Tension Sensorless Control For Web Transport Systems

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Abstract *— A tension regulation control problem is presented in the paper. Different from other studies, the paper proposes a simple tension observer that can effectively replace tension measurement devices. Derived tension information is used for designing a control based on backstepping control design that is integrated with sliding mode control for better robustness to system uncertainties and disturbances. The system performances with the designed control show good tension tracking ability.*

Keywords *- Roll-to-roll system, tension sensorless control, backstepping-sliding mode control, Lyapunov's stability*

I. INTRODUCTION

This Tension control is essential in web-handling systems. Low tension lead to web winkle and non-compact rewinding web roll. Meanwhile, over-tension cause web breakage resulting in serious production efficiency and financial problems. With an effort to eliminate the requirement of tension measurement, Lin et al [1] propose a tension sensorless control algorithm incorporated with friction and inertia compensation, good system performance achieved compared to classical system with tension feedback. Another type of tension observer based on balance and acceleration torque is presented in [2]. In [3], backlash phenomenon of a roll-to-roll machine is analyzed, and the authors suggest an additional mechanical mechanism to reduce backlash effects. Classical PID control is widely used in tension regulation, however the approach presents several drawbacks in dealing with nonlinear system with varying parameters such as the web-handling system [4]. A fuzzy logic control with self-tuning function is introduced in [5], however, fuzzy logic heavily depends on designer's knowledge the system and is very difficult to generalized to other systems. Due to the ability of dealing with system disturbance, active disturbance rejection control is adopted in [6] and [7] for unwinding systems. Sliding mode control is also employed to solve tension control problem thank to its robust property in [8] and [9].

The paper introduces a new control approach to the web-handling system where backstepping control [10]-[12] is supported by a simple tension observer and sliding mode control. The control removes tedious calculation in pervious papers when deriving tension information. In addition, the existence of sliding mode control gives robust property to the controlled system. Comprehensive control design procedure and numerical simulations are given to support the prosed control.

II. MODEL OF WEB TRANSPORT SYSTEM

The system is considered in Figure.1 that contains unwinder, rewinder, a loadcell subsystem with idle roller and two dancer subsystems. The idle rollers guide the moving of web around the load cell in fix angel. The torque generated from unwinder and rewinder is the force for operating system and it is alsothe control signal to drive the system. The torque will be adjusted in order to keep web's speed and tension at desired values during operating.

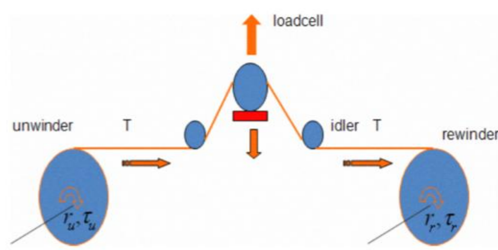

Fig.1: Roll-to-roll system

A. Dynamic model of unwinder and rewinder

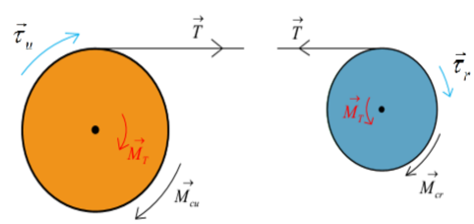

Fig.2: Unwinder and rewinder with torque and force

The dynamic equation on unwinder are:

$$\vec{\tau}_u + \vec{M}_{Tu} + \vec{M}_{cu} = J_u \vec{\dot{\omega}}_u \qquad (1)$$

Projecting on the axis, we have:

$$\tau_u + R_u T - B_u \omega_u = J_u \dot{\omega}_u \qquad (2)$$

Then we extract the formular of derivative of unwinder angular velocity as:

$$\dot{\omega}_u = c_1\omega_u + c_2 T + c_3\tau_u \qquad (3)$$

Where: $c_1 = \dfrac{B_u}{J_u}$, $c_2 = \dfrac{R_u}{J_u}$, $c_3 = -\dfrac{1}{J_u}$.

The same with the unwinder, base on dynamic equation, we have formular that show dynamic property of rewinder as following:

$$\dot{\omega}_r = c_7 T + c_8\omega_r + c_9\tau_r \qquad (4)$$

Where: $c_7 = \dfrac{R_r}{J_r}$, $c_8 = -\dfrac{B_r}{J_r}$, and $c_9 = \dfrac{1}{J_r}$.

B. *Relationship between web tension and angular velocites*

Assume that not considering the influence of loadcell subsystem, assuming that web's slippage and deformation do not occur, and the web property complies strictly Hook's law. The formular that represents for tension property of the web transport system will be found through that theory.
Firstly, considering in initial state of web transport system when the web does not be affected by the tenson as shown as following figure:

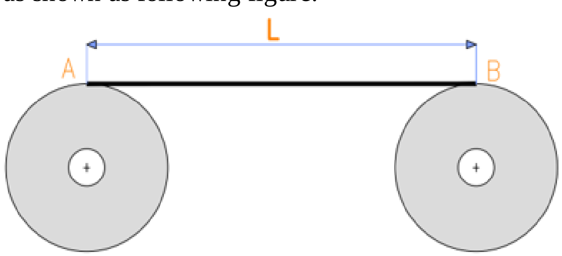

Fig.3: Initial state of the web without tension impact

According to the Hook's law, tension on the web can be computed as:

$$T = \frac{E.S}{l_o}(l - l_o) = \frac{E.S}{l_o}(L - l_o) \qquad (5)$$

Where l_o is the initial length of web, this parameter is the same with L that is the distance between two center of the rolls. l is the length of web after deformation.

After initial state, at the time $t + dt$ after that, web is stretched on unwinder side because of unwinder's rotation. Meanwhile, on the rewinder side, this roll is considered motionless.

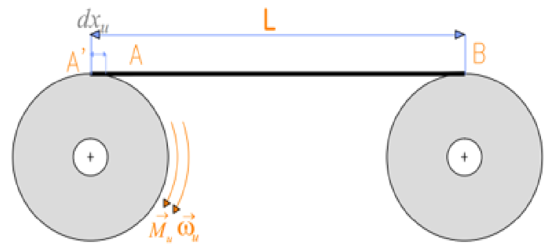

Fig.4: The web transport system with deformation on unwider web side

At this moment, according to Hook's law, the tension is shown as:

$$T(t + dt) = \frac{E.S}{l_o}(L - l_o - dx) = \frac{E.S}{l_o}(L - l_o - R_u d\theta_u) \quad (6)$$

With condition that the dt is very small time, so the change of unwinder's radius equals zero.

Then, let consider the next state when the rewinder rotate $d\theta_r$, now considering the unwinder is motionless as the following figure:

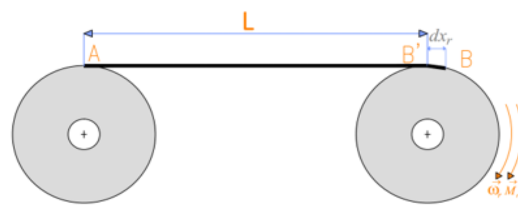

Fig.5: The web transport system with deformation on rewider web side.

At this time, with condition that radius of rewinder does not change $dR_r = 0$, we obtain the tension of web through Hook's law as:

$$\begin{aligned} T(t + dt) &= \frac{E.S}{l_o}(L + dx - l_o) \\ &= \frac{E.S}{l_o}(L + R_r d\theta_r - l_o) \end{aligned} \qquad (7)$$

Combining two aforementioned state that the unwinder rotate $d\theta_u$ and the web strentch dx_u on unwinder side, the rewinder rotate $d\theta_r$ and the web strentch dx_r on rewinder side as the following:

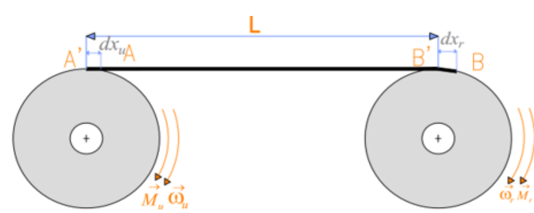

Fig.6: The general state of web transport system

From equation (6) and (7), according to Hook's law, we extract the tension at the time $t+dt$ as :

$$T(t+dt)=E.S\frac{(L+R_r d\theta_r)-(l_o+R_u d\theta_u)}{l_o+R_u d\theta_u} \tag{8}$$

Using (5) and (8), we have:

$$\begin{aligned} dT &= T(t+dt)-T(t) \\ &= E.S\frac{L+R_r d\theta_r-l_o-R_u d\theta_u}{l_o+R_u d\theta_u}-E.S\frac{L-l_o}{l_o} \\ &= ES\frac{l_o R_r d\theta_r-LR_u d\theta_u}{(l_o+R_u d\theta_u)l_o} \end{aligned} \tag{9}$$

Due to the negligible value of $d\theta_u$, so $R_u d\theta_u << l_0$, then we can rewritten the equation (9) as:

$$dT=ES\frac{R_r l_o d\theta_r-LR_u d\theta_u}{l_o^{\,2}} \tag{10}$$

From equation (5), we obtain:

$$T=E.S\frac{L-l_o}{l_o}\rightarrow L=\frac{T+E.S}{E.S}l_o \tag{11}$$

Replacing in (10), we have:

$$LdT=(T+E.S)R_r d\theta_r-\frac{(T+E.S)^2}{E.S}R_u d\theta_u \tag{12}$$

Due to $T << E.S$, then equation (12) can rewritten approximately as:

$$\dot{T}=\frac{(T+E.S)}{L}R_r\omega_r-\frac{(2T+E.S)}{L}R_u\omega_u \tag{13}$$

Defining $K=\frac{(2T+E.S)}{L}$, $c_4=-KR_u$, $c_5=-\frac{R_r}{L}$, $c_6=KR_r$. The ralationship between web tension and the angular velocities is expressed as:

$$\dot{T}=c_4\omega_u+c\omega_5 T\omega_r+c_{6r}. \tag{14}$$

C. The mathematical model of web transport system

From (3), (4) and (14), we have the fully mathematical model of the the system is shown as:

$$\begin{cases} \dot{\omega}_u=c_1\omega_u+c_2T+c_3\tau_u, \\ \dot{T}=c_4\omega_u+c\omega_5T\omega_r+c_{6r}, \\ \dot{\omega}_r=c_7T+c_8\omega_r+c_9\tau_r. \end{cases} \tag{15}$$

Where: $c_1=\frac{B_u}{J_u}$, $c_2=\frac{R_u}{J_u}$, $c_3=-\frac{1}{J_u}$,

$c_4=-KR_u$, $c_5=-\frac{R_r}{L}$, $c_6=KR_r$, $c_7=\frac{R_r}{J_r}$,

$c_8=-\frac{B_r}{J_r}$, and $c_9=\frac{1}{J_r}$.

The relevance among total moment of inertia, operating radius and the thickness of the rollsare shown in following equations:

$$R_u(t)=R_{u0}-\frac{\theta_u h}{2\pi} \tag{16}$$

$$R_r(t)=R_{r0}+\frac{\theta_r h}{2\pi} \tag{17}$$

$$J_u=J_{u0}+\pi\rho\omega\frac{\left(R_u^4-R_{u0}^4\right)}{2} \tag{18}$$

$$J_r=J_{r0}+\pi\rho\omega\frac{\left(R_r^4-R_{r0}^4\right)}{2} \tag{19}$$

Where T is web tension; J_u ismoment of inertia of the unwind roll and motor; J_r is moment of inertia ofthe rewind roll and motor; R_u is radius of the unwind roll; R_r is radius of the rewind roll; B_u iscoefficient of vicious friction of the unwind roll; B_r is coefficient of vicious friction of the unwind roll; K isspring constant of web; L is total length of web; h is the thickness of web; ρ isthe density of web; ω is the width of web

III. BACKSTEPPING-SLIDING MODE CONTROL DESIGN WITH TENSION OBSERVER

The controller aims to keep web tension and web speed at references. Basing on backstepping control design and integrating with sliding mode control, the controller is designed in order to achieve better system performance under system uncertainties and external disturbances. Defining tracking error variables below:

$$\begin{cases} \bar{T}=T-T_d, \\ \bar{\omega}_u=\omega_u-\alpha, \\ \bar{\omega}_r=\omega_r-\omega_{rd}. \end{cases} \tag{20}$$

Where T_d, ω_{rd} are desired web tension and desired rewind angular velocity of web, respectively. The ideal is using virtual control signal α generated through back-stepping technique in order to $T\rightarrow T_d$. Then, calculating control signal τ_u,τ_r by sliding mode method such that $\bar{\omega}_u$ and $\bar{\omega}_r$ are asymptotically stable.Proposing the Lyapunov candidate function as:

$$V_1=\frac{1}{2}\bar{T}^2 \tag{21}$$

The Lyapunov candidate funtion above is choosen so that it is positive definite and contains the ingredients that we need to consider, in this situation, that component is error between reference of tension and it's response.

Taking time derivative of (21) gives: $\dot{V}_1 = \bar{T}\dot{\bar{T}} = \bar{T}\left(\dot{T} - \dot{T}_d\right)$ (22)

According to Lyapunov stability theory, if derivative of Lyapunov candidate function is negative definite, error of tension will drive to zero. That means the response of tension will approach the desired value. Now considering the derivative of Lyapunov candidate function, from (15) and(22), rewriting $\dot{V}_1$ as:

$$\dot{V}_1 = \bar{T}\left(c_4\left(\bar{\omega}_u + \alpha\right) + c_5 T\omega_r + c_6\omega_r - \dot{T}_d\right) \quad (23)$$

The paremeter α is virtual control signal that we can interfere with, so we choose α as:

$$\alpha = \frac{1}{c_4}\left(-c_5 T\omega_r - c_6\omega_r + \dot{T}_d - k_1\bar{T}\right)$$

Where k_1 is a positive gain.

Assuming that $\bar{\omega}_u$ will be driven to zero, virtual control signal chosen above will drive the error of tension to zero. Replacing that virtual control signal in equation (23), we obtain:

$$\dot{V}_1 = -k_1\bar{T}^2 \le 0 \quad (24)$$

With derivative of Lyapunov funtion is negative definite as above demonstrate, signal $\bar{T}$ will aprroach to zero as Lyapunov stability theory.

In order to guarantee that the virtual control signal drive the response tension to reference, we need a condition that error between angular velocity equals zero. Now, τ_u is chosen to drive $\bar{\omega}_u$ to zero. Sliding mode control method will be apply to achieve that condition, this method will give hight robost property for the system.

Choosing the sliding surface as: $S_u = \omega_u - \alpha$ (25)

Taking time derivative above equation and combining with (1), we obtain:

$$\dot{S}_u = c_1\omega_u + c_2 T + c_3\tau_u - \dot{\alpha} \quad (26)$$

The control signal τ_u includes two components: τ_{ueq} will drive the sliding surface to zero and τ_{usw} will lead the system state on the sliding surface. Thus, the control signal can be rewritten as:

$$\tau_u = \tau_{ueq} + \tau_{usw} \quad (27)$$

From (26), it is straightforward to get τ_{ueq} as:

$$\tau_{ueq} = \frac{1}{c_3}\left(-c_1\omega_u - c_2 T + \dot{\alpha}\right) \quad (28)$$

In order to make $S_u \to 0$, we need signal τ_{usw} so that $S_u\dot{S}_u < 0.$ So we choose τ_{usw} in traditional Sliding Surface Control approach as:

$$\tau_{usw} = \frac{1}{c_3}\left(-k_{u1}\text{sign}\left(S_u\right) - k_{u2}.S_u\right) \quad (29)$$

With the control signal designed, the stability of the system will be considered according to Lyapunov stable standards. The second Lyapunov candidate function is proposed for proving the accuracy of the calculated control law:

$$V_u = \frac{1}{2}S_u^{\ 2} \quad (30)$$

Take the derivative of the second candidate function and using (26), (27), (28) and (29) we obtain:

$$\dot{V}_u = -k_{u1}S_u\text{sign}\left(S_u\right) - k_{u2}S_u^{\ 2} \le 0 \quad (31)$$

The next work is designing control signal τ_r so that $\bar{\omega}_r \to 0.$ Defining sliding surface for rewind angular speed as:

$$S_r = \omega_r - \omega_{rd} \quad (32)$$

Combine(15)and(32), we have:

$$\dot{S}_r = c_7 T + c_8\omega_r + c_9\tau_r - \dot{\omega}_{rd} \quad (33)$$

The same as designing control signal τ_u for unwind angular speed above, control signal τ_r is designed base on two components: τ_{req} and τ_{rsw}. From (33), with k_{r1}, k_{r2} are positive gains, we choose these elements as:

$$\tau_{req} = \frac{1}{c_9}\left(-c_8\omega_r - c_7 T + \dot{\omega}_{rd}\right) \quad (34)$$

$$\tau_{rsw} = \frac{1}{c_9}\left(-k_{r1}\text{sign}\left(S_r\right) - k_{r2}.S_r\right) \quad (35)$$

Control signals chosen above ensure rejecting bounded disturbance but the signum function will generatechattering chattering that will reduce effect of controller. Thus, we propose replace signum element by a saturation linear function defined as:

$$y = \text{sat}\left(x\right) \Leftrightarrow \begin{cases} y = -1 & \text{if } x \le -1 \\ y = x & \text{if } -1 < x < 1 \\ y = 1 & \text{if } x \ge -1 \end{cases} \quad (36)$$

The control signals are written as:

$$\tau_u = \frac{1}{c_3}\left(-c_1\omega_u - c_2 T + \dot{\alpha} - k_{u1}.\text{sat}\left(S_u\right) - k_{u2}.S_u\right) \quad (37)$$

$$\tau_r = \frac{1}{c_9}\left(-c_8\omega_r - c_7 T + \dot{\omega}_{rd} - k_{r1}.\text{sat}\left(S_r\right) - k_{r2}.S_r\right) \quad (38)$$

Basing on properties of the system shown through mathematical model, the superlative derivative of tension can be calculated by other system parameters as:

$$\dot{T} = c_4\omega_u + c_5 T\omega_r + c_6\omega_r \quad (39)$$

Assume that the initial tension $T(0) = 1\text{N}$, we have the fomular to compute web tension as:

$$T(k) = \tau.\dot{T}(k) + T(k-1)$$
$$= c_4\omega_u(\text{k}) + c_5 T\omega_r(k) + c_6\omega_r(k) + T(k-1) \quad (40)$$

The system performance with computed tension will be illustrated in next section.

IV. SIMULATION RESULTS

In this section, the performance of the proposed controller will be evaluated through a numerical simulations. In order to test the quality of the designed controller with a simple tension observer, the reference angular velocity of rewinding is changed in two cases and the simulation results of the presented controller is compared to the reference. The parameters of the Roll to Roll system are chosen as: $\text{R}_{u0} = 0.04\text{m}$; $\text{R}_{r0} = 0.015\text{m}$; $\text{J}_{u0} = \text{J}_{r0} = 1\text{kg/m/s}$; $\text{L} = 0.3\text{m}$; $\text{K} = 200\text{kg/m}$; $\text{B}_u = \text{B}_r = 2.533\times10^{-5}\text{kgms/rad}$; $\text{h} = 2\times10^{-5}\text{m}$; $\text{w} = 1\text{m}$. The parameters used in simulation for the proposed controller are selected as: $\text{k}_{u1} = \text{k}_{u2} = 10$; $\text{k}_{u3} = 5$.

Firstly, the system responses for design is examined with the difference cases are the reference that is kept at the desired speed and the sudden change of the reference angular velocity of rewinding from 50rad/s to 70rad/s respectively,the desired web tension is chosen as 5N. The results of the proposed controller are shown in these following figures

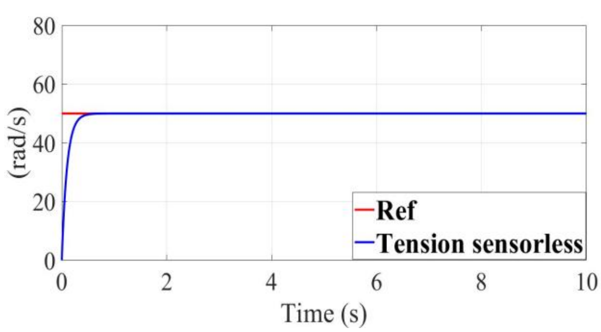

Fig.7:The angular velocity responsewith constant reference

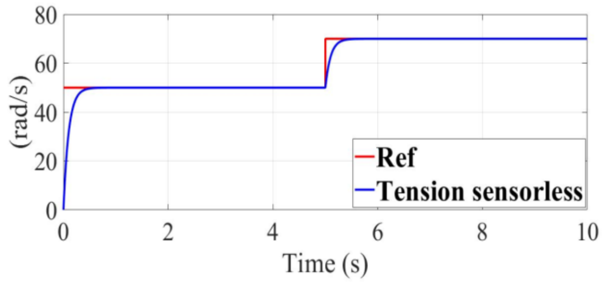

Fig.8: The angular velocity response with change reference

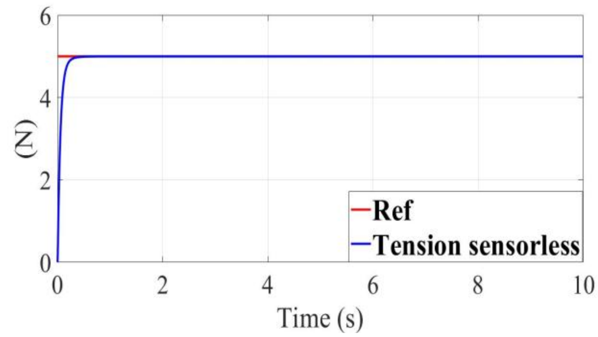

Fig.9: The tension response with constant reference

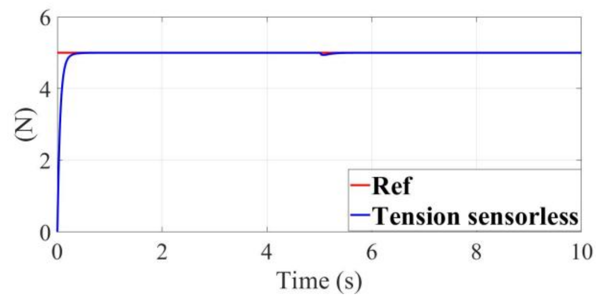

Fig.10: The tension response with change reference

As the simulation results, the responses of the rewinder angular velocity and the tension are shown in figures 7-10 respectively. It can be seen that with the reference speed and the tension are kept at the desired values, the angular velocity response and the tension response can track the reference and converge to the desired value in a short time at approximately 0.4s. Meanwhile, there is unremarkable different in the performance of the controller once the reference angular velocity is suddenly changed from the initial value 50rad/s to the final value 70rad/s, the Tension sensorless controller ensures the good quality of the roll to roll system, that presented method shows the good effectiveness of diminishing the vibration at steady state as well as reducing the settling time of system.

In order to adequately evaluate the tracking quality of the proposed control, figure 11 and 12show the simulation results of the tension response compared to the tension value calculated through the mathematical model

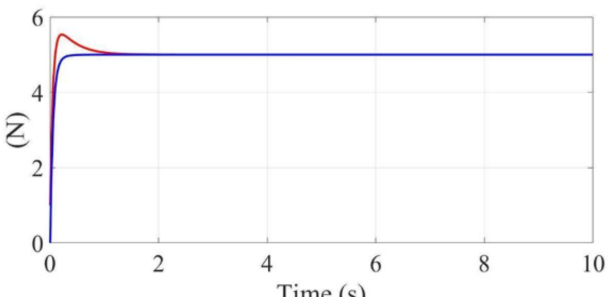

Fig.11: The estimated tension with constant reference

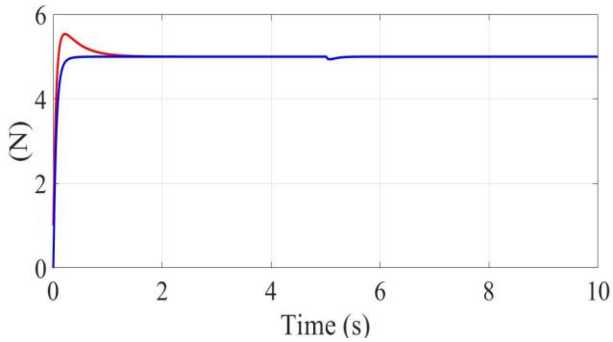

Fig.12: The estimated tension with change reference

V. CONCLUSION

Notice that the relationship between tension and other measured system information, the paper design the integral-type tension observer to accurately calculate the web tension. The backstepping-sliding mode control is employed for the tension control. Numerically, the control with tension sensorless is comparable with system perfomance in the presence of tension sensors.

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