Rules of Change of Weight Matrix in objective Function in Model Predictive Control in order to Consider Stability for Twin Rotor MIMO System Based on Bellman's Dynamic Programming Method

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Abstract

A paper [1] indicated that the survey results apply Model Predictive Control for the TRMS based on Bellman's dynamic programming method in order to consider the stability with the predictive horizon goes to infinity. The results of [1] has just proved the stability of system but they did not satisfy constraint conditions of state parameters. In this article, author provide rules of change of weight matrix in objective function in order to satisfy constraint conditions of the inputs, the outputs and the state parameters of this system. All of the state parameters reach to zero and satisfied with constraint conditions, the stable global of the system when the predictive horizon (N_P) goes to infinity proved a reasonable rule with simulation results.

Keywords *Constraint conditions, dynamic programming, model predictive control, stability, state parameters, Twin rotor MIMO system (TRMS).*

I. INTRODUCTION

Optimization of the model predictive control is a problem that is researching by many scientists. Until now, it was mainly used line search methods with finite predictive horizon for solving to optimize the model predictive control [2], [3], [8] because these methods are quite favorable for contrainted optimal problems. Moreover, there have few other optimization methods as Levenberg-Marquardt or trust region. However, all above methods were only used for finite predictive horizons. Therefore, these do not ensure the global optimization. So, the system is difficult to be stable [4].

The dynamic programming method has outstanding advantages when applying to solve multivariable optimal problems with ensuring the global of the optimal solution. However, this method is just applied to solve the optimal problem for linear systems with constant parameters or parameters changing over time.

Besides the above outstanding advantages, this method also reveals certain limitations that it does not directly satisfy the constraint conditions. In this study, the author applies the dynamic programming method to solve the optimal problem for the system with parameters depend on the TRMS state. At the same time, propose a rule to change the weight matrix of the objective function to overcome the above limitation in order to improve control quality.

II. THE TRMS MODEL

Considering the TRMS was given in figure 1, the predictive model as follows:

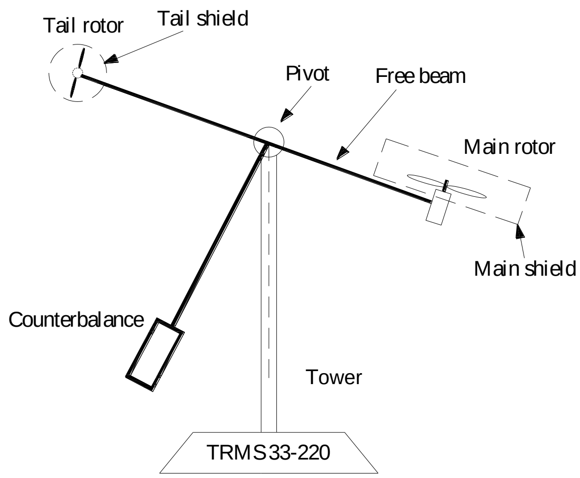

Fig 1. The TRMS

$$\begin{cases} \hat{x}(k+i+1|k) = A(x(k+i|k))\hat{x}(k+i|k) + B(x(k+i|k))\hat{u}(k+i|k) \\ \hat{y}(k+i|k) = C(x(k+i|k))\hat{x}(k+i|k) \end{cases} \quad (1)$$
$$i = 0,1,\dots,N_p - 1$$

In order to determine the control signal $\underline{u}_k$ at the current predictive window, such that the effect of model deviations $\underline{\varepsilon}_k$ to the stable quality $\underline{x}_k \to 0$ is minimal respectively with the predictive model (1), we will use the quadratic objective function [1]:

$$J = \sum_{i=0}^{N_P-1} \left\|\underline{x}_{k+i}\right\|^2_{Q_k} + \left\|\underline{u}_{k+i}\right\|^2_{R_k} \rightarrow \min \quad (2)$$

Where: $\left\|\underline{x}_{k+i}\right\|^2_{Q_k} = \underline{x}^T_{k+i} Q_k \underline{x}_{k+i}$ and

$\left\|\underline{u}_{k+i}\right\|^2_{R} = \underline{u}^T_{k+i} R_k \underline{u}_{k+i}$

with Q_k, R_k are two arbitrary positive definite symmetric matrixes. To increase the flexibility of the controller, we can change Q_k, R_k follow k, it means to change along the time axis $t = kT_a$.

When the predictive window is infinite ($N_P = \infty$), the optimization is done as follows: the objective function (2) will be rewritten to:

$$J = \sum_{i=0}^{\infty} \left\|\underline{x}_{k+i}\right\|^2_{Q_k} + \left\|\underline{u}_{k+i}\right\|^2_{R_k} \rightarrow \min \quad (3)$$

The dynamic programming method gives the results [1]:

$$\underline{u}_k = -\left(R_k + B_k^T L B_k\right)^{-1} B_k^T L A_k \underline{x}_k \quad (4)$$

where L is symmetric solution of:

$$L = Q_k + A_k^T L \left[I - B_k \left(R_k + B_k^T L B_k\right)^{-1} B_k^T L \right] A_k \quad (5)$$

The TRMS is a bilinear system with two inputs and two outputs. It can be described by the continuous model:

$$\begin{cases} \dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = C(\boldsymbol{x})\boldsymbol{x} \end{cases} \quad (6)$$

State variables, inputs, outputs, respectively are:

$$x = [\omega_h, S_h, \alpha_h, \omega_v, S_v, \alpha_v]^T \quad (7)$$

$$u = \begin{bmatrix} U_h & U_v \end{bmatrix}^T \quad (8)$$

$$y = \begin{bmatrix} \alpha_h & \alpha_v \end{bmatrix}^T \quad (9)$$

Where:

ω_h: Rotational velocity of the tail rotor (*rad/s*)

S_h: Angular velocity of the TRMS beam in the horizontal plane without effect of the main rotor (*rad/s*)

α_h: Yaw angle of the TRMS beam (*rad*)

ω_v: Rotational velocity of main rotor(*rad/s*)

U_v: Input voltage signal of the main motor (*V*)

U_h: Input voltage signal of the tail motor (*V*)

S_v: Angular velocity of the TRMS beam in the vertical plane without effect of the tail rotor (*rad/s*)

α_v: Pitch angle of the TRMS beam (*rad*)

In which:

$R_{ah}, L_{ah}, k_{ah}\varphi_h, J_{tr}, B_{tr}, l_t, D, E, F, k_m, R_{av}, L_{av}, k_{av}\varphi_v, J_{mr}, B_{mr}, l_m, k_g, g, A, B, C, H, J_v, k_t$

are positive constants, Ω_h and Ω_v is defined by

$$\Omega_h = S_h + \frac{k_m \omega_v \cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \quad (10)$$

$$\Omega_v = S_v + \frac{k_t \omega_h}{J_v} \quad (11)$$

The nonlinear continuous state space equations of the TRMS are expressed in [5], [6], [7],[9] as (12):

$$\frac{d}{dt}\begin{bmatrix} \omega_h \\ S_h \\ \alpha_h \\ \omega_v \\ S_v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -\dfrac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \dfrac{B_{tr}}{J_{tr}}\omega_h - \dfrac{f_1(\omega_h)}{J_{tr}} + \dfrac{k_{ah}\varphi_h}{J_{tr}R_{ah}} f_6(U_h) \\ \dfrac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \dfrac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\dfrac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \dfrac{B_{mr}}{J_{mr}}\omega_v - \dfrac{f_4(\omega_v)}{J_{mr}} + \dfrac{k_{av}\varphi_v}{J_{mr}R_{av}} f_8(U_v) \\ \dfrac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} + \\ \dfrac{g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \dfrac{k_t}{J_v}\omega_h \end{bmatrix} \quad (12)$$

III. DESIGNING PREDICTIVE CONTROLER FOR THE TRMS BASED ON DYNAMIC PROGRAMMING METHOD

Consider the state-dependent space model of the TRMS:

$$\frac{d\underline{x}}{dt} = A(\underline{x})\underline{x} + B(\underline{x})\underline{u} \quad (13)$$

in $t = k.Ts$

In which: Ts is sample time (small enough)

$x(t) = x_k$ with $(k-1)T_s \le t < kT_s$

$u(t) = u_k$ with $(k-1)T_s \le t < kTs$

inferred: $\dfrac{d\underline{x}}{dt} = A(\underline{x}_k)\underline{x} + B(\underline{x}_k)\underline{u}$ with

$(k-1)T_s \le t < kT_s$

Replace $\frac{d\underline{x}}{dt} = \frac{\underline{x}_{k+1} - \underline{x}_k}{T_s}$ We have:

$$\begin{cases} \underline{x}_{k+1} = \underline{x}_k + T_s A(\underline{x}_k)\underline{x}_k + T_s B(\underline{x}_k)\underline{u}_k \\ = [\mathrm{I} + T_s A(\underline{x}_k)]\underline{x}_k + T_s B(\underline{x}_k)\underline{u}_k \\ = \hat{A}(\underline{x}_k)\underline{x}_k + \hat{B}(\underline{x}_k)\underline{u}_k \\ = A_k \underline{x}_k + B_k \underline{u}_k \end{cases} \tag{14}$$

Algorithm diagrams of dynamic programming method is shown in Figure 2.

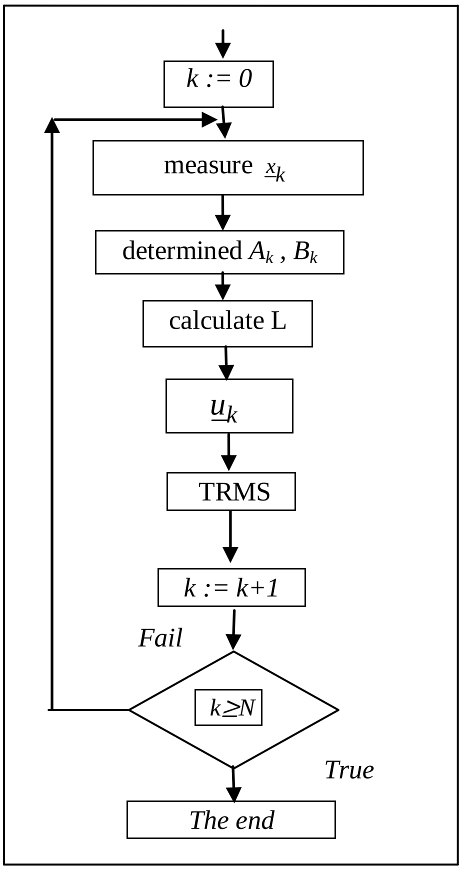

Fig 2. Algorithm diagrams of dynamic programming method

IV. SIMULATION RESULTS

With the mathematical model of TRMS was built in (12) and the parameters of the system as shown in table 1, the algorithm diagrams shown in figure 2, the simulation on Matlab obtained the time response of the state variables of the TRMS are shown from figure 3 to figure 8.

Table 1: System parameters

Parameters	Cost	Parameters	Cost
$l_t\,(m)$	0,282	$R_a\,(\Omega)$	8
$l_m\,(m)$	0,246	$L_a\,(mH)$	0,86
$l_b\,(m)$	0,290	$k_{av}\varphi_v\,(Nm/A)$	0,0202
$l_{cb}\,(m)$	0,276	$J_{mr}\,(gcm^2)$	1272
$r_{ms}\,(m)$	0,155	$J_{tr}\,(gcm^2)$	248
$r_{ts}\,(m)$	0,100	$B_{tr}\,(kgm^2/s)$	$2{,}3\times10^{-5}$
$m_{tr}\,(kg)$	0,221	$B_{mr}\,(kgm^2/s)$	$4{,}5\times10^{-5}$
$m_{mr}\,(kg)$	0,236	k_{th}	$3{,}6\times10^{-7}$
$m_{cb}\,(kg)$	0,068	k_{tv}	$8{,}7\times10^{-7}$
$m_t\,(kg)$	0,015	k_{fhp}	$1{,}84\times10^{-6}$
$m_m\,(kg)$	0,014	k_{fhn}	$2{,}20\times10^{-7}$
$m_b\,(kg)$	0,022	k_{fvp}	$1{,}62\times10^{-5}$
$m_{ts}\,(kg)$	0,119	k_{fvn}	$1{,}08\times10^{-5}$
$m_{ms}\,(kg)$	0,219	k_t	$2{,}6\times10^{-5}$
kg	0,2	k_m	2×10^{-4}
$k_{ah}\varphi_h\,(Nm/A)$	0.0202		

Sample time and weight matrixes have parameters:

$$T = 0.2,\ R_k = [0.01\ 0;\ 0\ 0.01],$$

$$Q_k = [1\ 0\ 0\ 0\ 0\ 0;\ 0\ 1\ 0\ 0\ 0\ 0;\ 0\ 0\ 0.01\ 0\ 0\ 0;\ 0\ 0\ 0\ 1\ 0\ 0;\ 0\ 0\ 0\ 0\ 1\ 0;\ 0\ 0\ 0\ 0\ 0\ 0.01]$$

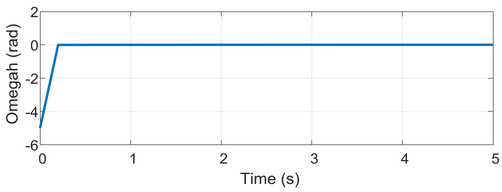

Fig 3. The response of the first state variable

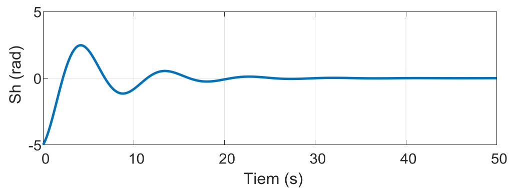

Fig 4. The response of the second state variable

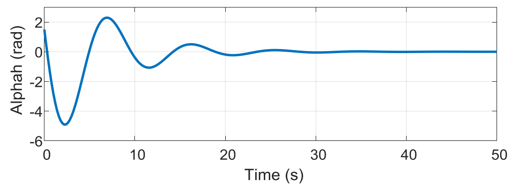

Fig 5. The response of the third state variable

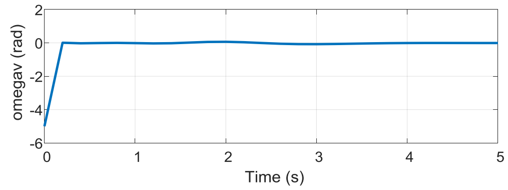

Fig 6. The response of the fourth state variable

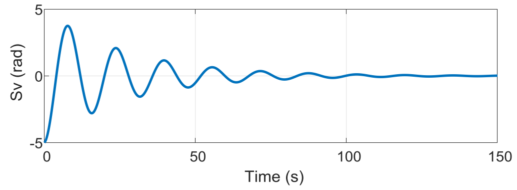

Fig 7. The response of the fifth state variable

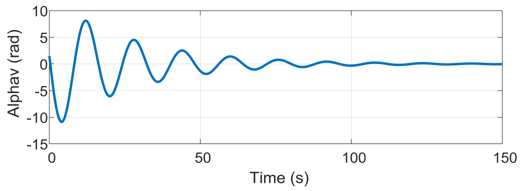

Fig 8. The response of the sixth state variable

With a simulation time of 5s, the simulation results show: the first and fourth state variables (ω_h, ω_v) move to 0 at 0.2s of the simulation process and satisfy their state constraint condition. When the simulation time is 200s the second and third state variables (S_h, α_h) go to 0 at 20th seconds, the fifth and sixth state variables (S_v, α_v) also go to 0 at the simulation time of the 100th second. So when the predictive window (*Np*) goes to infinity, all 6 status parameters of the TRMS system are approaching 0, which proves that the system is globally stable. However, state variables S_h, S_v, α_h and α_v are the relatively large adjustment, which shows that the method has not directly solved the input constraint conditions of the state variables, the fifth and sixth state variables have a lot of fluctuations. So we propose to change the weight matrix in the objective function as follows: keeping the matrix unchange R_k and change the value of the weight matrix Q_k as a rule $Q_{k+1} = i^2 Q_k$ in which $i = 1 \div 2$ to satisfy the constraint conditions of system state variables given from figure 9 to figure 14.

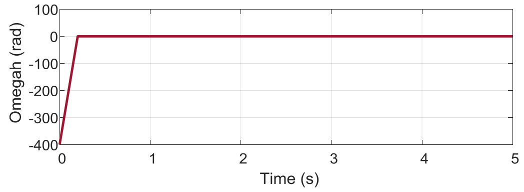

Fig 9. The response of the first state variable

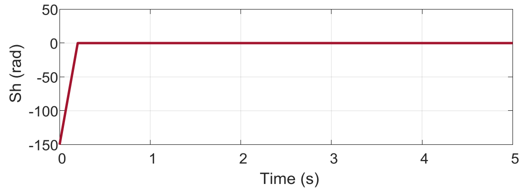

Fig 10. The response of the second state variable

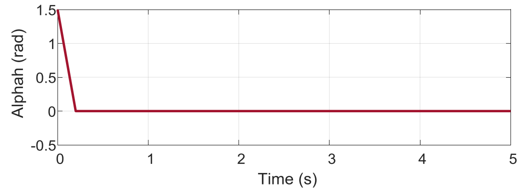

Fig 11. The response of the third state variable

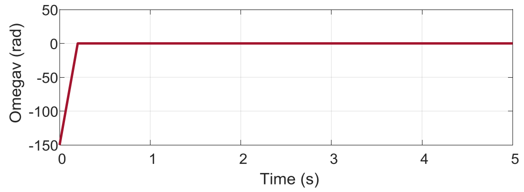

Fig 12. The response of the fourth state variable

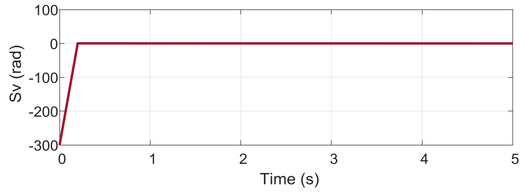

Fig 13. The response of the fifth state variable

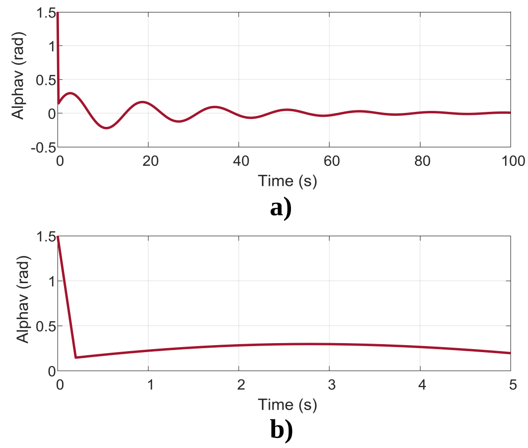

Fig 14. The response of the sixth state variable

Comment: With simulation time of 5s, the simulation results from Figure 9 to Figure 14 show: state variables ω_h, S_h, α_h, ω_v and S_v are all going to zero at 0.2 second during the simulation and satisfy the input and output constraints of the state parameters compared to them that were given from Figure 3 to Figure 7. In particular, the 6th state parameter

α_v although it has not reached 0 in the first second of the simulation process like other state variables, the adjustment is only from - 0.2 rad to 0.35 rad compared to the actual constraint conditions. Infact, the output of the object is -2 rad to 2 rad.

When we keeping the matrix unchange R_k and change the value of the weight matrix Q_k as a rule $Q_{k+1} = i^{\frac{1}{4}} Q_k$ in which $i = 1 \div 3$ to satisfy the constraint conditions of system state variables given from Figure 15 to Figure 20.

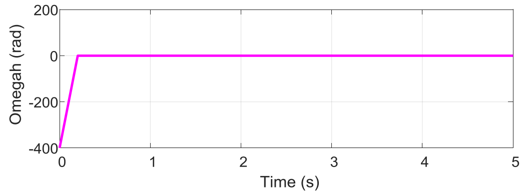

Fig 15. The response of the first state variable

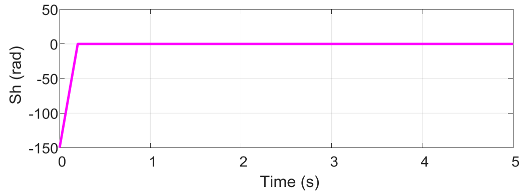

Fig 16. The response of the second state variable

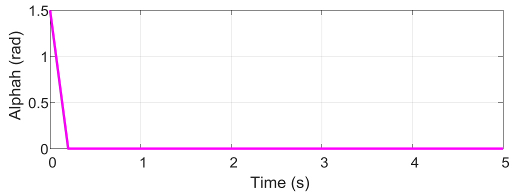

Fig 17. The response of the third state variable

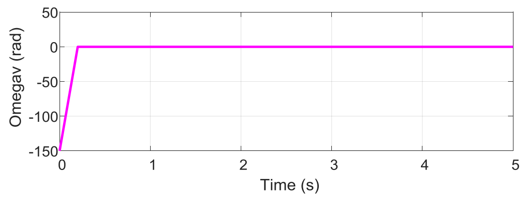

Fig 18. The response of the fourth state variable

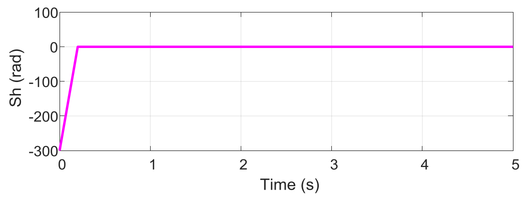

Fig 19. The response of the fifth state variable

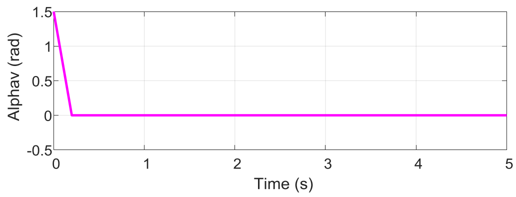

Fig 20. The response of the sixth state variable

Comment: With simulation time of 5s, the simulation results from Figure 15 to Figure 20 show: state variables ω_h , S_h , α_h , ω_v , S_v and α_v are all going to 0 at 0.2 second during the simulation and satisfy the input and output constraints of the state parameters compared to them that were given from Figure 3 to Figure 8 and from Figure 9 to Figure 14.

V. CONCLUSION

By using Bellman's dynamic programming method, we have built a predictive controller for the TRMS to consider the stability of the system when the predictive window is infinite. At the same time, overcome the limitations of this method when proposing rules to change the weight matrixes in the objective function to satisfy the constraint conditions of the state parameters of the system. The simulation results on Matlab prove that the system is global stable and also resolves the constraint conditions. This results also prove the rationale for changing the weight matrix has been proposed.

VI. ACKNOWLEDGEMENTS

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