suspended object dealt with NAC\_ISS and AC\_ISS is presented. Responses of both NAC\_ISS and AC\_ISS are almost same (see Fig. 5). However, it is clear that the tracking error for the NAC\_ISS due to reference position is much less than those of AC\_ISS (see Fig. 4) and the adaptive gains of NAC\_ISS automatically alter when disturbance impacts on the system (see Fig. 3).

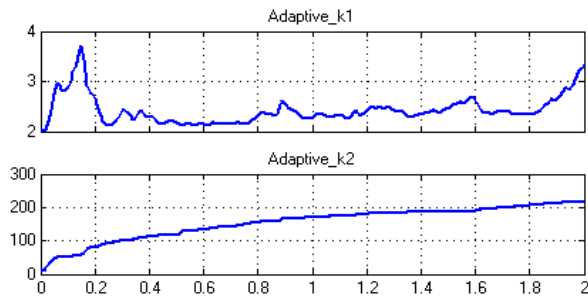


Fig. 3. Adaptive gains of NAC\_ISS

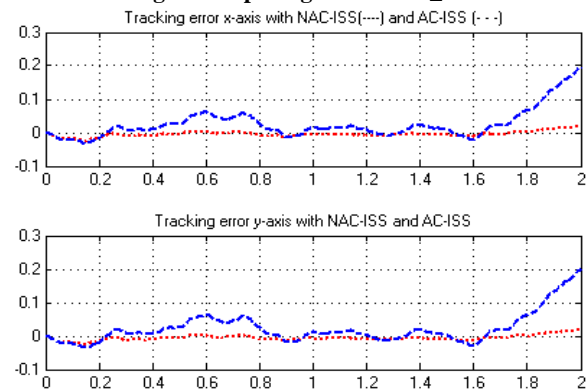


Fig. 4. Tracking error of NAC\_ISS (----) and AC\_ISS (- - -)

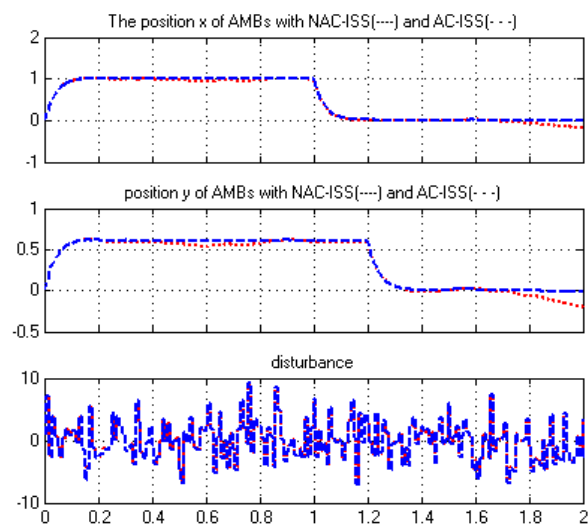


Fig. 5. Transient response of NAC\_ISS (----) and AC\_ISS (- - -) with disturbances

#### IV. CONCLUSIONS

Our approaches is applied for a class of nonlinear MIMO systems involving plan external disturbances. The tracking performances are greatly improved by the use the ISS stabilization and adaptive law of controller's parameters. We investigate the effect of the controller from the simulation results. Compared to the case with NAC\_ISS and AC\_ISS in the illustrated examples, active magnetic bearing system, for instance, can do the following: (a) Greatly improve the transient behavior of the system; (b) Eliminate steady-state errors; and (d) Eliminate the influence of

lumped disturbances. Strong properties achieved via the proposed methods confirm that NAC\_ISS and is an attractive approaches for controlling nonlinear MIMO systems

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