Nonlinear Control of a Gantry Crane System with Limited Payload Angle

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Abstract

The paper deals with tracking and vibration suppression problem of a gantry system. Based on flatness property of the gantry, a controller that ensures zero tracking error of the payload and minimizes payload fluctuation is designed. In addition, a PI controller is integrated to the system to drive the payload-swinging angle to a certain range. Numerical simulations are also given to prove the effectiveness of the proposed controller.

Keywords - *Flatness control, Reference generation, Gantry crane, PI control.*

I. INTRODUCTION

Due to the flexibility in handling loads, gantry crane systems are essential in industrial and logistic applications. Expected operating condition of the gantry system is the desired positions of the trolley and the payload are coinciding. In practice, this is impossible because of swinging motion of the payload. Swinging payload phenomenon slows down goods handling operations and can be a potential threat to human and surrounding devices. Certain types of payload can ignite multi-modes or double-link pendulum effects [1] -[4]. In addition, char-acterized as a class of under-actuated systems, precisely controlling trolley position and suppressing payload vibration simultaneously pose many challenges for control engineers.

In order to tackle the aforementioned control problem, a decoupling control law is proposed in [5] to asymptotically stabilize trolley position and swing angle of the payload. Actually, the designed control only guarantee bounded swing angle. An improvement is made in [6] with a gantry system with varying rope length. A switching control action is derived based on feedback linearization technique. Position control and vibration suppression of gantry crane is considered in [7], the control problem is partly solved with the coupling effect between trolley and payload motions are taken into account. However, the obtained results are relatively limited in practice since the variance of system’s parameters and actuator’s dynamics are not considered. Practically, parameters of a gantry system is varying and challenging to identify due to hydrodynamic forces acting on the payload and the variance of payload mass and geometry. In order to deal with system uncertainty, an adaptive mechanism is integrated in proposed control law suggested in [8]. Well-known with its robustness against system uncertainty and disturbances, sliding mode control is applied in gantry control in [9]. However, it is need to cooperate with a pre-shape input to

gain better performances [10]. Several adaptive schemes for gantry control also presented in [11] and [12].

Although having some promising results, feedback control seems to be suitable with automated gantry systems (where desired payload positions are pre-defined via a human machine interface). The reason is when in manual operating modes, human-in-the-loop actions may interfere with control decision of the feedback controller and deteriorate system responses. Instead of feedback controls, control actions from operators are modified before sending to the gantry actuators as shown in [13] and [14]. The advantage of pre-shape input technique over feedback control is that measurement of system states is not required but a full knowledge of the system must be available. To rectify this drawback, pre-shape input method can be hybridised with a robust control as indicated in [14]-[16]. A brief literature review above shows that the limitation of aforementioned researches rooted in the modeling step. Gantry control problem are solved with an assumption of pendulum motions of the payload that results in a system of ordinary differential equations govern system motions. Practice has shown that it is not the case, and gantry cable actually considered as a flexible system whose motions are modeled as a system of partial differential equations.

The paper designs a gantry control algorithm based on flatness property of the system. Trolley position and swinging payload suppression are simultaneously controlled, the payload swing angle in restricted in a define range. Finally, the effectiveness of the closed-loop system is proven numerically

II. MATHEMATICAL MODEL

The gantry system is illustrated in Fig. 1A trolley of mass M rolls along OX axis of the overhead crane. The trolley is actuated by a motor that produces a horizontal force of intensity F through a transmission system. The trolley carries a winch of radius ρ around which is a winding of hoisting cable with the payload attached at its end

A load of mass m attached to the coordinates (ξ, ζ). The torque exerted on the winch by a second motor is denoted by C. The cable length, its tension and the angle of the cable with respect to the vertical are denoted by R, T and θ, respectively.

The working space is limited to R < R0 (R0 is a strictly positive number) to avoid that the load touches the ground, and we assume that the tension T of the cable is always positive, i.e. the cable does not experience slack. We use the angular sign convention for θ as follows: θ ≤ 0 if ξ ≤ x and θ > 0 otherwise.

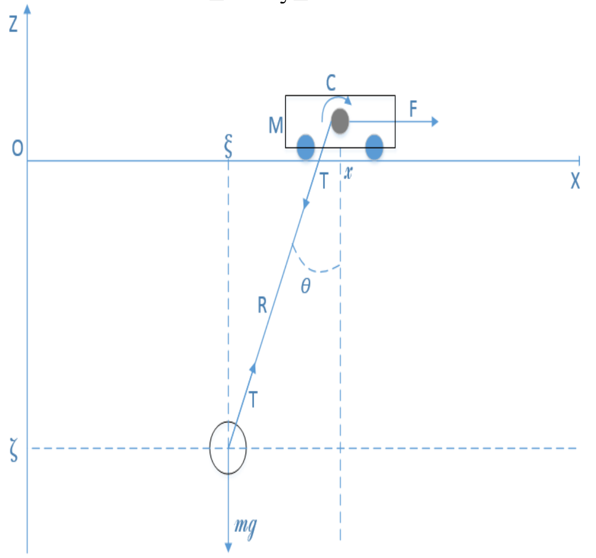

Fig.1. Two – dimensional overhead crane

We also assume that a viscous friction force, noted $\gamma_1(\dot{x})$, including the aerodynamic friction of the cable, is opposing to the trolley's displacement and that another viscous friction torque, noted $\gamma_2(\dot{R})$, resists to the winch motion. The functions γ_1 and γ_2 are non-negative and such that $\gamma_i(0)=0, i=1,2$

According to Newton's law, we get the equation describing crane system

$$\begin{cases} m\ddot{\xi} = -T\sin\theta \\ m\ddot{\zeta} = T\cos\theta - mg \\ M\ddot{x} = -\gamma_1(\dot{x}) + F + T\sin\theta \\ \dfrac{J}{\rho} = -\gamma_2(\dot{R}) - C + T\rho \end{cases} \qquad (1)$$

The geometric constraints between the coordinates of the trolley and the load is given by

$$\begin{cases} \xi = x + R\sin\theta \\ \zeta = -R\cos\theta \end{cases} \qquad (2)$$

From (1) and (2) we can obtain the mathematic model of the crane:

$$\begin{pmatrix} \left(\dfrac{M}{m} + \sin^2\theta\right) & \sin\theta & 0 \\ \sin\theta & \left(\dfrac{J}{m\rho^2} + 1\right) & 0 \\ \cos\theta & 0 & R \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{R} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} \left(R\dot{\theta}^2 + g\cos\theta\right)\sin\theta - \dfrac{1}{m}\gamma_1(\dot{x}) + \dfrac{F}{m} \\ R\dot{\theta}^2 + g\cos\theta - \dfrac{1}{m\rho}\gamma_2(\dot{R}) - \dfrac{C}{m\rho} \\ -2\dot{R}\dot{\theta} - g\sin\theta \end{pmatrix}$$

(3)

Where it is assumed that friction forces are proportional to velocity $\gamma_1(\dot{x}) = \Gamma_1.\dot{x}$ $\gamma_2(\dot{R}) = \Gamma_2.\dot{R}$

III. FLATNESS PROPERTY OF THE GANTRY CRANE SYSTEM

From the system dynamics given in (3), the tension force T from the two first equations of (1) through the following mathematical transformation

$$\tan\theta = -\frac{\ddot{\xi}}{\ddot{\zeta}+g} \qquad (3)$$

Using (2), we can rewrite equation (4) as

$$\tan\theta = -\frac{\xi - x}{\zeta}, (\xi - x)^2 + \zeta^2 = R^2 \qquad (4)$$

Finally, eliminating $\tan q$ from (3) and (4), we get the differential – algebraic relation below

$$\begin{cases} (\ddot{\zeta} - g)(\xi - x) = \ddot{\xi}\zeta \\ (\xi - x)^2 + \zeta^2 = R^2 \end{cases} \qquad (5)$$

Using (6) it is straightforward to show that

$$x = \xi - \frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}, R^2 = \zeta^2 + \left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)^2$$

$$\theta = \arctan\left(\frac{\ddot{\xi}}{\ddot{\zeta}+g}\right), T^2 = m^2\left(\ddot{\xi}^2 + \left(\ddot{\zeta}+g\right)^2\right). \qquad (6)$$

From the first three equations of (1), the expression of driving force F can be calculated as

$$F = M\ddot{x} + \gamma_1(\dot{x}) - T\sin\theta$$

$$= M\left(\ddot{\xi} - \frac{d^2}{dt^2}\left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)\right) + \gamma_1\left(\dot{\xi} - \frac{d}{dt}\left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)\right) + m\ddot{\xi}$$

$$= (M+m)\ddot{\xi} - M\frac{d^2}{dt^2}\left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right) + \gamma_1\left(\dot{\xi} - \frac{d}{dt}\left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)\right)$$

(7)

Similarly, from the last equations of (1), we can show that the torque C acting on the winch can be given as

$$C = -\frac{J}{\rho}\ddot{R} - \gamma_2\left(\dot{R}\right) + T\rho$$

$$= -\frac{J}{\rho}\left(\frac{d^2}{dt^2}\sqrt{\zeta^2 + \left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)^2}\right) - \gamma_2\left(\frac{d}{dt}\sqrt{\zeta^2 + \left(\frac{\ddot{\xi}\zeta}{\ddot{\zeta}+g}\right)^2}\right)$$

$$+ m\rho\sqrt{\ddot{\xi}^2 + \left(\ddot{\zeta}+g\right)^2} \tag{8}$$

It is easy to see that all variables of the system denoting by $(x, R, \theta, \xi, \zeta, T, F, C)$ can be expressed in function of ξ and ζ (the coordinates of the load) and of their derivatives up to the fourth order, this result is compatible with the principle of flatness.

IV. TRAJECTORIES GENERATION

It is assumed that the angle q remains sufficiently small, as well as its angular speed $\dot{\theta}$, so that, according to [17], we have $x \gg X$ and $z \approx R$. We want to drive the payload follow a straight line starting from the initial position $\xi_i = x_i$, $\zeta_i = -R_i$.

At time t_i, with $\theta(t_i) = \dot{\theta}(ti) = 0,\ \dot{\xi}(t_i) = \dot{x}(t_i) = 0,\ \dot{R}(t_i) = -\dot{\zeta}(t_i) = 0$

and arriving at the final position $\xi_f = x_f$, $\zeta_f = -R_f$

At time t_f, with $\theta(t_f) = \dot{\theta}(t_f) = 0,\ \dot{\xi}(t_f) = \dot{x}(t_f) = 0,\ \dot{R}(t_f) = -\dot{\zeta}(t_f) = 0$

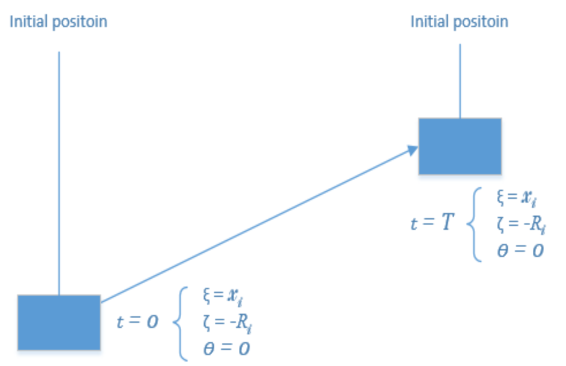

Fig.2. Load displacement in straight line crane

Motion plant: $t \mapsto \left(x(t), R(t)\right)$ with $t \in \left[t_i, t_f\right]$

Start position $t = 0 \begin{cases} \xi = x_i \\ \zeta = -R_i \\ \theta = 0 \end{cases}$

and final position $t = T \begin{cases} \xi = x_f \\ \zeta = -R_f \\ \theta = 0 \end{cases}$

According to Rest-to-Rest Trajectories, we have

$$\xi(t_i) = \varphi_0\left(x(t_i), 0, \ldots, 0\right),\ \zeta(t_i) = \varphi_0\left(R(t_i), 0, \ldots, 0\right)$$

$$\xi(t_f) = \varphi_1\left(x(t_f), 0, \ldots, 0\right),\ \zeta(t_f) = \varphi_1\left(R(t_f), 0, \ldots, 0\right)$$

The polynomial rest-to-rest trajectories are of the form

$$\xi_{ref}(t) = x(t_i) + \left(x(t_f) - x(t_i)\right)\left(\frac{t - t_i}{t_f - t_i}\right)^{r+2}\left(\sum_{k=0}^{r+1}\alpha_k\left(\frac{t - t_i}{t_f - t_i}\right)^k\right)$$

$$k = 0, \ldots\ldots, r+1. \tag{9}$$

With $\alpha_0, \ldots, \alpha_{r+1}$ solution of

$$\begin{pmatrix} \frac{1}{r+2} & \frac{1}{r+3} & \cdots & \frac{1}{2r+3} \\ \frac{1}{(r+1)(r+2)} & \frac{1}{(r+2)(r+3)} & & \frac{2r+3}{(2r+2)(2r+3)} \\ \vdots & & & \vdots \\ (r+2) & \frac{(r+3)}{2} & \cdots & \frac{(2r+3)}{(r+2)} \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{r+1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{10}$$

Denoting by the transfer duration, motion planning of the load is given by

$$x_{ref}(t) = x_i + \left(x_f - x_i\right)\left(\frac{t}{T}\right)^5$$

$$\times\left(126 - 420\left(\frac{t}{T}\right) + 540\left(\frac{t}{T}\right)^2 - 315\left(\frac{t}{T}\right)^3 + 70\left(\frac{t}{T}\right)^4\right) \tag{11}$$

And:

$\zeta(\xi_i) = \zeta_i, \zeta(\xi_f) = \zeta_f$,

$\zeta(\xi) = \zeta_i + \left(\zeta_f - \zeta_i\right)\left(\frac{\xi - \xi_i}{\xi_f - \xi_i}\right)$.

We find:

$$\zeta_{ref}(t) = -R_i - \left(R_f - R_i\right)\left(\frac{\xi_{ref}(t) - x_i}{x_f - x_i}\right) \tag{12}$$

According to condition $x^*(t) = \xi_{ref}(t)$, $R^*(t) = -\zeta_{ref}(t)$, we use PD controller.

$$F = K_{D1}\left(\dot{x} - \dot{x}^*\right) + K_{P1}\left(x - x^*\right) \tag{13}$$

$$C = K_{D2}\left(\dot{R} - \dot{R}^*\right) + K_{P2}\left(R - R^*\right) \tag{14}$$

It is desired that the swing angle of the payload is limited to a certain range. This ability is essential when maneuvering the gantry crane in a narrow space. It is assumed that the angle θ remains sufficiently small, as well as its angular velocity, hence we will design motion trajectoris with limit angle constrains, then use a traditional PD controller to achieve the control objective.

We have the equation describing swing angle of the form:

$$\alpha = \alpha_0 \cos(\omega t + \varphi) \tag{15}$$

With the angular sign convention for $\xi \le x \to \theta \le 0$ and otherwise, this will lead to the following definition

$$\to \begin{Bmatrix} \alpha = \alpha_0 \cos\varphi = 0 \\ v < 0 \end{Bmatrix} \to \begin{Bmatrix} \cos\varphi = 0 \\ \sin\varphi > 0 \end{Bmatrix} \to \varphi = \frac{\pi}{2}$$

It is straightforward to show that

$$\alpha = \alpha_0 \cos\left(2\pi f t + \frac{\pi}{2}\right) \tag{16}$$

V. SIMULATION

In order to show the effectiveness of the proposed control algorithm, a set of simulations are carried out. In the first case, no swing angle limit is placed, the payload responses are given in Fig 3, 4 and 5. It can be seen that, the payload swing angle is quite large.

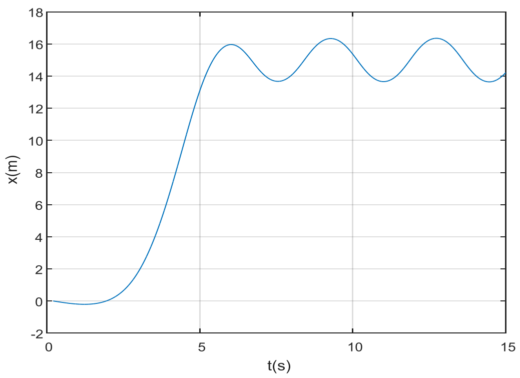

Fig 3. The position responses (x-axis)

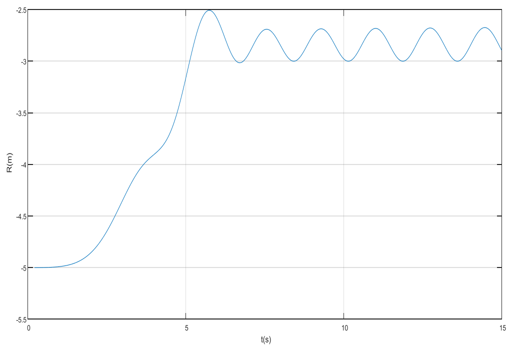

Fig 4. The position responses (z-axis)

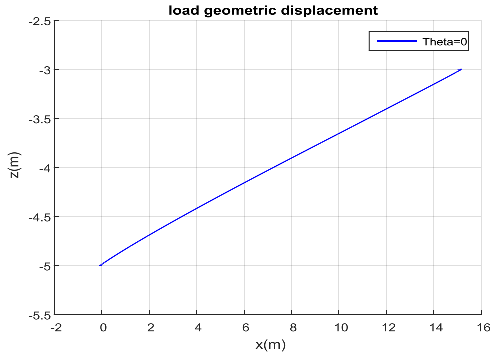

Fig 5. Load geometric displacement

Fig 6, 7, 8 and 9 demonstrate system responses when angle limit is considered,

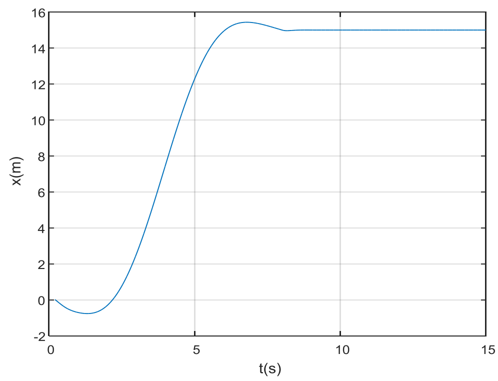

Fig 6. The position responses (x-axis)

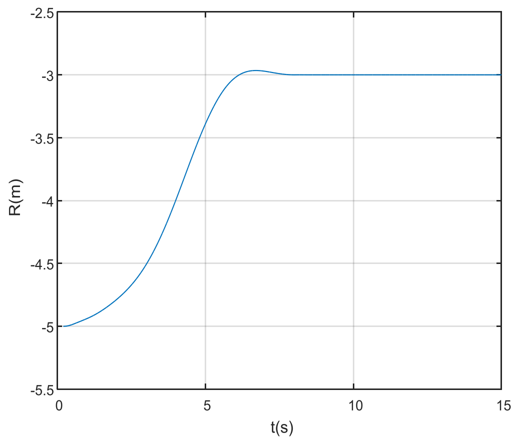

Fig 7. The position responses (z-axis)

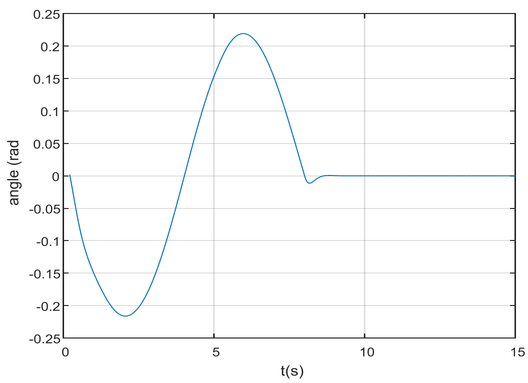

Fig 8. Motion planning angle

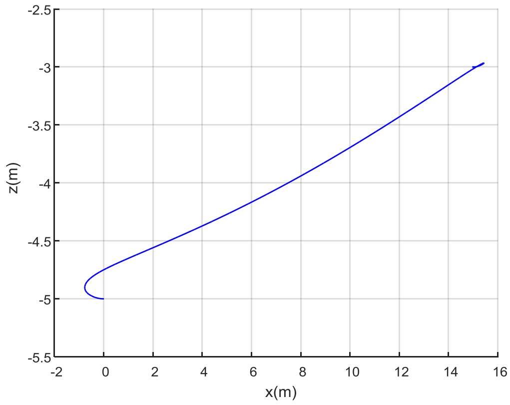

Fig 9. Load displacement

With the action of the additional PI controller, swing angle is restricted to a certain limit. The limit can be tighten which results in slower system responses, and the relaxed limit will posses faster responses.

VI. CONCLUSIONS

The tracking and vibration suppression problems are investigated in the paper. Based on flatness property's of the system a controller is designed to tackle the aforementioned problem. Payload trajectories are designed to achieve desired location with minimum vibrations. A set of simulation results are given to illustrate the control ability of the system.

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