Model Order Reduction using Routh Approximation Method, Factor Division Method and Genetic Algorithm

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Abstract

In this paper author proposed a mixed technique for reducing the order of the high order dynamic systems to lower order dynamic system. In this paper authors introducing three method, Routh approximation method, factor division method, genetic method. Here, all three methods proposed guarantees of the stability of the reduced model if the original model (High Order system) is stable. This proposed method is described the numerical example and also be find step response of the system and also find bode plot of the system. This is all system oriented program done on MATLAB.

Keywords - *Higher order dynamic system, Lower order dynamic system, Routh approximation method, factor division method, genetic method, stability, Transfer Function, Step Response, Bode plot, impulse response*

I. INTRODUCTION

Mathematical description of many physical systems when obtained using theoretical methods Here, in this paper author represent the considerations often results in high-order system. To be more precise, in the time domain or state space representation, the modeling procedure leads to a high-order state space model and a high-order transfer function model in frequency domain representation in the lower order in frequency domain.

Model order reduction techniques to reduce the higher order to lower order of the system, and this is very challenging area of the control system. The mathematical models of higher order dynamic systems can be introduce in state space form or in transfer function form which are also called frequency domain representations. In the state space representation a physical system is represented by set of first order differential equations. Similarly in the transfer function representation a physical system is represented as a rational function.

Many physical systems are translated into mathematical model through higher order differential equations. It is usually recommended to reduce the order of the model retaining the dominant behavior of the original system. This will help to make better understanding of the physical system, reduce computational complexity, reduce hardware complexity and simplifies the controller design. The proposed research works deals with the methods of approximating the transfer function of high order system by one of lower order system.

Mixed methods of model order reduction are proposed in frequency domain to reduce the higher order system into various lower order models. In mixed methods , the reduced order model is obtained by one of the stability based reduction methods such as Factor Division method, Routh Approximation Method and Genetic Method. The method preserves steady state value and stability of original system in the reduced order models for stable systems. The response of reduced order model obtained is compared on the basis of unit step response and frequency response. Also some of these methods are extended for order reduction of multi input and multi output system and discrete time system.

A new generation controller has been designed on the basis of approximate model matching, based on Linear or non-linear approaches, using the many method available. In directly, in this paper introduces many method converter the higher order transfer function to lower order transfer function. Actually, Controller are design very high performance and it is complex, for the high order system and open loop system is not good for the any system so that always the closed loop response of original high order system and high order controller with unity feedback is reduced and compared with reference model. The performance comparison of various models has been carried out using MATLAB.

II. METHODS USED

A. Routh Approximation Method

In this method, the alpha and beta tables are constructed in a way. For the-transfer function H (s) given in the form, the first two rows of the two tables are rearranged as indicated below:

$$alpha\ I\ row: \quad \alpha_i^0 = a_i \quad \alpha_i^0 = a_{n-i}$$

$$table\ II\ row: \quad \alpha_i^1 = a_{i+1} \quad \alpha_i^1 = a_{n-(i+1)}$$

$$beta\ I\ row: \quad b_i^1 = b_{i+1} \quad b_i^1 = b_{n-i}$$

$$table\ II\ row: \quad b_i^2 = b_{i+2} \quad b_i^2 = b_{n-(i+1)} \tag{1}$$

If the transfer function is expressed as

$$H(s) = \frac{b_n s^{n-1} + \cdots + b_1}{a_n s^n + \cdots + a_0} \tag{2}$$

the alpha and beta tables need not be rearranged.
The new algorithms necessary to compute the numerator and denominator functions, $B_k(s)$ and $A_k(s)$, respectively, of the k^{th} Routh approximans $H_k(s)$ with the computed values of alphas and betas are

$$B_1(s) = \beta_1$$

$$A_1(s) = s + \alpha_1$$

$$B_2(s) = \beta_2 s + \alpha_2\beta_1$$

$$A_2(s) = s^2 + \alpha_2 s + \alpha_1\alpha_2$$

$$B_3(s) = (\beta_1 + \beta_3)s^2 + \beta_2\alpha_3 s + \beta_1\alpha_2\alpha_3$$

$$A_3(s) = s^3 + (\alpha_1 + \alpha_3)s^2 + \alpha_2\alpha_3 s + \alpha_1\alpha_2\alpha_3$$

In general, $B_k(s)$ and $A_k(s)$ are given by

$$B_k(s) = \beta_k s^{k-1} + s^2 B_{k-2}(s) + \alpha_k B_{k-1}(s) \tag{3}$$

$$A_k(s) = s^2 A_{k-2}(s) + \alpha_k A_{k-1}(s) \tag{4}$$

With $B_{-1}(s)$=O, $B_0(s)$=O, $A_{-1}(s)$=l/s and $A_0(s)$=l.
Extension of Routh Approximation Method to multiple-input-multiple-output systems is straightforward as illustrated below for a system with two outputs and a single input.

B. Factor Division Method

In this factor division technique, the higher order polynomial equation can be reduce using this method. The higher order linear variant or invariant equation can be reduce b using modified cauer form. The cauer form reduce the denominator polynomial equation and factor division method can reduce nominator equation. This technique is easy and reduces the complex equation in simple. This technique provide stable system for high order system.

Reduction Method

Let the higher order transfer function of the equation with n^{th} order and k^{th} order are:

$$G_n(s) = \frac{b_{11} + b_{12}s + b_{13}s^3 + \cdots \ldots \ldots + b_{1n}s^{n-1}}{a_{11} + a_{12}s + a_{13}s^2 \ldots \ldots + a_{13}s^{n-1} + s^n} \tag{5}$$

Where; b_{1i} & a_{1i} , $1 \le i \le n$ are known scalar constants

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{b_{k,1} + b_{k,2}s + \cdots \ldots \ldots + b_{k,k}s^{k-1}}{a_{k+1,1} + a_{k+1,2}s + \cdots \ldots \ldots + a_{k+1,k}s^{k-1} + s^k} \tag{6}$$

Here, polynomial higher order reduction is in objective of the K^{th} order reduced in the model in the form (6) from the original system is (5).

This technique for solving consists of the following two steps:

Step-1: Determine of the denominator polynomial $D_k(s)$ for higher order polynomial system where k^{th} is the reduced order of the system using technique cauer form.
The given higher order system $G_n(s)$
Here, Original equation can reduce (high power of s^n equation) equation (5) and (6) . the coefficient of highest power can always made unity.

$$Gn(s) = \cfrac{1}{\cfrac{a11}{b11} + \cfrac{s}{\cfrac{b1n}{b2n} + \cfrac{1}{\cfrac{a21}{b2n} + \cdots \cdots \cfrac{s}{\cfrac{b2,n-1}{a3,n-1} + \cfrac{1}{\ldots \ldots \ldots \ldots}}}}}$$

Where $$a21 = a_{12} - \frac{a_{11}b_{12}}{b_{11}}$$

$$b21 = b_{11} - \frac{b_{1n}a_{21}}{a_{2n}} \tag{7}$$

$$a_{2,n-1} = a_{1,n} - \frac{a_{11} b_{1n}}{b_{11}} \qquad b_{2,n-1} = b_{1,n-1} - \frac{b_{1n} a_{2,n-1}}{a_{2,n}}$$

$$a_{2,n} = 1$$

By continuing the above sequence of expansion we get the following form

$$Gn(s) = \cfrac{1}{\hslash 1 + \cfrac{s}{H1 + \cfrac{1}{\hslash 2 + \cfrac{s}{H2 + \cfrac{1}{\cdots\cdots}}}}} \qquad (8)$$

Where the quotients h1, h2, ………… H2, H1…….. are evaluated from the following modified array

a_{11}	a_{12}	$a_{1,n-1}$	$a_{1,n}$	1
b_{11}	b_{12}	$a_{1,n-1}$	$b_{1,n}$	
a_{21}	a_{22}	$a_{2,n-1}$	1	
b_{21}	b_{22}	$b_{2,n-1}$		
a_{31}	a_{32}	1		

(9)

$b_{n-1,1}$ $b_{n-1,2}$

$a_{n,1}$

$b_{n,1}$

1

$h_1 = \frac{a_{11}}{b_{11}}, \quad \hslash_2 = \frac{a_{21}}{b_{21}}, \ldots\ldots\ldots.\hslash_n = \frac{a_{n,1}}{b_{n,1}}$

$H_1 = b_{1,n}$

$H_2 = b_{2,n-1}$

$H_{n-1} = b_{n-1,2}$

$H_n = b_{n,1}$

(10)

Here the coefficients in equation (9) form the first two rows and remaining elements starting with third row are obtained from the recursive relations.

$$a_{j+1,k} = a_{j,k+1} - \hslash_j b_{j,k+1}$$

And

$$b_{j+1,k} = b_{j,k} - H_j a_{j+1,k} \quad j = 1,2,\ldots.n-1$$

$$where \quad \hslash_j = \frac{a_{j,1}}{b_{j,1}}$$

$$H_j = \frac{b_{j,n+1-j}}{a_{j+1,n+1-j}} \quad j = 1,2\ldots.n \qquad (11)$$

It is to be noted that the end elements of all the odd rows can be written directly as

$$a_{1,n+1} = a_{2,n} = a_{3,n-1} = \cdots\ldots\ldots. = a_{n+1,1} = 1 \qquad (12)$$

After reducing the order, is obtained from truncating equation (9) and after that transfer function find out h_j and H_j derived from equation (11) and constructing the inversion table as follows:

A_{11}				
B_{11}				
A_{21}	1			
B_{21}	B_{22}			
………………				
………………				
$A_{k,1}$	$A_{k,2}$	…..	$A_{k,k-1}$	1
B_{k-1}	$B_{k,2}$	…..	$B_{k,k-1}$	$B_{k,k}$
$A_{k+1,1}$	$A_{k+1,2}$	……	$A_{k+1,k}$	1

$$H_k = \frac{a_{11}}{b_{11}} \qquad \hslash_k = \frac{b_{11}}{a_{21}}$$

$$H_{k-1} = \frac{1}{b_{22}} \qquad \hslash_{k-1} = \frac{b_{21}}{a_{23}}$$

………………… …………

………………… …………

$$H_1 = \frac{1}{b_{k,k}} \qquad \hslash_1 = \frac{b_{k,1}}{a_{k+1,1}}$$

Here, it is a_{11}=1, and element evaluated second and third and subsequent rows are evaluated follows:

$$a_{j+1,1} = \hslash_{k+1-j} b_{j,1} \qquad j = 1,2\ldots..m$$

$$a_{j,m} = a_{j-1,k-1} + \hslash_{k+2-j} b_{j-1,m} \qquad m=2,3,\ldots j-1$$

& $b_{j,m} = b_{j-1,m} + H_{k+1-j}\, a_{j,m}$

j=2,3………and m=1, 2……………j-1

here, all end element can written by inspection according:

A_{jj}=1 j=1,2,3,……….k+1

$B_{jj}=H_{k+1-j}$ j=1,2,……………k

Here, must be complete rows(2k+1)

$$D_k(s) = a_k + a_{k+1,k}s^{k-1} + \ \dots\dots\dots .+a_{k+1,2}s + a_{k+1,1}$$

Step-2 here, determination of k[th] order form find in n[th] higher order. For find the nominator obtaining the factor division method. Also obtaining the denominator and numerator of the reduction model is derived as follows:

$$\widetilde{N(s)} = \frac{N(s)}{D(s)} \times D_k(s) = \frac{N(s)}{D(s)\big/D_k(s)}$$

Where $D_k(s)$ is the order reduction demoninator

For determining of numerator of reduce model:

(i) By peing the product of N(s) and Dk(s) as the first row of factor division algorithm and Dk(s) as the second row up to sk-1 terms are needed in both rows.

(ii) By expressing N(s)Dk(s)/D(s) as N(s)/[D(s)/Dk(s)] and using factor division algorithm twice; the first time to find the term up to sk-1 inthe expansion of D(s)/Dk(s)(i.e. put D(s) in the first row and Dk(s) in the second row, using only terms up to sk-1), and second time with N(s) in the first row and the expansion [D(s)/Dk(s)] in the second row.

Therefore the numerator Nk(s) of the reduced order model (Rk(s)) will be the series expansion of

$$\frac{N(s)}{D_8(s)\big/D_k(s)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k-1} d_i s^i}$$

About s=0 up to term of order sk-1

.

$$g_i = d_{i+1} - \alpha_0 * f_{i+1} \ \ and \ \ l_i = g_{i+1} \ \ i = 0,1,2, \dots.$$

$$q_0 = p1 - \alpha_{k-2} * f_1$$

$$\propto_0 = \frac{d_0}{f_0} \ < \ \begin{matrix} d_0 & d_1 & d_2 & \dots\dots\dots\dots\dots d_{k-1} \\ f_0 & f_1 & f_2 & \dots\dots\dots\dots\dots .f_{k-1} \end{matrix}$$

$$\propto_1 = \frac{g_0}{f_0} \ < \ \begin{matrix} g_0 & g_1 & g_2 & \dots\dots\dots\dots\dots g_{k-2} \\ f_0 & f_1 & f_2 & \dots\dots\dots\dots\dots .f_{k-2} \end{matrix}$$

$$\propto_2 = \frac{l_0}{f_0} \ < \ \begin{matrix} l_0 & l_1 & l_2 & \dots\dots\dots\dots\dots l_{k-3} \\ f_0 & f_1 & f_2 & \dots\dots\dots\dots\dots .f_{k-3} \end{matrix}$$

……………………………………..

Where

$$\propto_{k-2} = \frac{p_0}{f_0} \ < \ \begin{matrix} p_0 & p_1 \\ f_0 & f_1 \end{matrix}$$

$$\propto_{k-1} = \frac{q_0}{f_0} \ < \ \begin{matrix} q_0 \\ f_0 \end{matrix}$$

C. Genetic Algorithm Method

Let present higher level reduction method system of the transfer function matrix of order 'n' having 'p' inputs and 'm' outputs be:

$$[G(s)] = \frac{1}{D(s)}\begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) & \cdots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) & \cdots & a_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1}(s) & a_{m2}(s) & a_{m3}(s) & \cdots & a_{mp}(s) \end{bmatrix}$$

$[G(s)]=[g_{ij}(s)]$, i=1,2,……..m; j=1,2,………p

M x p is a Transfer Matrix.

$$g_{ij}(s) = \frac{a_{ij}(s)}{D(s)}$$

$$= \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1}s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_{n-1}s^{n-1} + s^n}$$

$$g_{ij}(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1}s^{n-1}}{(s+\lambda_1)(s+\lambda_2)\dots(s+\lambda_n)}$$

where, $-\lambda 1 < -\lambda 2 < \dots\dots < -\lambda r$ are poles of the higher order system.

Let, the transfer function matrix of low order system of order 'r'and input 'p'and output 'm'

$$[R(s)] = \frac{1}{\tilde{D}(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) & \cdots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) & \cdots & b_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ b_{m1}(s) & b_{m2}(s) & b_{m3}(s) & \cdots & b_{mp}(s) \end{bmatrix}$$

M x p is a transfer matrix

In general form of r_{ij} of R(s)

$$r_{ij}(s) = \frac{b_{ij}}{\widetilde{D(s)}}$$

$$= \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{r-1} s^{r-1}}{d_0 + d_1 s + d_2 s^2 + \cdots + d_{r-1} s^{r-1} + s^r}$$

$$r_{ij}(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1}}{(s+\lambda_1)(s+\lambda_2)\ldots(s+\lambda_r)}$$

Numerical Examples

Example 1: The transfer function of the fourth order equation:

$$G_4(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^2}$$

For getting a 2nd order model find from first denominator

D2(s)= s2+1.786s+1.149

Then 2nd step factor division algorithm

N(s)D2(s)=(2400+1800s+496s^2+28s^3)*(1.149+1.786s+ s^2)

N(s)D$_2$(s)=2757.6+6354.6s+6184.7s^2+2718s^3 +546.01s^4+28s^5

Also

+2s^4 D(s)=240+360s+204s^2+36s^3

N(s)=9.241 s+11.49

Therefore, finally second order model

$$G(s) = \frac{9.241\,s + 11.49}{s\hat{}2 + 1.786\,s + 1.149}$$

Example 2: the 8th order transfer function G(s)

$$G_8(s) = \frac{18s^7 + 514s^6 + 5982\,s^5 + 36380s^4 + 122664s\hat{}3 + 222088s\hat{}2 + 185760s + 40320}{s^8 + 36\,s^7 + 546\,s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124\,s^2 + 1.096e06\,s + 40320}$$

The above equation is higher order transfer function and following low order transfer function derived

This is the 2nd order of the nominator of N(s)=1.606s+0.3486

And G(s)= $s\hat{}2 + 9.476\,s + 0.3486$

And transfer function

$$G_2(s) = \frac{1.606\,s + 0.3486}{s\hat{}2 + 9.476\,s + 0.3486}$$

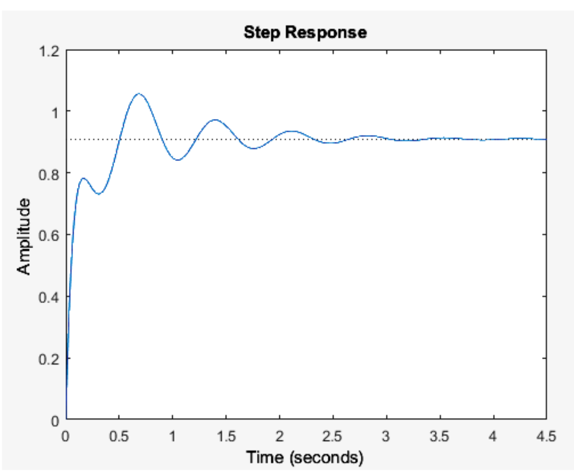

Fig 1: step response of the higher order reduction

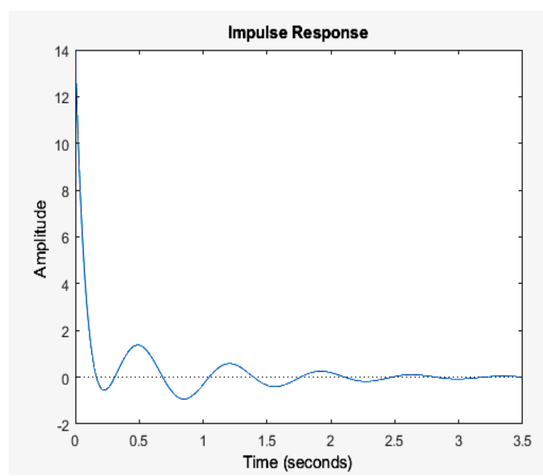

Fig 4: Impulse response of the higher order reduction

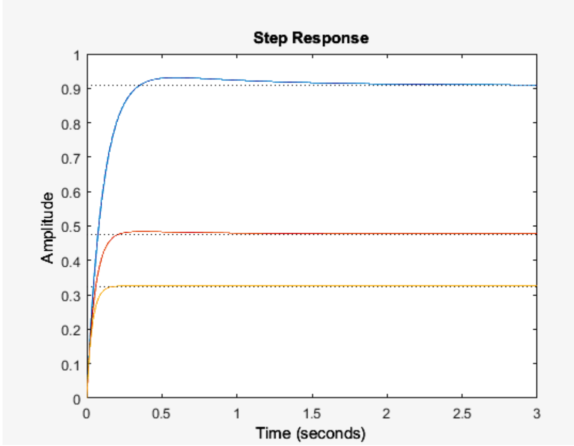

Fig 2: step response of the low order reduction of the routh approximation, factor division method and Genetic algorithm.

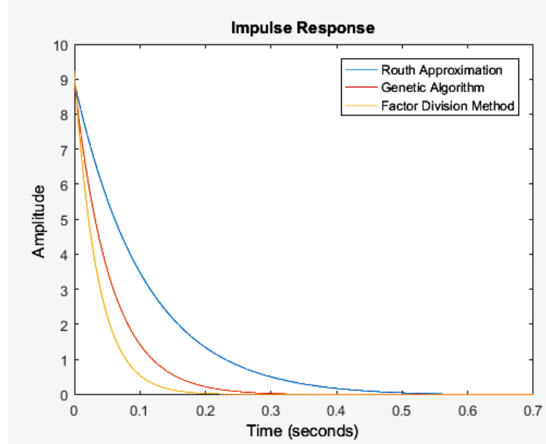

Fig 5: step response of the low order reduction of the routh approximation, factor division method and Genetic algorithm.

	Higher Order	**Routh Approximation**	**Genetic Algorithm**	**Factor Division Method**
Rise Time	0.4289	0.2026	0.1121	0.0757
Settling Time	2.2147	0.7496	0.1812	0.1293
Settling Min	0.8220	0.8252	0.4314	0.2926
Settling Max	1.0556	0.9310	0.4837	0.3246
Over shoot	16.1775	2.2308	1.1417	0
Under shoot	0	0	0	0
Peak	1.0556	0.9310	0.4837	0.3246
Peak Time	0.6823	0.5951	0.3662	0.1858

	$G_4(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^4}$	$G_8(s) = \frac{18s^7 + 514s^6 + 5982\,s^5 + 36380s^4 + 122664s\text{^}3 + 222088s\text{^}2 + 185760s + 403}{s^8 + 36\,s^7 + 546\,s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124\,s^2 + 1.096e06\,s\ +\ 403}$	**ISE**
Routh Appximation	$\frac{8.911s + 11.88}{s^2 + 1.782s + 1.188}$	$\frac{1.606s + 0.3486}{s^2 + 9.476s + 0.03486}$	0.0410
Factor Division	$\frac{9.241s + 11.49}{s^2 + 1.786s + 1.149}$	$\frac{-0.1174s - 0.03424}{s^2 + 0.8903s + 0.03424}$	0.0018
Genetic Algorithm	$\frac{8.911s + 11.88}{s^2 + 1.782s + 1.188}$	$\frac{1.606s + 0.3486}{s^2 + 9.476s + 0.03486}$	0.0060

III.CONCLUSIONS

The author proposed an order reduction method for linear SISO of the HOS. Determine of low order transfer function applying three different method Routh Appximation, Factor division method, Genetic algorithm. These are simple algorithm can apply on any stable equation. Here, two example are shown in the paper. Those are stable system and also implemented on a figure. in this figure, the author proposed step response of the transfer function and bode response of the transfer function. Here, All three method are solving and representing in transfer function.

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