Kinematics and Control A Three Wheeled Omnidirectional Mobile Robot

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Abstract *- A three-wheeled omnidirectional mobile robot is a mobile robot can solve constraint of common design a mobile robot which uses two-wheel drive with differential steering and a free balancing wheel (Non-holonomic). By found kinematic model of a three-wheeled omnidirectional mobile robot, we describe inverse kinematics and inverse jacobian matrix to control and the simulated motion of a mobile robot for optimizing mobility and maneuvers ability of a mobile robot . It has a past advantage over a conventional design (non-holonomic) in term of mobility and maneuvering in a congested environment. In this paper, we are presentinganalysis and simulated kinematic model a three-wheeled omnidirectional mobile robot, velocities of coordinates, and trajectory from initial pose to the desired pose. The simulation of the trajectory robot is used to describe the ability of a system to move instantaneously in any direction from any configuration. Modeling mobile robot with three wheels Omni-directional describe optimizing motion capability in differential geometric point of view. This research is will be developing for visual servoing law method research in robotic field.*

Keywords — *Omnidirectional, Mobile robot, Kinematic, Mobility, Constraint*.

I. INTRODUCTION

Autonomous mobile robot has many applied in industry and various environments [1].In the education fields, most of student interestinthe implementation of autonomous mobile robotics [11].The problem of autonomous wheeled mobile robot has been presented a lot of research in the field of robotics. Various methods are using for mobile robots navigation such as obstacle avoidance [2] [3], dead reckoning method [4] [5], using odometry [6] has been presented. All methods using formake efficient, fast, and obstacle avoidance but the constrain of mobility with the conventional(differential drive)design of mobile a robot [12] can cause the performance of a robot is not maximal. In mobile robotics, we need to understand the mechanical the behavior of the robot both to design appropriate mobile robots for tasks and to understand how to create control for instant mobile robot hardware [7]. The process of understanding the motion of a robot begins with the process of describing the contribution each wheel provides for motion. Each wheel has a role in enabling the whole robot to move. Demonstrate the construction of simple forward kinematic models of motion, describing how the robot as whole moves as a function of its geometry and individual wheel behavior [8].

In the modern automated industry, the mobile robot is more flexible and can perform more tasks effectively. An omnidirectional wheeled mobile robot is a special class of mobile robots is nows a day. This robot can drive translational and angular movement [9]. Kinematic is the most basic study of how mechanical systems behave. By combine of a kinematic model and inverse jacobian matrix of a mobile robot, it's can use for effective motion control of holonomic three wheels omnidirectional mobile robot [10]. The papers consist of describing a simulation of three wheels omnidirectional mobile robot can be controlled by using the model kinematic and inverse jacobian matrix of a mobile robot.By use model kinematic of the robot has an advantage in our approach here outlines:

- Optimizing control of mobility and maneuvers of mobile robot
- Mobility constraint in conventional design (nonholonomic) of the wheeled mobile robot can be solved
- By found model kinematic of three wheels omnidirectional MR, It's capable to control each wheel independently

The paper is organized as follows. Section II introduces an omnidirectional mobile robot and it's designed. In section III, Kinematics of three wheels omnidirectional mobile robot. The simulation and measurement of velocity mobile robot is presented insection IV. in section V, we provide some concluding remarks highlighting the main contribution of the paper.

II. OMNIDIRECTIONAL ROBOT

The robotic vehicle is often designed for planar motion. Most of the non-holonomic or conventional vehicle is not capable of controlling these three degrees of freedom independently, because of so-called non-holonomic constraints. it may require complicated maneuvers and complex path planning to do so, this is the case both human or robot-controlled vehicles. By contrast, a vehicle that is not hampered by these constraints is capable of Omni-directional mobility. It can travel any direction under any orientation. In many cases where mobile robots are put into action, especially in confined or congested space, omnidirectional mobility is very high advantageous. Omnidirectional provide excellent mobility with a static or dynamic obstacle in congested areas. The 3D mechanics design of the three wheels omnidirectional mobile robot is shown in fig1.

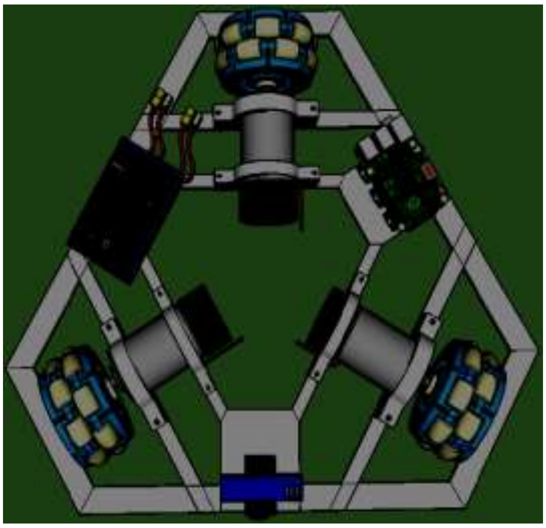

Fig.1. Mechanics design of robot - Top view

III. KINEMATICS MODEL OF ROBOT

Omni-directional mobile robots have developed and investigated by several kinds of research. Holonomic wheeled platforms that feature full omnidirectionally with simultaneous and independent controlled and translational capabilities. The angular position and velocities of the wheels shafts needed to know as a variable control. Here we describe the kinematic model of the robot. The base of the mechanic's design of a mobile robot has three wheels omnidirectional. The local frame *(*x*l*, *yl*), and global frame *(xg ,yg)* for reference to a robot is given by the global coordinate (x,y,θ). The global velocity of the robot can be written φ1, φ2, and φ3. The center of the local frame coincides with the center of gravity of the robot. The three Omni wheels are located at an angle αi, where (i = 1,2,3) relative to the local frame. If we take the local axis *(*x*l ,*y*l*) as a reference point of axis robot.

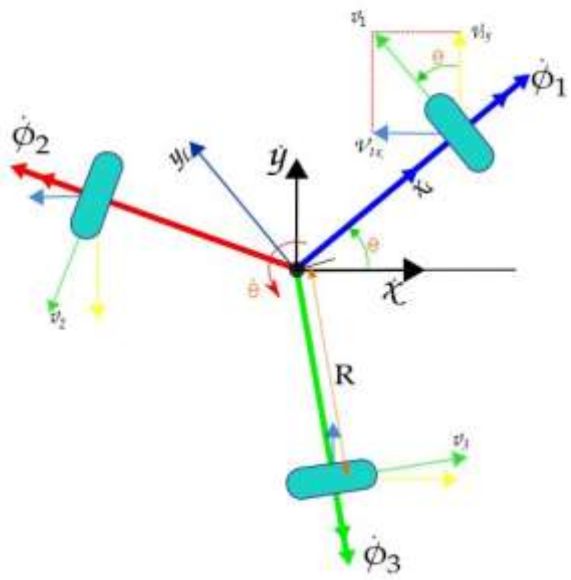

Fig.2. Kinematic of three wheels Omni-directional

and we start count clockwise, we get. The translational velocity of the wheel hub is consist of translation of the robot and rotation of the robot :

$$v_i = v_{trans} + v_{rot} \quad (1)$$

where $v_1 = v_{trans\,.1} = y_l$

$$v_{trans} = -\sin\theta.\dot{x} + \cos\theta\,.\dot{y} \quad (2)$$

we can generalized vector mapping for all wheels when we take consideration is positioned at an angle (θ+α_i), from this we can write:

$$V_{trans} = -\sin(\theta+\alpha_i).\dot{x} + \cos(\theta+\alpha_i)\,.\dot{y} \quad (3)$$

R represents the distance from the center of the body robot to the center of the wheels, along a radial path. when the rotation platform executes, the speed rotation should be :

$$V_{rot} = R.\theta \quad (4)$$

when equation (3) and equation (4) is substituted to equation 1, we get:

$$v_i = -\sin(\theta+\alpha_i).\dot{x} + \cos(\theta+\alpha_i).\dot{y} + R.\theta (5)$$

The translation velocity is related to the angular velocity θ_i of the wheels through:

$$v_i = r.\theta_i \quad (6)$$

Where r is the radius of an omnidirectional wheel,

$$-\sin(\theta+\alpha_i).\dot{x} + \cos(\theta+\alpha_i).\dot{y} + R.\theta = r.\theta_i (7)$$

$$\theta_i = \frac{1}{r}.((-\sin(\theta+\alpha_i).\dot{x}) + (\cos(\theta+\alpha_i).\dot{y}) + (R.\theta)) \quad (8)$$

Using the inverse jacobian matrix, the wheels shaft rotational velocity can be derived when both wheel radius and platform dimensions (r, R) are known. We can transform (9) to matrix representation (10) and (11)

$$\dot{\theta} = J_{inv}.\dot{u} \tag{9}$$

In this relation, is the relationship between the angular velocity of the wheels and global velocity $\dot{u}$.

$$\begin{bmatrix}\dot{\boldsymbol{\theta}}_1\\ \dot{\boldsymbol{\theta}}_2\\ \dot{\boldsymbol{\theta}}_3\end{bmatrix} = \frac{\mathbf{1}}{\boldsymbol{r}}.\begin{bmatrix}-\boldsymbol{sin(\theta)} & \boldsymbol{cos(\theta)} & \boldsymbol{R}\\ -\boldsymbol{sin(\theta+\alpha_2)} & \boldsymbol{cos(\theta+\alpha_2)} & \boldsymbol{R}\\ -\boldsymbol{sin(\theta+\alpha_3)} & \boldsymbol{cos(\theta+\alpha_3)} & \boldsymbol{R}\end{bmatrix}.\begin{bmatrix}\dot{\boldsymbol{x}}\\ \dot{\boldsymbol{y}}\\ \dot{\boldsymbol{\theta}}\end{bmatrix} \tag{10}$$

To steer robot in local coordinates, we can convert global coordinate to local coordinates with the following equation :

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos(\theta) & 0 & 0\\ 0 & \cos(\theta) & 0\\ 0 & 0 & \cos(\theta)\end{bmatrix}.\begin{bmatrix}\dot{x}_l\\ \dot{y}_l\\ \dot{\theta}\end{bmatrix} \tag{11}$$

after we substituting equation (10) and (11), we get the following equation :

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = \frac{1}{r}.\begin{bmatrix}-sin(\theta) & cos(\theta) & R\\ -sin(\theta+\alpha_2) & cos(\theta+\alpha_2) & R\\ -sin(\theta+\alpha_3) & cos(\theta+\alpha_3) & R\end{bmatrix}. \begin{bmatrix}\cos(\theta) & 0 & 0\\ 0 & \cos(\theta) & 0\\ 0 & 0 & \cos(\theta)\end{bmatrix}.\begin{bmatrix}\dot{x}_l\\ \dot{y}_l\\ \dot{\theta}\end{bmatrix} \tag{12}$$

For easy implementation in programming application, we can write this matrix relation to three separate equation for $\dot{\theta}_1$,$\dot{\theta}_2$,$\dot{\theta}_3$:

$$\dot{\theta}_1 = (-sin(\theta).cos(\theta).\dot{x}_l + cos2(\theta).\dot{y}_l + R.\dot{\theta})/r \tag{13}$$

$$\dot{\theta}_2 = (-sin(\theta+\alpha_2).cos(\theta).\dot{x}_l + cos(\theta+\alpha_2).cos(\theta).\dot{y}_l + R.\dot{\theta})/r \tag{14}$$

$$\dot{\theta}_3 = (-sin(\theta+\alpha_3).cos(\theta).\dot{x}_l + cos(\theta+\alpha_3).cos(\theta).\dot{y}_l + R.\,.\dot{\theta})/r \tag{15}$$

IV. SIMULATION AND RESULT

General coordinates of the mobile robot are denoted, q_1 , q_2 …, q_n in the joint space and $X, X_2 \ldots, X_m$ in the task space.

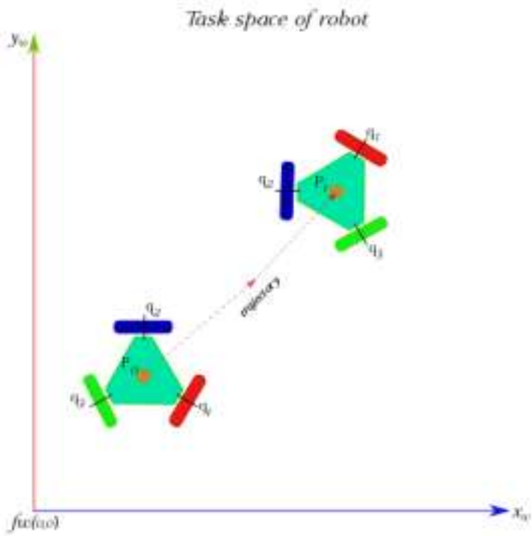

Fig.3. Task space from initial pose (P_0) to the desired pose (P_1) and coordinates of the robot

Define the vector from fig. 3:

$$\boldsymbol{q}_{[n]} = \begin{bmatrix}q_1\\ q_2\\ q_3\\ .\\ .\\ .\\ .\\ q_n\end{bmatrix} \text{ and } \boldsymbol{p}_{[m]} = \begin{bmatrix}p_1\\ p_2\\ p_3\\ .\\ .\\ .\\ .\\ p_m\end{bmatrix} \tag{16}$$

the direct kinematic models for the robot is determined **p** knowing **q**,

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) \tag{17}$$

Where,

$$f(\boldsymbol{q}) = \begin{bmatrix}f_1(q)\\ f_2(q)\\ f_3(q)\\ .\\ .\\ .\\ .\\ f_m(q)\end{bmatrix} \tag{18}$$

if we determine **q** knowing **p** is called the inverse kinematics models for the robot :

$$\boldsymbol{p} = \boldsymbol{f}^{-1}(\boldsymbol{q}) \tag{19}$$

the kinematic equation depends on the fixed geometry of the robot in the fixed world coordinate frame. To get motions of bodies without referring moments of inertia and torque, we need to find the differential relation of **q** and **p**, where :

$$dp = Jd_q \tag{20}$$

$$dp_m = \begin{bmatrix} dp_1 \\ dp_2 \\ dp_3 \\ . \\ . \\ . \\ . \\ dp_m \end{bmatrix} \text{ and } dq_n = \begin{bmatrix} dq_1 \\ dq_2 \\ dq_3 \\ . \\ . \\ . \\ . \\ dq_n \end{bmatrix} \tag{21}$$

and Jacobian matrix are *mxn* matrix:

$$J = \begin{bmatrix} & \frac{\delta x_1}{\delta q_1} \frac{\delta x_1}{\delta q_2} \frac{\delta x_1}{\delta q_n} & \\ . & . & . \\ . & . & . \\ . & . & . \\ . & . & . \\ . & . & . \\ . & . & . \\ & \frac{\delta x_m}{\delta q_1} \frac{\delta x_m}{\delta q_2} \frac{\delta x_m}{\delta q_n} & \end{bmatrix} \tag{22}$$

Jacobian matrix represents the relation of the displacements of the joint with the displacement of the position of the robot in the task space. The velocity in the joint and task spaces described :

$$\dot{q} = [\dot{q}_1, \dot{q}_2, \dots \dot{q}_n]^T \text{and } \dot{p} = [\dot{p}_1, \dot{p}_2, \dots \dot{p}_m]^T \tag{23}$$

dividing relation **p** and **q** by *dt*, we get direct jacobian kinematics:

$$dp = \mathbf{J}d_q \text{ , } \frac{dp}{dt} = \mathbf{J}\frac{df(q)}{dt} \tag{24}$$

because of matrix $m = n$ (***J*** square matrix), the inverse jacobian matrix we get:

$$\dot{\mathbf{p}} = \mathbf{J}\dot{\mathbf{q}} \tag{25}$$

where , the inverse jacobian is

$$\dot{\mathbf{P}}\dot{\mathbf{J}}^{-1} = \mathbf{J}.\mathbf{J}^{-1}\dot{\mathbf{q}} \tag{26}$$

if $\mathbf{J.J^{-1}}$ is invertible, we get inverse differential kinematics:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1}\dot{\mathbf{p}} \tag{27}$$

if error $\mathbf{E} = ||\mathbf{p}_{0,0} - \mathbf{p}_{des}||$, to get differensial of error,itcandividing by *dt*:

$$\frac{dE}{dt} = \frac{dP_{0,0}}{dt} - \frac{dP_{des}}{dt} \tag{28}$$

if $\mathbf{E} = \dot{\boldsymbol{P}} = ||\mathbf{p}_{0,0} - \mathbf{p}_{des}||$, and $\mathbf{E} = e^{-\lambda t}$, $\dot{\boldsymbol{E}} = -\lambda e^{-\lambda t}$:

$$\dot{\boldsymbol{E}} = -\lambda \boldsymbol{E} = -\lambda \dot{\boldsymbol{P}} \tag{29}$$

So we get :

$$\dot{\mathbf{q}} = \mathbf{J}^{-1}\,\lambda \boldsymbol{E}$$

to solve problem how to get $\dot{\boldsymbol{p}}_k$ and $\dot{\boldsymbol{q}}_k$, it can used differential equation $\dot{P} = \frac{\Delta P}{\Delta t} = \frac{(p_k) - (p_{k-1})}{ts}$, finally we get :

$$\boldsymbol{p_k} = \boldsymbol{P_{k-1}} + \boldsymbol{ts}.\dot{\boldsymbol{P}}_{\boldsymbol{k}} \tag{30}$$

The rotation angle θ are $\theta_{1,2,3}$concerning x,y, and z respectively, in mobile robots moving on a horizontal plane or the robot is rotating only concerning the vertical axis z. Direct trigonometric derivation of the rotation matrix (**R**) with respect to axis z are :

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} \tag{31}$$

type of simulation for three wheels omnidirectional mobile robot refer to equation found. Rotation matrix of x,y and θ respect to time *t* are shown fig.4, fig.5, and fig.6:

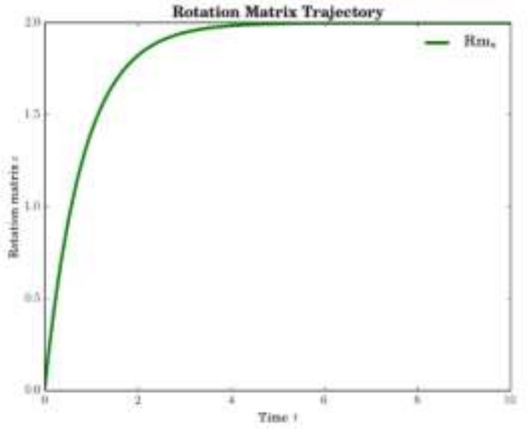

Fig.4. Rotation matrix *x* respect to time *t*

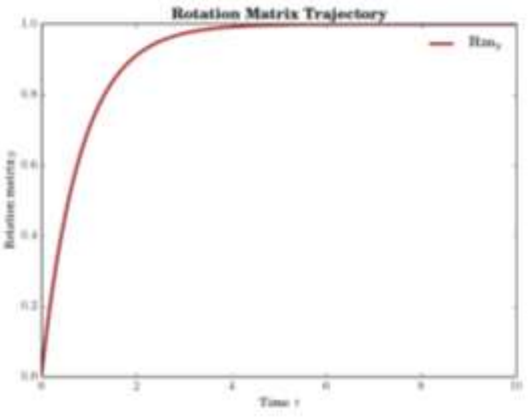

Fig.5. Rotation matrix *y* respect to time *t*

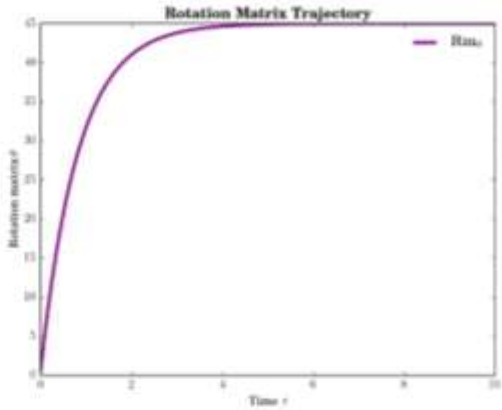

Fig.6. Rotation matrix θ respect to time t

if we describe radius(r) = 5cm and distance of center mass of robot and center of wheels are 20cm, the velocity $\dot{q}_1, \dot{q}_2, \dot{q}_3$, of robot shown in fig. 7.

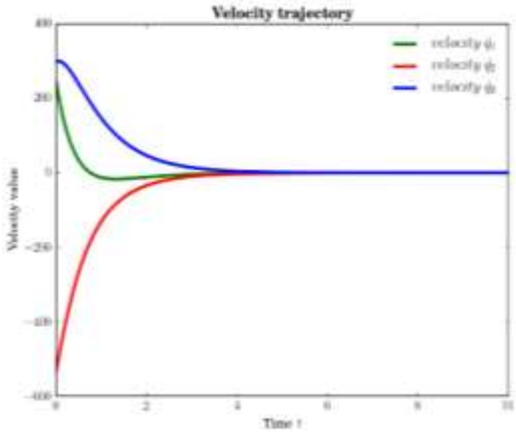

Fig.7. velocity of $\dot{q}_1, \dot{q}_2, \dot{q}_3$ respect to time t

The error of the trajectory robot is used minimizing the squared norm of the coordinate $\mathbf{e}_{norm}$, $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_\theta$ trajectory error, also the collect error of the coordinates are shown in fig.8. and fig.9. below:

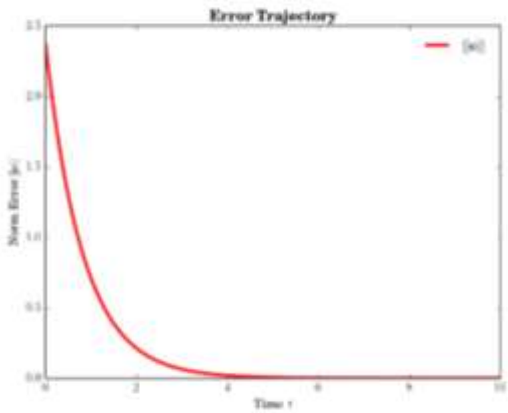

Fig.8. Trajectory of $\boldsymbol{e}_{norm}$ respect to time $t$respect to time t

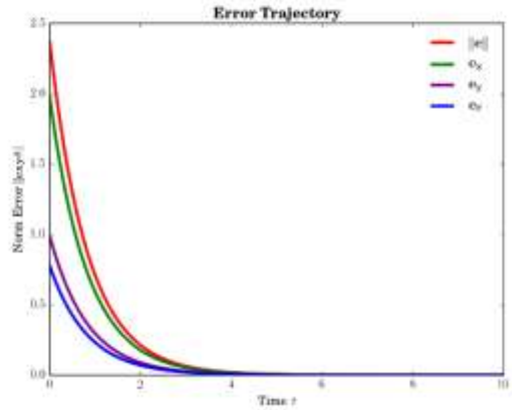

Fig.9. Trajectory of $\boldsymbol{e}_{norm}$, $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and$\boldsymbol{e}_\theta$respect to time $t$respect to time t

finally, from the simulation we get the figure of the trajectory of the robot, the θ of trajectory is shown by the arrow of the line:

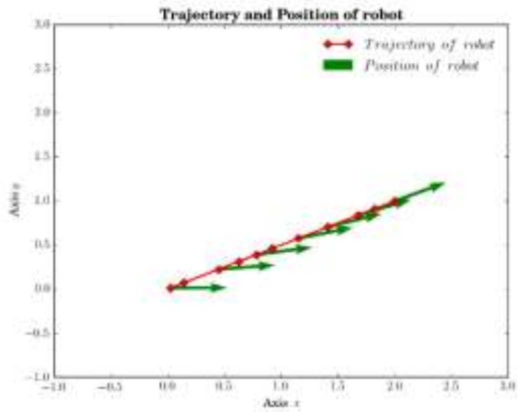

Fig.10. Robot trajectory on task space $\dot{\theta}$ = 25°

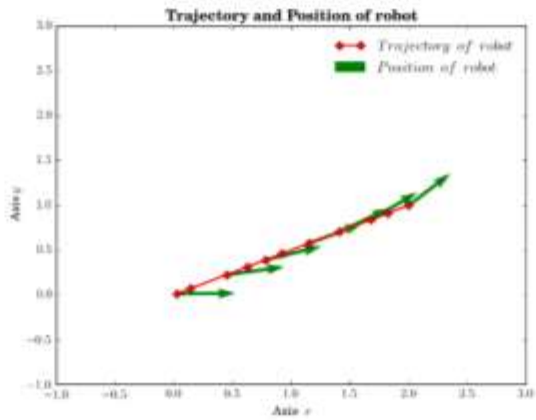

Fig.11. Robot trajectory on task space $\dot{\theta}$ = 45°

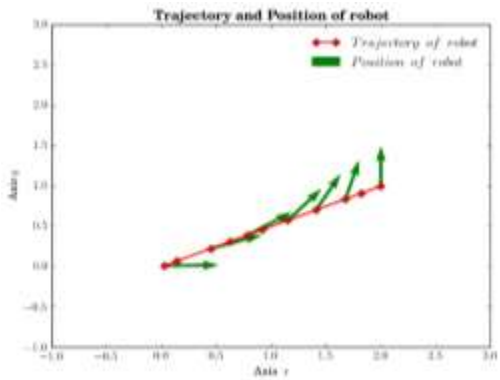

Fig.12. Robot trajectory on task space $\dot{\theta}$ = 90°

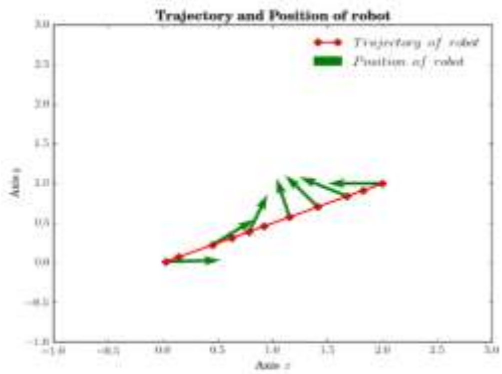

Fig.13. Robot trajectory on task space $\dot{\theta}$ = 180°

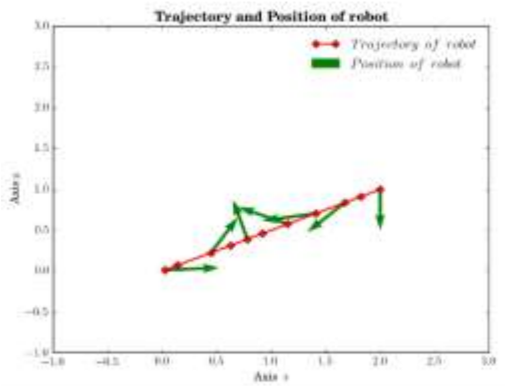

Fig.14. Robot trajectory on task space $\dot{\theta}$ = 270°

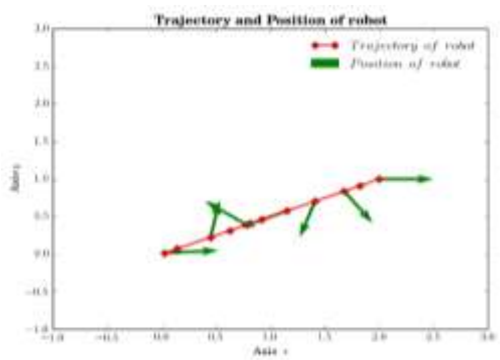

Fig.15. Robot trajectory on task space $\dot{\theta}$ = 360°

V. CONCLUSIONS

This paper has been discussed and simulated of kinematic a three-wheeled omnidirectional mobile robot performance. Using translation velocity and angular velocity of the robot, the jacobian inverse used for robot best ways trajectory. The trajectory of the robot from an initial position (**p**) and desired position (**p*des***) has been simulated and discussed, the motion of trajectory with the desired angle of the robot has been simulated. The norm error velocities are decreased exponentially.

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