Comparison of Optimal Problem Solving Methods in Model Predictive Control for Twin Rotor MIMO System

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Abstract

There are many methods to optimize the installation of service search algorithm optimized for testing optimization problem $U^* = \arg\min_{U \in U} J(U)$ *of model predictive control, these algorithms called the nonlinear programming [2], [7]. In Model Predictive Control, when dealing with the binding conditions attached to the nonlinear programming to plan is often used to test optimization problems like: Quadratic Programming (QP), Sequential quadratic programming (SQP), interior point, or Genetic algorithm (GA). When the control problem is in the form* $Q(u) = \int_0^T g(x,u,t)dt \to \min_u$ *,we can propose optimal control methods such as: the Bellman's dynamic programming, Pontragin's maximal principle or variation methods [7], [8]. Each method has its advantages and disadvantages. This paper compares two methods of solving optimal problems in model predictive control that apply the TRMS object control as Sequential quadratic programming (SQP), variational method.*

Keywords: *Optimization problem, Model predictive control, Sequential quadratic programming, Variational method.*

I. INTRODUCTION

Optimization of the model predictive control is a problem that is researching by many scientists. Until now, it was mainly used line search methods with finite predictive horizon for solving to optimize the model predictive control [1], [2], because these methods are quite favorable for contrainted optimal problems. Moreover, there have few other optimization methods as Levenberg-Marquardt or trust region. However, all above methods were only used for finite predictive horizons. Therefore, these do not ensure the global optimization. So, the system is difficult to be stable [3].

The dynamic programming method is an effective one for solving optimal problems in multivariation with ensuring the global of the optimal solution. Currently, this method is just applied to solve the optimal problem for linear systems with constant parameters or parameters changing over time.

In [8], we present the tracking model predictive control method to get desired output and an infinite predictive horizon for bilinear continuous systems by using the variational method. In this paper, we will present the advantages and disadvantages of the SQP and variational methods for solving optimal problems in model predictive control that apply the TRMS.

II. THE TRMS MODEL

The TRMS was given in figure 1

The TRMS is a bilinear system with two inputs and two outputs. It can be described by the continuous model:

$$\begin{cases} \dot{\mathbf{x}} = A(\mathbf{x})\mathbf{x} + B(\mathbf{x})\mathbf{u} \\ \mathbf{y} = C(\mathbf{x})\mathbf{x} \end{cases} \quad (1)$$

State variables, inputs, outputs, respectively are:

$$x = [\omega_h, S_h, \alpha_h, \omega_v, S_v, \alpha_v]^T \quad (2)$$

$$u = [U_h \quad U_v]^T \quad (3)$$

$$y = [\alpha_h \quad \alpha_v]^T \quad (4)$$

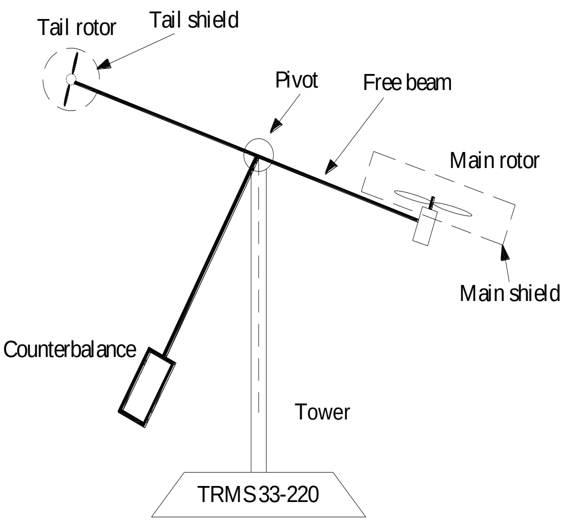

Fig 1. The TRMS

Where:

ω_h: Rotational velocity of the tail rotor (*rad/s*)

S_h :Angular velocity of the TRMS beam in the horizontal plane without affect of the main rotor (*rad/s*)

α_h **:** Yaw angle of the TRMS beam (*rad*)

ω_v: Rotational velocity of main rotor(*rad/s*)

S_v: Angular velocity of the TRMS beam in the vertical plane without affect of the tail rotor (*rad/s*)

α_v: Pitch angle of the TRMS beam (*rad*)

U_h: Input voltage signal of the tail motor (*V*)

U_v: Input voltage signal of the main motor (*V*)

The nonlinear continuous state space equations of the TRMS are expressed in [4], [5], [6] as (7):

Where:

$R_{ah}, L_{ah}, k_{ah}\varphi_h, J_{tr}, B_{tr}, l_t, D, E, F, k_m, R_{av}, L_{av}, k_{av}\varphi_v, J_{mr}, B_{mr}, l_m, k_g, g, A, B, C, H, J_v, k_t$

are positive constants, Ω_h and Ω_v is defined by

$$\Omega_h = S_h + \frac{k_m \omega_v \cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \qquad (5)$$

$$\Omega_v = S_v + \frac{k_t \omega_h}{J_v} \qquad (6)$$

$$\frac{d}{dt}\begin{bmatrix} \omega_h \\ S_h \\ \alpha_h \\ \omega_v \\ S_v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -\dfrac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \dfrac{B_{tr}}{J_{tr}}\omega_h - \dfrac{f_1(\omega_h)}{J_{tr}} + \dfrac{k_{ah}\varphi_h}{J_{tr}R_{ah}} f_6(U_h) \\ \dfrac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \dfrac{k_m \omega_v \cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\dfrac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \dfrac{B_{mr}}{J_{mr}}\omega_v - \dfrac{f_4(\omega_v)}{J_{mr}} + \dfrac{k_{av}\varphi_v}{J_{mr}R_{av}} f_8(U_v) \\ \dfrac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} + \dfrac{g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \dfrac{k_t}{J_v}\omega_h \end{bmatrix} \qquad (7)$$

III. INSTALLATION OF NONLINEAR MODEL PREDICTIVE CONTROL FOR THE TRMS

A. Control algorithm applying for SQP

Based on the mathematical models of the above TRMS, in this part, the author will present the installation of the constraint system applied to TRMS system which uses SQP and the variational method.

Considering the constant bilinear system model as follows:

$$\begin{cases} \dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u}, \ \boldsymbol{x} \in \mathbb{R}^n, \ \boldsymbol{u} \in \mathbb{R}^m \\ \boldsymbol{y} = C(\boldsymbol{x})\boldsymbol{x}, \ \boldsymbol{y} \in \mathbb{R}^m \end{cases} \qquad (8)$$

If that model is rewritten by sampling signals with sampling cycle T and approximations for the derivative formula is used, we have:

$$\frac{d\boldsymbol{x}}{dt} \approx \frac{\boldsymbol{x}_{k+1} - \boldsymbol{x}_k}{T}, \ \boldsymbol{x}_k = \boldsymbol{x}(kT),$$
$$\boldsymbol{u}_k = \boldsymbol{u}(kT), \ \boldsymbol{y}_k = \boldsymbol{y}(kT) \qquad (9)$$

A uncontinuous model is found:

$$\begin{cases} \boldsymbol{x}_{k+1} = \mathrm{A}(\boldsymbol{x}_k)\boldsymbol{x}_k + \mathrm{B}(\boldsymbol{x}_k)\boldsymbol{u}_k \\ \boldsymbol{y}_k = \mathrm{C}(\boldsymbol{x}_k)\boldsymbol{x}_k \end{cases} \qquad (10)$$

where:

$\mathrm{A}(\boldsymbol{x}_k) = I + TA(\boldsymbol{x}_k), \ \mathrm{B}(\boldsymbol{x}_k) = TB(\boldsymbol{x}_k), \ \mathrm{C}(\boldsymbol{x}_k) = C(\boldsymbol{x}_k)$

Hence, we get the predictive model for bilinear systems throughout the current predictive window $\left[k, k+N_p\right)$ as follows:

$$\begin{cases} \hat{\boldsymbol{x}}(k+i+1|k) = \mathrm{A}(k+i)\hat{\boldsymbol{x}}(k+i|k) + \mathrm{B}(k+i)\hat{\boldsymbol{u}}(k+i|k) \\ \hat{\boldsymbol{y}}(k+i|k) = \mathrm{C}(k+i)\hat{\boldsymbol{x}}(k+i|k) \end{cases} \qquad (11)$$

Block of predictive model

Through regression calculations by the predictive model (11) as follows:

$$\begin{aligned}
\hat{\boldsymbol{x}}(k+1|k) &= \mathrm{A}(k)\boldsymbol{x}(k|k) + \mathrm{B}(k)\hat{\boldsymbol{u}}(k|k)\\
\hat{\boldsymbol{x}}(k+2|k) &= \mathrm{A}(k+1)\hat{\boldsymbol{x}}(k+1|k) + \mathrm{B}(k+1)\hat{\boldsymbol{u}}(k+1|k)\\
&= \mathrm{A}(k+1)\left[\mathrm{A}(k)\boldsymbol{x}(k|k) + \mathrm{B}(k)\hat{\boldsymbol{u}}(k|k)\right] + \mathrm{B}(k+1)\hat{\boldsymbol{u}}(k+1|k)\\
&= \mathrm{A}(k+1)\mathrm{A}(k)\boldsymbol{x}(k|k) + \mathrm{A}(k+1)\mathrm{B}(k)\hat{\boldsymbol{u}}(k|k) + \mathrm{B}(k+1)\hat{\boldsymbol{u}}(k+1|k) \qquad (12)
\end{aligned}$$

and

$$\begin{aligned}
\hat{\boldsymbol{x}}(k+N_p|k) = \mathrm{A}(k+N_p-1)\cdots\mathrm{A}(k+1)\mathrm{A}(k)\boldsymbol{x}(k|k) +&\\
+\mathrm{A}(k+N_p-1)\cdots\mathrm{A}(k+1)\mathrm{B}(k)\hat{\boldsymbol{u}}(k|k) +&\\
+\mathrm{A}(k+N_p-1)\cdots\mathrm{A}(k+2)\mathrm{B}(k+1)\hat{\boldsymbol{u}}(k+1|k) +&\\
\vdots&\\
+\mathrm{A}(k+N_p-1)\mathrm{B}(k+N_p-2)\hat{\boldsymbol{u}}(k+N_p-2|k) +&\\
+\mathrm{B}(k+N_p-1)\hat{\boldsymbol{u}}(k+N_p-1|k)& \qquad (13)
\end{aligned}$$

It will be modified through a range of the predictive output values which are in the current predictive window $[k, k+N_p)$. Due to the status $\boldsymbol{x}_k = \boldsymbol{x}(k|k)$ of the system at the time of k (it is made by using sensors measurement or observe, $\hat{\mathrm{Y}}$ only depends on $\hat{\mathrm{U}}$:

$$\hat{\mathrm{Y}} = M(\hat{\mathrm{U}})\boldsymbol{x}(k|k) + N(\hat{\mathrm{U}})\hat{\mathrm{U}} = M(\hat{\mathrm{U}})\boldsymbol{x}_k + N(\hat{\mathrm{U}})\hat{\mathrm{U}} = \qquad (14)$$

Constructing the objective function

If we exchange the given relationship in the formula (12):

$$\begin{aligned}
\hat{\boldsymbol{x}}(k+N_p|k) &= \mathrm{A}_k^{k+N_p-1}\boldsymbol{x}(k|k) + \mathrm{A}_{k+1}^{k+N_p-1}\mathrm{B}(k)\hat{\boldsymbol{u}}(k|k) + \cdots +\\
&+ \mathrm{A}_{k+N_p-1}^{k+N_p-1}\mathrm{B}(k+N_p-2)\hat{\boldsymbol{u}}(k+N_p-2|k) + \mathrm{B}(k+N_p-1)\hat{\boldsymbol{u}}(k+N_p-1|k)\\
&= \mathrm{A}_k^{k+N_p-1}\boldsymbol{x}(k|k) + \left(\mathrm{A}_{k+1}^{k+N_p-1}\mathrm{B}(k), \ldots, \mathrm{A}_{k+N_p-1}^{k+N_p-1}\mathrm{B}(k+N_p-2), \mathrm{B}(k+N_p-1)\right)\hat{\mathrm{U}} \qquad (15)
\end{aligned}$$

into the formula of penetration function $s(\hat{x}(k+N_p|_k))$, then: as $x_k = x(k|k)$ is supposed to have been known through the measurement by sensors or observation, we have:

$s(\hat{\boldsymbol{x}}(k+N_p|k)) = s(\hat{\mathrm{U}})$ just depends on $\hat{\mathrm{U}}$. Therefore, when we use more symbols:

$$\mathrm{Y}_{ref} = col\left(\boldsymbol{y}_{ref}((k+1)T), \boldsymbol{y}_{ref}((k+2)T), \ldots, \boldsymbol{y}_{ref}((k+N_p)T)\right)$$

$$Q = diag(Q_i)$$

$$R = diag(R_j)$$

where "col" is column

The corresponding objective function for TRMS system will be:

$$\begin{aligned}
J(\hat{\mathrm{U}}) &= \sum_{i=1}^{N_p}\left(\mathrm{Y}(k+i) - \hat{\mathrm{Y}}(k+i|k)\right)^T Q_i\left(\mathrm{Y}(k+i) - \hat{\mathrm{Y}}(k+i|k)\right) +\\
&\sum_{j=0}^{N_c-1}\hat{\mathrm{U}}(k+j-1)^T R_j\hat{\mathrm{U}}(k+j-1) + s\left(\hat{\boldsymbol{x}}(k+N_p|k)\right)\\
&= \left(\mathrm{Y}_{ref} - \hat{\mathrm{Y}}\right)^T Q\left(\mathrm{Y}_{ref} - \hat{\mathrm{Y}}\right) + \hat{\mathrm{U}}^T R\hat{\mathrm{U}} + s(\hat{\mathrm{U}})\\
&= \left[\mathrm{Y}_{ref} - \left(M(\hat{\mathrm{U}})\boldsymbol{x}_k + N(\hat{\mathrm{U}})\hat{\mathrm{U}}\right)\right]^T Q\left[\mathrm{Y}_{ref} - \left(M(\hat{\mathrm{U}})\boldsymbol{x}_k + N(\hat{\mathrm{U}})\hat{\mathrm{U}}\right)\right] +\\
&+ \hat{\mathrm{U}}^T R\hat{\mathrm{U}} + s(\hat{\mathrm{U}}) \qquad (16)
\end{aligned}$$

Constraints conditions

In case there are the constraints conditions, the algorithm is installed:

$$y_j^{\min} \le y_j(k+i|k) \le y_j^{\max},\ i = 1, 2, \ldots, N_p \qquad (17)$$

$$u_j^{\min} \le u_j(k+i-1|k) \le u_j^{\max},\ i = 1, 2, \ldots, N_c \qquad (18)$$

$$\Delta u_j^{\min} \le \Delta u_j(k+i-1|k) \le \Delta_j^{\max},\ i = 1, 2, \ldots, N_c \qquad (19)$$

where $j = 1, 2$ is the number of elements of vector $\boldsymbol{y}(k+i|k)$, $\boldsymbol{u}(k+i|k)$ and $\Delta\boldsymbol{u}(k+i|k)$ at the time

$k+i$ in the future and belongs to the k^{th} predictive window k :

$$y(k+i|k) = \left(y_1(k+i|k)\,,\, y_2(k+i|k)\right)^T = \left(\alpha_h((k+i)T)\,,\,\alpha_v((k+i)T)\right)^T$$

$$\boldsymbol{u}(k+i|k) = \left(u_1(k+i|k)\,,\,u_2(k+i|k)\right)^T = \left(U_h((k+i)T)\,,\,U_v((k+i)T)\right)^T$$

$$\Delta\boldsymbol{u}(k+i|k) = \left(\Delta u_1(k+i|k)\,,\,\Delta u_2(k+i|k)\right)^T = \left(\Delta U_h((k+i)T)\,,\,\Delta U_v((k+i)T)\right)^T \quad (20)$$

As $y_1 = \alpha_h$ is the third element, $y_2 = \alpha_v$ is the sixth element and it is two elements of $\boldsymbol{u}$ that is the control signal. Therefore, the formula presents the constraint system will be:

$$U = \left\{ \hat{U} \in \mathbb{R}^{2N_p} \;\middle|\; \begin{array}{l} \Delta u_j^{\min} \le \Delta u_j(k+i-1|k) \le \Delta_j^{\max},\ i = 1,2,\ \dots\ ,N_c \\ y_j^{\min} \le \hat{\boldsymbol{a}}_j^T(i)\hat{\boldsymbol{x}}_k + \hat{\boldsymbol{f}}_j^T(i)\hat{U} \le y_j^{\max},\ j = 3,6 \text{ and } i = 1,2,\ \dots\ ,N_p \\ u_j^{\min} \le \hat{\boldsymbol{a}}_j^T(i)\hat{\boldsymbol{x}}_k + \hat{\boldsymbol{f}}_j^T(i)\hat{U} \le u_j^{\max},\ j = 7,8 \text{ and } i = 1,2,\ \dots\ ,N_c \end{array} \right\} \quad (21)$$

B.Control algorithm applying for variational method

Consider a bilinear system MIMO with the same input and output signals, presented by continuous model:

$$\begin{cases} \dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = C(\boldsymbol{x})\boldsymbol{x} \end{cases} \quad (22)$$

where $\boldsymbol{u} \in \mathbb{R}^m$ are vector of m the input signals, $\boldsymbol{y} \in \mathbb{R}^m$ are vector of m the output signals, and $\boldsymbol{x} \in \mathbb{R}^n$ are vector of n the state variables. $A(\boldsymbol{x}),\ B(\boldsymbol{x})$ and $C(\boldsymbol{x})$ are state dependent matrices. In general, the model (22) has $n \ge m$.

Assuming that the system is controlled by the model predictive controller with the interval $T_k = t_{k+1} - t_k$ of predictive horizon which is also the sample time signal. If T_k is small enough, the matrices $A(\boldsymbol{x}),\ B(\boldsymbol{x}),\ C(\boldsymbol{x})$ can be approximated by constant matrices:

$$A(\boldsymbol{x}) \approx A_k,\ B(\boldsymbol{x}) \approx B_k,\ C(\boldsymbol{x}) \approx C_k \quad (23)$$

when $t_k \le t < t_{k+1}$. An in this case, the system can be approximated by a linear model with constant parameters:

$$\begin{cases} \dot{\boldsymbol{x}} = A_k\boldsymbol{x} + B_k\boldsymbol{u} \\ \boldsymbol{y} = C_k\boldsymbol{x} \end{cases} \quad (24)$$

Let $\boldsymbol{y}_{ref}$ be the sample output signals that system must follow. Assume that the sample signals are constants (or may be segment constants) as well as under the influence of constant signals, state feedback closed-loop system (24) will tend to the steady state, with steady state $\boldsymbol{x}_e$, i. e, constant signals $\dot{\boldsymbol{x}}_e = \boldsymbol{0}$, and input signals are also in steady state. Now, the established values of this system will satisfy:

$$\begin{cases} \boldsymbol{0}_n = A_k\boldsymbol{x}_e + B_k\boldsymbol{u}_e \\ \boldsymbol{y}_{ref} = C_k\boldsymbol{x}_e \end{cases} \quad (25)$$

Hence, we get a system of $n+m$ equations with unknowns $n+m$: $(\boldsymbol{x}_e,\boldsymbol{u}_e)$ as follows:

$$\begin{pmatrix} A_k\boldsymbol{x}_e + B_k\boldsymbol{u}_e \\ C_k\boldsymbol{x}_e \end{pmatrix} = \begin{pmatrix} \boldsymbol{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} A_k & B_k \\ C_k & \Theta \end{pmatrix}\begin{pmatrix} \boldsymbol{x}_e \\ \boldsymbol{u}_e \end{pmatrix} = \begin{pmatrix} \boldsymbol{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \boldsymbol{x}_e \\ \boldsymbol{u}_e \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C_k & \Theta \end{pmatrix}^{-1}\begin{pmatrix} \boldsymbol{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix} \quad (26)$$

This system allow us to find the steady state values $(\boldsymbol{x}_e,\boldsymbol{u}_e)$ from the sample output signals $\boldsymbol{y}_{ref}$. Put $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{x}_e$ and $\boldsymbol{\rho} = \boldsymbol{u} - \boldsymbol{u}_e$. Since $(\boldsymbol{x}_e,\boldsymbol{u}_e)$ are constant vectors, and in (24) and (25), we have the equivalent model in transitional process as the following:

$$\dot{\boldsymbol{\delta}} = A_k\boldsymbol{\delta} + B_k\boldsymbol{\rho} \quad (27)$$

In order to design model predictive control for the continuous system (22) to get stable tracking, i.e., $\boldsymbol{y} \to \boldsymbol{y}_{ref}$, we will control the system (27) to achieve $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{x}_e \to \boldsymbol{0}$ and $\boldsymbol{\rho} = \boldsymbol{u} - \boldsymbol{u}_e \to \boldsymbol{0}$ by using the optimal control method LQR for the translated step of the $k-th$ predictive horizon with the infinite predictive horizon. That means, we minimize the objective function

$$J_k(\boldsymbol{\rho}) = \frac{1}{2}\int_{t_k}^{\infty}\left(\boldsymbol{\delta}^T Q_k\boldsymbol{\delta} + \boldsymbol{\rho}^T R_k\boldsymbol{\rho}\right)dt. \quad (28)$$

where $Q_k,\ R_k$ are two arbitrary symmetric positive definite matrices, which can be changed at each translated step of the predictive horizon.

Using the variation method to find solution ρ^* of the optimal problem in transitional process, given by (27), (28). We will have [7]:

$$L_k B_k R_k^{-1} B_k^T L_k - L_k A_k - A_k^T L_k = Q_k \quad (29)$$

and

$$\boldsymbol{\rho}^* = R_k^{-1}B_k^T L_k\boldsymbol{\delta} = R_k^{-1}B_k^T L_k(\boldsymbol{x} - \boldsymbol{x}_e) \quad (30)$$

From these, we get optimal control signals:

$$u^*(t) = \rho^*(t) + u_e \quad \text{với } t_k \le t < t_{k+1} \tag{31}$$

for bilinear continuous systems (22) in the current predictive horizon.

Summarily, model predictive control with infinite predictive horizon to apply for bilinear continuous systems (22), will work well with an algorithm including iterative steps as follows:

Algorithm: The state feedback model predictive control so that the output signals track the reference for bilinear continuous systems with an infinite predictive horizon.

1. Choose the appreciate symmetric positive definite weight matrices Q_k, R_k. Take $t_0 = 0$ and $k = 0$.

2. Sample x_k and approximate $A(x), B(x), C(x)$ by A_k, B_k, C_k as (23).

3. Determine (x_e, u_e) from y_{ref} by (26).

4. Find L_k that is a symmetric positive semidefinite solution of Riccati equation (29).

Find ρ^* corresponding L_k by (30), then find u^* from (10).

5. Choose t_{k+1} so that $\left\| e^{\hat{A}_k (t_{k+1} - t_k)} \right\| < 1$ with $\hat{A}_k = A_k - B_k R_k^{-1} B_k^T L_k$. Put u^* as the input of (22) in the interval $t_k \le t < t_{k+1}$ and assign $k := k+1$, then go back to step 2. Here, Q_k and R_k are the arbitrary weight matrices, which can be changed at each step of the translation of the predictive horizon, i. e they depend on k such that the solution of the optimal problem satisfies the bounded condition

$$|\rho| \le \Omega \in \mathbb{R} \tag{32}$$

with Ω is the given upper bounded value whereas $\delta = x - x_e \to 0$. Moreover, to set up the algorithm, we have to use a method for solving the Riccati equation (29). We can find some methods to seek L_k effectively in [7].

A model predictive control working in the way of this algorithm was illustrated Figure 5. This close-loop system works basing on the state feedback principles and is not a discrete system. In each control horizon with a infinite width $[k, \infty]$, $u^*(t)$ are used to control the plant only in an sample time interval $t_k \le t < t_{k+1}$. Thus, we denote $u^*(t)$ instead of $u_k^*(t)$. In the whole control proccess, control signals $u(t)$ will be a sequence of continuous signals $u_k^*(t), k = 0,1,\ldots$ Therefore, this close-loop system is one of the sampled data systems [1].

IV. SIMULATION RESULTS

With the object model, objective function and constraint conditions discussed above when installed into model prediction controllers using SQP and variational method to solve the problem. I obtain the following results:

A. Using SQP method

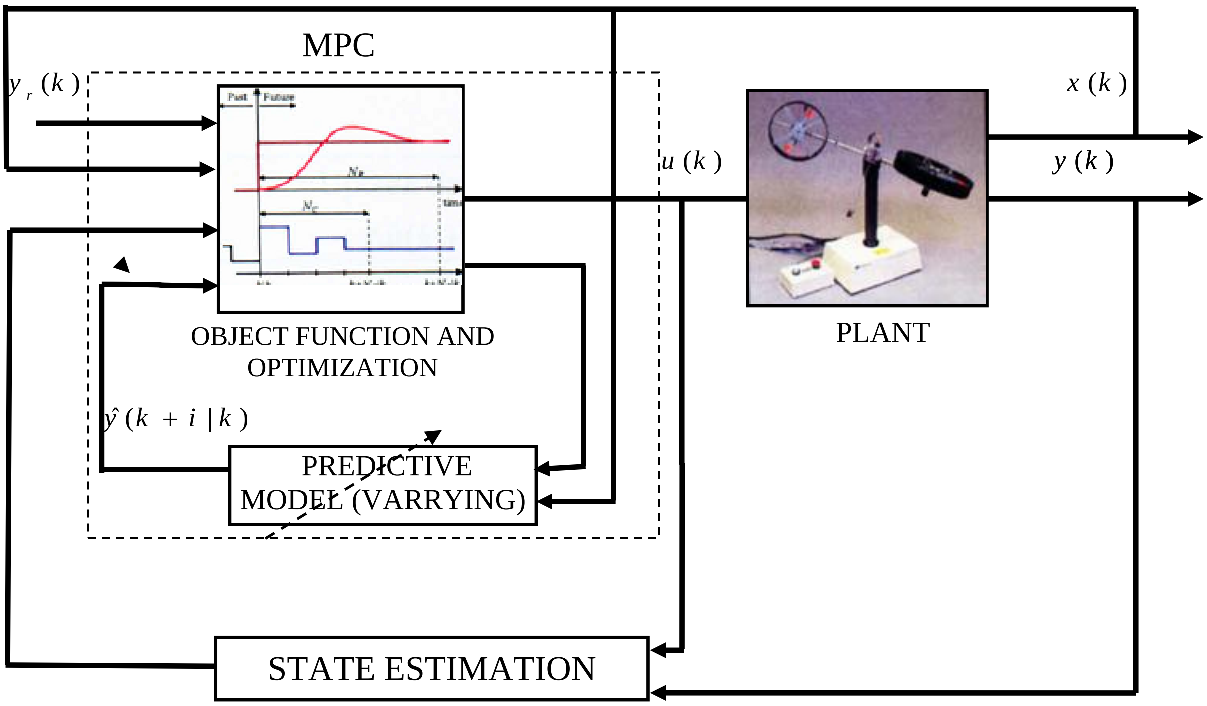

Fig 2. Structure of Model Predictive Control to Applying SQP

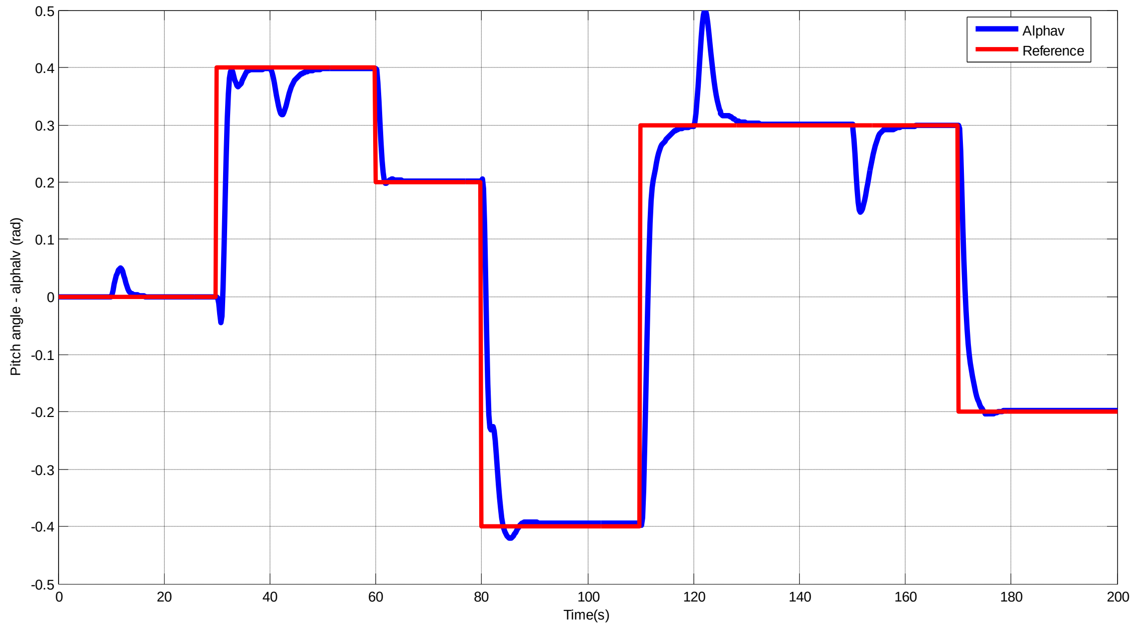

Fig 3. The response of Pitch angle as reference signal is the substep

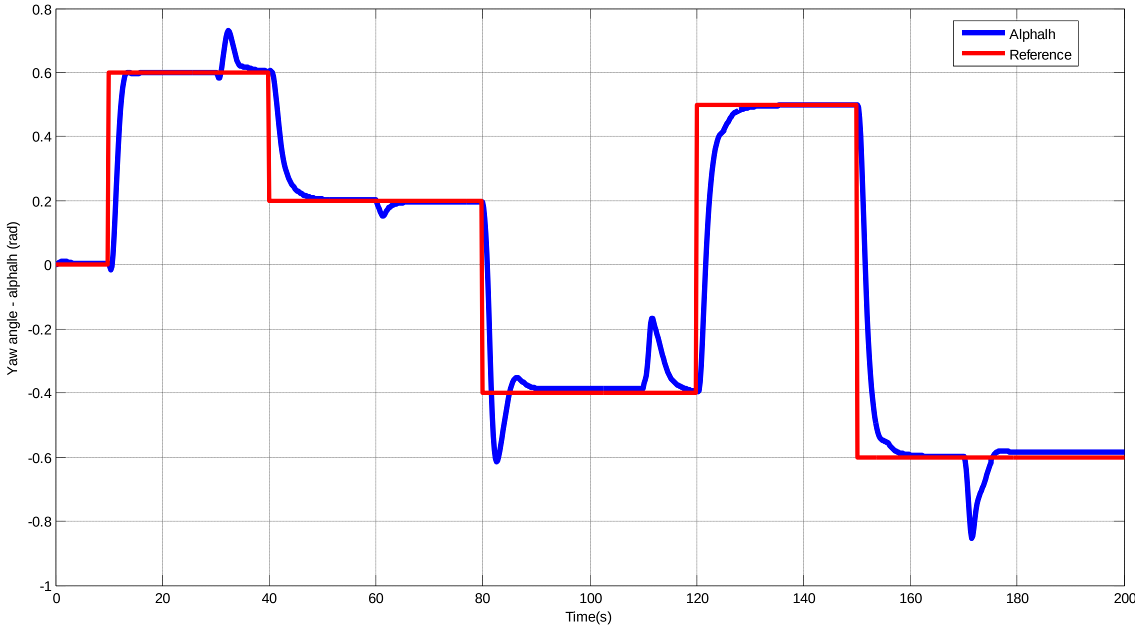

Fig 4. The response of Yaw angle as reference signal is the substep

C. Using variational method

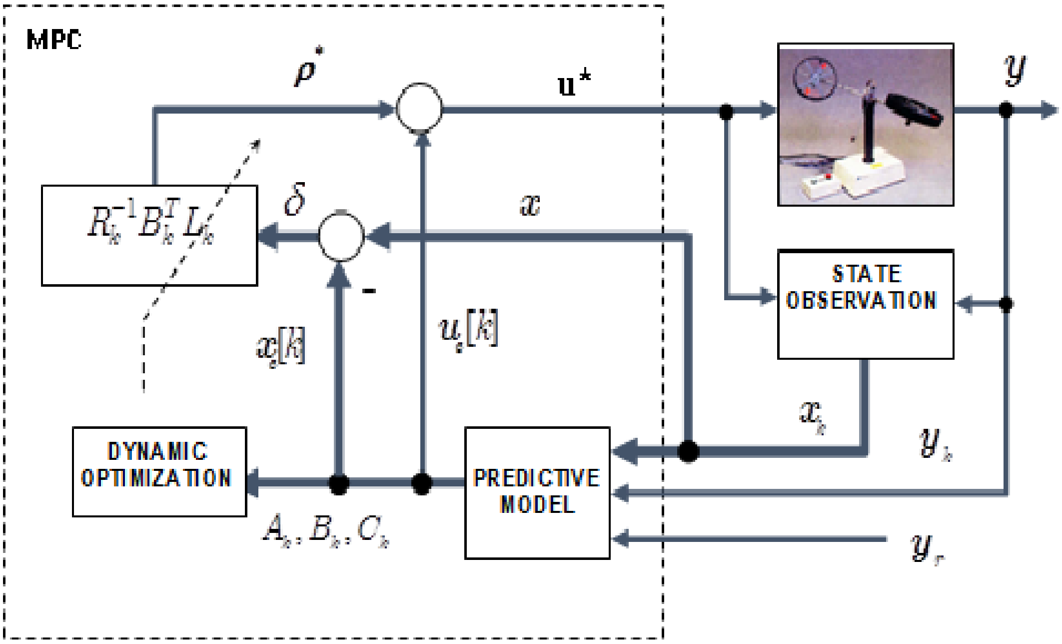

Fig 5. Structure of Model Predictive Control to Applying variational method

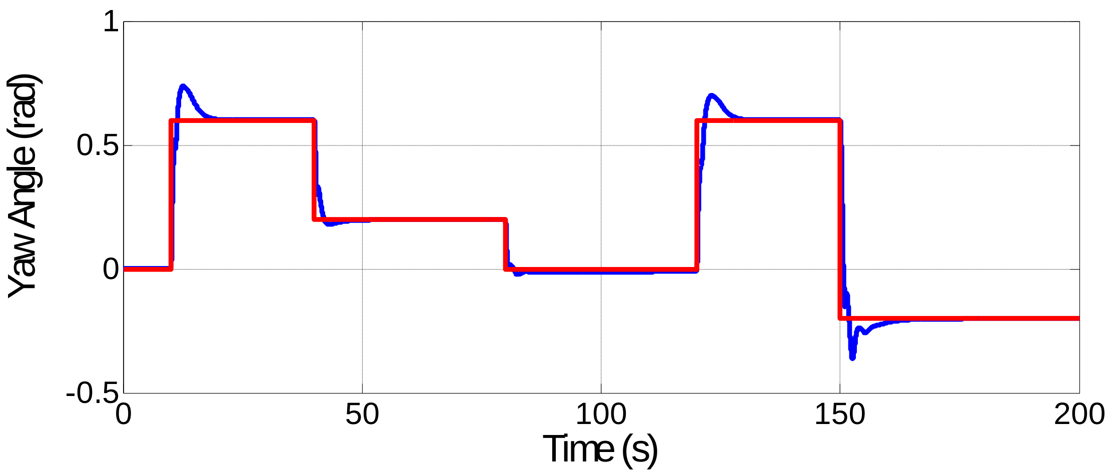

Fig 6. The response of the Yaw angle control loop with respect to a substep

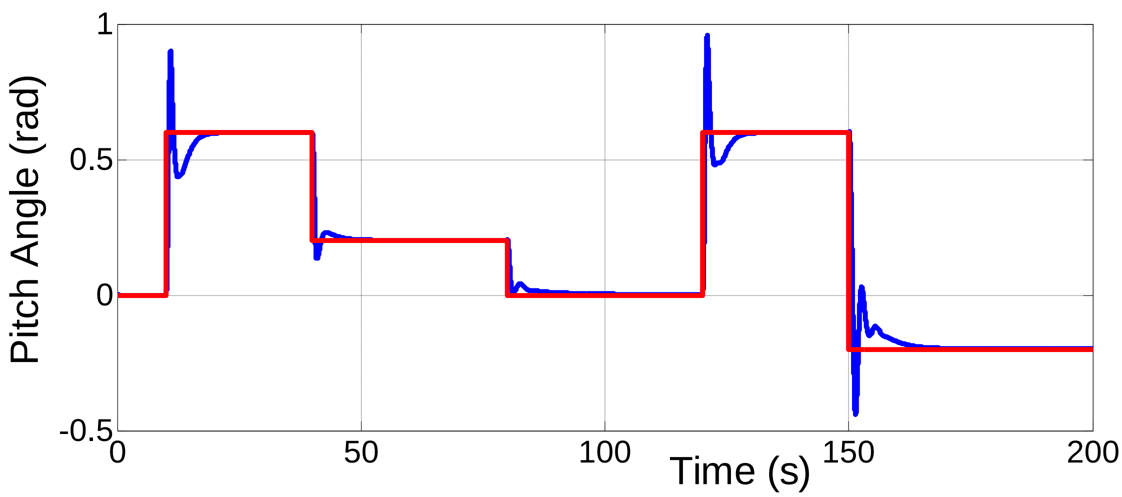

Fig 7. The response of the pitch angle control loop with respect to a substep

Comparison and evaluation of quality

Advantages, disadvantages of methods:

Advantages of SQP: Satisfies constraints (including constraints on status and input constraints, output constraints).

Disadvantages of SQP: It is imperative to use a discrete model, a long time to compute, an online calculation, a finite predictive horizon, stable quality of the system depends on the penalty $s(x(k+N_c|k))$ is choosen, while there is not an optimal penalty, it is difficult to apply for reallity.

To determine the restriction of SQP method, this paper offer the variable method.

Advantages of variational method: calculating time is faster, using a continuous model, easy to install and practical application, works with infinite predictive horizon, definitely ensure stability.

Disadvantages of variational method: The complexity of the constraints can not be directly dealt with.

V. CONCLUSION

In the Model Predictive Control, one of the important jobs must be implemented is to build the model and to solve the optimal problem so that the object function is minimized, and satisfy the constraints condition. This paper, author has compared two methods to solve optimal problem, analyze the advantages and disadvantages of each method, compare simulation runtime and real runtime on the same computer to see the advantages of the variational method. However, to overcome the limitation of the variational method is difficult to solve directly the complex constraint conditions. The author proposed the law of changing the weight matrix in the objective function, the constraint conditions will be satisfied. It will study in future work.

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