# Design a Hybrid PI – Hedge Algebraic Controller for Controlling Brushless DC Motor

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***Abstract***

*Hedge Algebra (HA) is an algebraic structure for computing, simulating semantics of language, so it can be considered as the basis of fuzzy logic. Using HA for designing the controller can create an algebraic structure in the form of a functional relationship, which allows to form a large set of linguistic values to describe input and output relationships. HA-based research is a new approach in the calculation of fuzzy controllers, so researchers have had ideas applying it in the field of control. The remarkable success of the HA is based on functional reasoning methods that contain a wide variety of open elements. The user can optionally select different approaches in each step of the method. The paper proposes the Hybrid PI – Hedge Algebraic controller applied for the BLDC motor. The control system includes an inner PI controller loop and an outer hedge algebraic controller loop.*

**Keywords** - *BLDC motor, DC-Link, PI controller, HA, HAC.*

## I. INTRODUCTION

Hedge Algebra (HA) has been studied since the 1990s arming at inheriting and developing the advantages of fuzzy systems [4]. HA has solved effectively the problem of identification, diagnosis, and objects which model difficultly. Therefore, its applications are becoming popular in the field of control and automation [3], [8]. However, the use of HA in industrial systems is still a new problem.

Recently, the BLDC motors are used in many applications such as optical drives, radiator cooling fans of laptops, household appliances and office automation. In these applications, control circuits are designed simply and reliably. With the development of semiconductor switching technology and the design of high power converters, the performance of the electric drive systems using BLDC motors is better than that of others using DC motors as well as synchronous motors. Take mobile vehicles consuming independent DC voltage sources from batteries or solar energy an example, especially drive systems of electric vehicles, cars and aircrafts with the capacity from several W to hundreds of kW.

There is a variety of proposed papers in terms of different speed control methods of BLDC motors. They work on tuning PID parameters as based on genetic algorithm, Ziegler Nicholas tuning methods and other work on adjusting PID parameters by fuzzy optimized algorithm or using fractional order PID controller. The BLDC motor is non-linear and multi variable system due to temperature or load, so that it is difficult to drive it with accurate and reliable control response using the classical controlling methods.

The paper proposes the hedge algebraic controller for the outer speed control loop and the PI controller for the inner current control loop.

## II. THE INTRODUCTION OF HEDGE ALGEBRA

### A. *Hedge Algebra*

In fact, the value of linguistic variables has certain orders semantically. For example, it is obvious that young is smaller than old, fast is larger than slow.

Hedge Algebra consists of four parts including “X, G, H,≤”. With “X” is a domain of the linguistic variable; “≤”” is a semantically ordering relation; “G” is the set of generators; “H” is the set of linguistic hedges (H can be negative or positive hedges).

As a result, semantics of words represented by the structure HA is likely to be determined by their relative position in ordering arrangement between words in linguistic domain, based on their natural meaning. Linguistic value is quantified by a real value in [0, 1].

Example: the linguistic variable “SPEED” can be considered as an algebraic structure, singed AX = (X, G, H, ≤), where: “X” is the set of values (Fast, Slow, very Fast, very Slow, more Fast, more Slow, little Fast, little Slow, approximately Fast, approximately Slow, less Fast, less Slow, very more Fast, very more Slow, very possible Fast, very possible Slow,…) ; “G” is the set of generators (Fast, Slow); “H” is the set of linguistic hedges (very, more, little, less,…); “≤” is an semantically ordering relation (Slow ≤ Fast, more Slow ≤ more Fast, very Slow ≤ very Fast, less Slow ≤ less Fast, little Slow ≤ little Fast,…)

### B. Selection of HA parameters

A set of rules is shown as following equations, with $X_1$, $X_2$,…, $X_m$; linguistic variables Y and proportional values $A_{ij}$, $B_i$ (i = 1… n; j = 1… m).

If $X_1 = A_{11}$ and … and $X_m = A_{1m}$ then $Y = B_1$

If $X_1 = A_{21}$ and … and $X_m = A_{2m}$ then $Y = B_2$

……

If $X_1 = A_{n1}$ and … and $X_m = A_{nm}$ then $Y = B_n$

The main idea of this method is that each clause "if … then"will determine a point in Decac space Dom ($X_1$)*…*Dom ($X_m$)* Dom (Y), where Dom ($X_i$), Dom (Y) are proportional linguistic domains of linguistic variables $X_i$ and Y seen as HAs.

Recently outstanding successes of HA are based on functional reasoning method including open elements to help researchers can develop and choose different accesses depending on study purpose. A number of studies using HA mainly focus on information technology and control sectors. For example, control problems for inverted pendulums, earthquake prediction or disease diagnosis… Hence, this paper analyzes and appreciates existing results of various studies, leading to the development of methods applied in the industry.

From the model (1), we see that this model includes (m+1) linguistic variables. HA parameters consist of the fuzzy measure of elements: $fm_{AXi}(c^-)$, $fm_{AXi}(c^+)$ in the relation: $fm_{AXi}(c^-) + fm_{AXi}(c^+) = 1$.

And the fuzzy measure of particles: $\mu_{AXi}(h_j)$ satisfies some functions:

$$\sum_{j=-q_i}^{-1} \mu_{AXi}(h_j) = \alpha, \sum_{j=1}^{p_i} \mu_{AXi}(h_j) = \beta, \alpha + \beta = 1$$

### C. The design method

The HA Controller designed from the block diagram as Fig. 1.

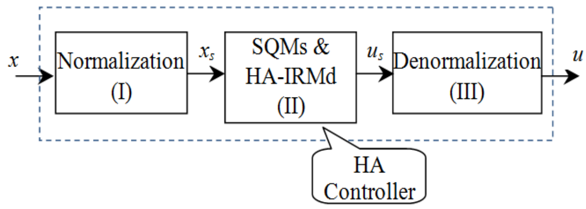


**Fig 1: The structure diagram of the HA controller**

Where, *x*denotes input value; $x_s$ means input semantic value; *u* is control value and $u_s$ represents semantic control value.

The HAC consists of the following blocks:

- Block I - Normalization: transforming *x* into $x_s$.

- Block II - SQMs & HA-IRMd (Semantically Quantifying Mappings & Hedge Algebra-based Interpolative Reasoning Method): implementing the semantic interpolation from $x_s$ to $u_s$based on quantitative semantic mapping and control rules.

- Block III - Denormalization: linear transformation$u_s$ into *u*.

## III. DESIGN THE HYBRID PI – HEDGE ALGEBRAIC CONTROLLER FOR THE BLDC MOTOR

### A. The mathematical model of the BLDC motor

The main parts of the system are shown in the figure 2, which consists of three-phase star-connected stator windings and permanent magnet rotor. The motor is driven by three-phase inverter with trigger signals generated by the controller. It depends on rotor position sensors connected to the switches from $Q_1$ to $Q_6$ in order to drive the motor with stator currents corresponding with the back emfs.

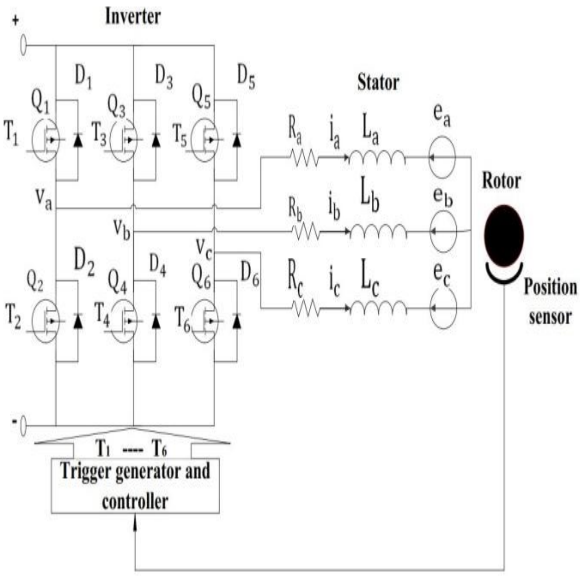


**Fig 2: The principle diagram of BLDC motor**

The stator phase voltages can be described by the following three equations (1):

$$\begin{aligned} V_a &= R_a i_a + L_a \frac{di_a}{dt} + M_{ab}\frac{di_b}{dt} + M_{ac}\frac{di_c}{dt} + e_a \\ V_b &= R_b i_b + L_b \frac{di_b}{dt} + M_{ba}\frac{di_a}{dt} + M_{bc}\frac{di_c}{dt} + e_b \\ V_c &= R_c i_c + L_c \frac{di_c}{dt} + M_{cb}\frac{di_b}{dt} + M_{ca}\frac{di_a}{dt} + e_c \end{aligned} \quad (1)$$

If three-phase system is balanced, we have:

$R_a = R_b = R_c = R$, $L_a = L_b = L_c = L$, $M_{ab} = M_{ac} = M_{ba} = M_{bc} = M_{cb} = M_{ca} = M$

Equations (1) become a set of equation (2)

$$V_a = Ri_a + L\frac{di_a}{dt} + M\frac{di_b}{dt} + M\frac{di_c}{dt} + e_a$$
$$V_b = Ri_b + L\frac{di_b}{dt} + M\frac{di_a}{dt} + M\frac{di_c}{dt} + e_b \quad (2)$$
$$V_c = Ri_c + L\frac{di_c}{dt} + M\frac{di_b}{dt} + M\frac{di_a}{dt} + e_c$$

Transforming equations (2), we have:

$$v_a = Ri_a + (L - M)\frac{di_a}{dt} + e_a$$
$$v_b = Ri_b + (L - M)\frac{di_b}{dt} + e_b \quad (3)$$
$$v_c = Ri_c + (L - M)\frac{di_c}{dt} + e_c$$

If neglecting mutual inductances then equations (3) are rearranged as:

$$v_a = Ri_a + L\frac{di_a}{dt} + e_a$$
$$v_b = Ri_b + L\frac{di_b}{dt} + e_b \quad (4)$$
$$v_c = Ri_c + L\frac{di_c}{dt} + e_c$$

The electromagnetic torque is defined as (5), (6):

$$T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{w_r} \quad (5) \quad ; \quad T_e = B_m w_r + J_m \frac{d}{dt} w_r + T_L \quad (6)$$

where $T_L$ is the load torque, $J_m$ is rotor inertial and B is friction constant.

### *B. The control structure*

In this paper, the pulse width modulation (PWM) is used for this cascade control system with the inner current control loop and the outer speed control loop shown in Fig. 3.

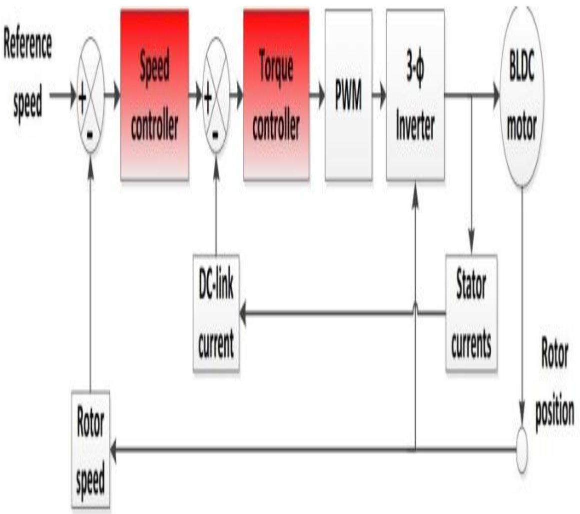


**Fig 3: The proposed block diagram for BLDC motor**

The model of the command system contains a BLDC motor, inverter to three phase, pulse width modulation (PWM) and speed and current controllers. The BLDC motor speed can be directly changed by the duty cycle of the inverter switches in which firing signals depend on the control error. The control system consists of current sensors to measure three phase currents to obtain the DC-link current. Contemporarily, the rotor position sensors give the desired commutation sequence.

The speed controller is PI controller and has equation (7) as:

$$I_d = K_{Pw}(w_d - w_f) + K_{Iw}\int (w_d - w_f)dt \quad (7)$$

In the speed control loop, the DC-link current is the control current due to it is also the three-phase currents and its sequence is chosen in table 1. When the system receives the phase current feedback, the phase voltage is calculated by equation (8):

$$V_d = K_{PI}(I_d - I) + K_{II}\int (I_d - I)dt \quad (8)$$

Pole position and speed signals are determined through a set of Hall-effect sensors [2], [9]. Decoding and choosing phases are described as Fig. 4 and table 1

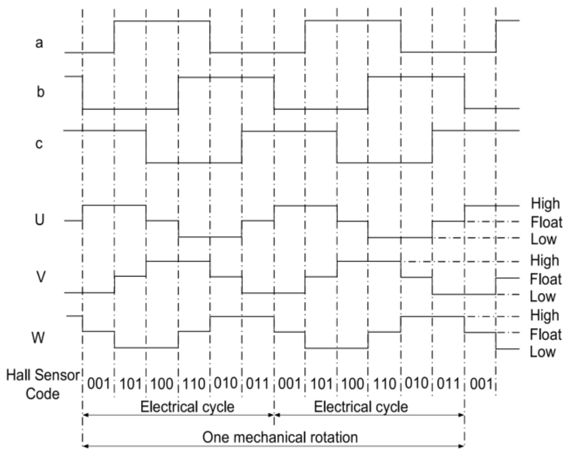


**Fig 4: The phase control rule based on Hall-effect sensors**

**Table 1. The distribution of fuel for heat areas**

| Voltage vector | (S1,S4) | (S3,S6) | (S5,S2) | Phase current |
|---|---|---|---|---|
| V1 | (1,0) | (0,0) | (0,1) | $i_a=i_{dc}$ |
| V2 | (1,0) | (1,0) | (0,1) | $i_c=-i_{dc}$ |
| V3 | (0,1) | (1,0) | (0,0) | $i_b=i_{dc}$ |
| V4 | (0,1) | (0,0) | (1,0) | $i_a=-i_{dc}$ |
| V5 | (0,0) | (0,1) | (1,0) | $i_c=i_{dc}$ |
| V6 | (1,0) | (0,1) | (0,0) | $i_b=-i_{dc}$ |

Because the armature back emfs when controlling current are kept constant, the structure diagram of the system is shown equivalently as Fig. 5.

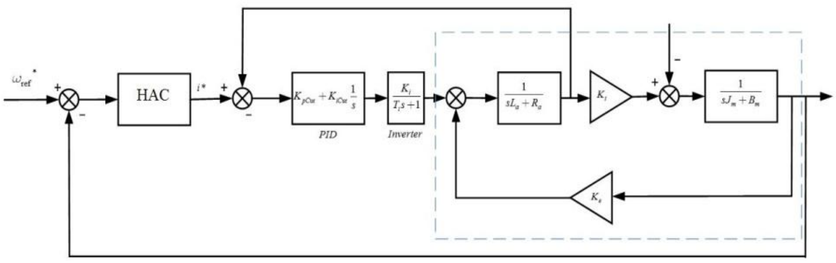


**Fig 5: The equivalent block diagram with the DC-link feedback current**

### *C. Design the PI controller for the current control loop*

With the above structure, it is possible to apply different methods of linear control algorithms applied to DC motor. According to the PI controller of the current control as [1], [5], [6], [8], the dynamics of the measurement and the inverter circuit is considered lower than that of current. Then, the simple equivalent model of current control loop is depicted as Fig. 6.

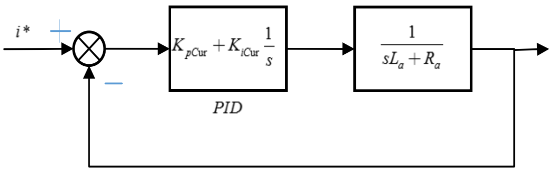


**Fig 6: The simple equivalent diagram of the current control loop**

The transfer function of the open loop from the model (6) is determined by equation (9):

$$G_h(s) = \frac{1}{s}\frac{K_{iCur} + K_{pCur}s}{R_a + L_a s} = \frac{K_{iCur}}{sR_a}\frac{1+Ks}{1+T_a s} \quad (9)$$

Controller parameters are selected based on the desired switching frequency of the closed system and reduced-order model as following: $w_{kCur} = \frac{K_{iCur}}{R_a}$, $\frac{K_{pCur}}{K_{iCur}} = \frac{L_a}{R_a}$

### *D. Design the PI controller for the speed control loop*

For the speed control loop, it is assumed that the current control loop is ideal, meaning that the transfer function of the closed current control loop is equal to 1. It is practicable since the dynamics of the current control loop is much faster than that of the speed control loop [2].

The HAC controller consists of two inputs and one output: the first input is the speed error - e(t), denoted by E, the second input is the derivative of the first input, denoted by dE and the output is represented by U.

Firstly, the symbol S, F, L and V mean Slow, Fast, Less and Very respectively.

Hence, we have:

G = {0, S, W, F, 1}

H– = {L} = {h–1}; q = 1; H+ = {V} = { h+1}; p = 1;

fm(S) = θ = 0.5; fm(F) = 1 - fm(S) = 0.5;

v(W) = θ = 0.5;

The HAC controller parameters are chosen as following table 2.

**Table 2.Selection of the E, dE and U values**

| | | Input1 (E) Input2 (dE) | | Output (U) | |
|---|---|---|---|---|---|
| H | | μ(h) | | μ(h) | |
| H- | Less (L) | α | 0.45 | α | 0.365 |
| H+ | Very (V) | β | 0.55 | β | 0.635 |

Secondly, a set of semantic value for E, dE and U is determined including different values as very slow (VS), slow (S), less slow (LS), less fast (LF), fast (F) and very fast (VF) accompanied with rules in table 3.

**Table 3.The control rules**

| U | | E | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | VS | S | LS | W | LF | F | VF |
| dE | VS | VS | VS | VS | VS | LS | LF | VF |
| | S | VS | VS | S | S | W | LF | VF |
| | LS | VS | VS | LS | LS | W | F | VF |
| | W | VS | VS | LS | W | LF | VF | VF |
| | LF | VS | S | W | LF | LF | VF | VF |
| | F | VS | LS | W | F | F | VF | VF |
| | VF | VS | LS | LF | VF | VF | VF | VF |

Semantic values of variables in Table 2 are calculated by equation

$$\upsilon(S) = \theta - \alpha fm(S) = 0.5 - (0.45)(0.5) = 0.275$$

$$\upsilon(VS) = \upsilon(S) + sign(VS)\left\{\sum_{i=1}^{1} fm(VS) - 0.5[1 + sign(VS)sign(VVS)(\beta - \alpha)]fm(VS)\right\}$$

$$= \upsilon(S) + sign(VS)\left\{\sum_{i=1}^{1} \beta fm(S) - 0.5[1 + sign(VS)sign(V,V)sign(VS)(\beta - \alpha)]\beta fm(S)\right\}$$

=0.275+(-1){(0.55)(0.5)-0.5[1+(-1)1(-1)(0.55-0.45)](0.55)(0.5)} = 0.1513

Similarly, the values of thesemantic variables E, dE and U are determined as following tables:

| VS | S | LS | W | LF | F | VF |
|---|---|---|---|---|---|---|
| 0.1513 | 0.275 | 0.3988 | 0.5 | 0.6012 | 0.725 | 0.8488 |

| VS | S | LS | W | LF | F | VF |
|---|---|---|---|---|---|---|
| 0.2016 | 0.3175 | 0.4334 | 0.5 | 0.5666 | 0.6825 | 0.7984 |

Combination of input variables (E and dE) with $w_1 = 0.6$; $w_2 = 0.4$ and the quantitative semantic curve obtained from the relationship between U and (E, dE) results in the control value us.

The control value u is found by solving us control problem. Quantifying real values and solving quantitative problems are taken as [4], [5] in limited range of Es, dEs and Us variables.

## IV. SIMULATION AND EXPERIMENTAL RESULTS

Simulating the cascade control method for the BLDC motor with the hybrid PI – hedge algebraic controller is shown as Fig. 7 and Fig. 8.

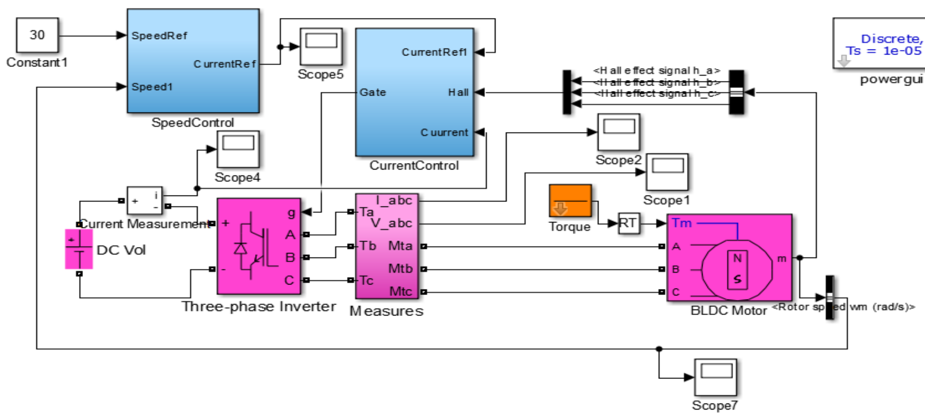


**Fig 7: The simulating model of the system in MATLAB/Simulink**

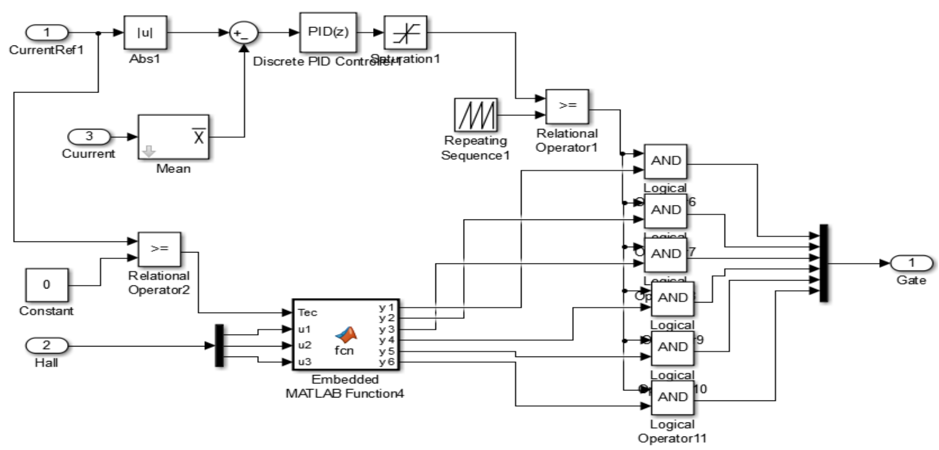


**Fig 8: The current control loop using the PI controller**

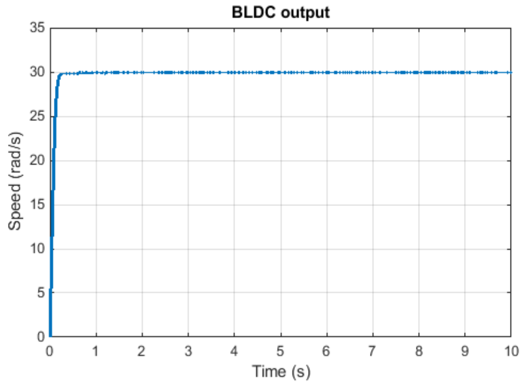


**Fig 9:The speed response with the unit step function**

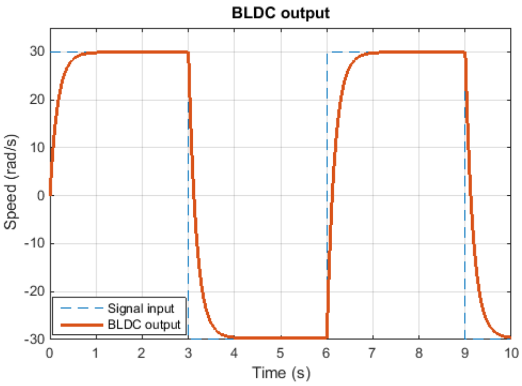


**Fig 10: The speed response with the pulse signal**

The simulation results show that the proposed controller proved the effective performance of the proposed controller in tracking required speed references, preserving the stability of the system under the conditions of the load disturbance and meeting quality requirements.

## V. CONCLUSIONS

By using the hybrid PI – hedge algebraic controller with the appropriate structure and parameters, the motor speed is remained stable and tracks the desired speed.

Simulation results represent that the algorithm and the way to construct the hybrid PI – hedge algebraic controller for the drive system is correct. As a result, this controller can meet the control requirements applied for the BLDC motor.

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