# Design-analysis and optimization of 10 MW permanent magnet surface mounted off-shore wind generator

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#### Abstract

With advancing technology, market environment for wind power generation systems has become highly competitive. Industry has been moving towards higher wind generator power ratings, in particular off-shore generator ratings. Current offshore wind turbine generators are in the power range of 10 to 12 MW. Unlike traditional induction motors, slow speed Permanent Magnet Surface Mounted (PMSM) high-power generators are relatively challenging and designed differently.

In this paper, PMSM generator design features have been discussed and analysed. The focus attention is on armature windings, harmonics and permanent magnet. For the power ratings under consideration, the generator air-gap diameters are in the range of 8 to 10 meters and active material weigh ~60 tons and above. Therefore, material weight becomes one of the critical parameters. Particle Swarm Optimization (PSO) technique is used for weight reduction and performance improvement. Four independent variables have been considered which are air gap diameter, stack length, magnet thickness and winding current density. To account for core and teeth saturation, preventing demagnetization effects due to short circuit armature currents and maintaining minimum efficiency, suitable penalty functions have been applied. To check for performance satisfaction, a detailed analysis and 2D flux plotting is done for the optimized design.

Keywords — Offshore wind generator, PMSM, PSO optimization.

#### TABLE I NOMENCLATURE

Symbol	Description					
A <sub>wire</sub>	Cross sectional area of the copper wire (mm <sup>2</sup> )					
B <sub>n</sub>	The n-th spatial harmonic component of the air-gap flux density.(T)					

- Residual flux density of the permanent magnet (T)
- c1 Self-acceleration constant
- c2 Global acceleration constant
- *CW* Concentrated winding
- D Stator inside diameter (mm)
- E1 The fundamental component of back EMF (V)
- *F* Machine operating frequency (Hz)
- *G* Physical air-gap length (mm)
- $H_d$  The field intensity value on the demagnetization curve
- $h_m$  Thickness of permanent magnet in radial direction (mm)
- *I*<sub>dgmrm</sub> Maximum permitted value of steady state stator current before demagnetization (A)
- $J_s$  Stator winding current density (A/mm<sup>2</sup>) Ratio of stator leakage inductance to armature reaction inductance to prevent demagnetization at three phase short circuit.
- $K_{wn}$ Winding factor for the n<sup>th</sup> harmonicLMachine axial length (mm)
- $L_{lsn}$  Leakage inductance to protect the magnet from demagnetization at short circuit (H)
- $M_n$  Magnetization vector of the permanent magnet
- $N_c$  Number of turns per phase for stator winding
- *N<sub>s</sub>* Number of stator slots
- *N* Rotational speed in revolutions per second.
- *p* Pole number
- $pelSat_t$  Penalty function for tooth saturation
- *q* Number of slots per pole per phase
- $X_{sl}$  Leakage reactance per phase ( $\Omega$ )
- $X_m$  Armature reactance per phase ( $\Omega$ )
- $\alpha_p$  Pole coverage coefficient
  - Angular position with reference to the center of magnet pole
- $\theta_{me}$  Magnet angle electrical
- $\mu_o$  Permeability of vacuum= 4  $\pi$  10<sup>-7</sup>
- φ Flux per pole (Wb)

 $\theta$ 

#### I. INTRODUCTION

Most wind turbines installed today are land based. However, trend is moving towards offshore installations to maximize the utilization of strong winds and reduce concerns over noise produced. Commonly used systems are - doubly fed induction generators, squirrel induction generators, synchronous generators with electrical excitation or permanent magnet generators with gear boxes of single stage or multi stage.

In offshore applications, due to higher wind speeds, Direct drive turbines which drive the generator shaft without any gear box are used. They are less complex than gear box turbines which are more vulnerable for gear box failures. With advancing technology, cost effective and lighter versions of direct drive turbines have been developed.

In this paper, different electrical design features of Permanent Magnet Surface Mounted (PMSM) direct drive generators are discussed. The electromagnetic material content is optimized to reduce the active material weight maintaining efficiency within the specified limits. Optimization technique, Particle Swarm Optimization (PSO) is used to reduce the active weight.

## II. FEATURES OF PMSM ELECTRICAL DESIGN

#### A. TORQUE

PMSM direct drive generators are slow speed generators (~14 rpm) in comparison to gear driven generators (~1500 rpm). As torque is inversely proportional to speed, the torque to be developed at slow speed is very high when compared to the high-speed generators. Calculations [1] have shown that direct drive synchronous generator size would have to be ~7 times that of equivalent high-speed generator.

#### **B.** EFFICIENCY

PMSM generators have higher efficiency than high speed generators with gear box. For generators >3 MW capacity, the number of stages are >3. Approximately 1% efficiency is lost in every stage of gear box. Therefore, the relative efficiency of PMSM is >3%.

#### C. WINDINGS

Fractional-slot winding configurations have attracted much attention primarily due to the availability of concentrated windings, possibility to control and reduce cogging torque. Concentrated windings have short end windings resulting in low material weight and low copper loss. Fig 1. shows the possible reduction in over hang winding length.

#### **D.** POLE AND SLOT COMBINATION

In concentrated windings, pole slot combination has influence on both cogging torque and fundamental winding factor. Careful selection of pole slot combination will help in reducing the low cogging torque [2]. If pole and slot combination have a large Least Common Multiple (LCM) [3], then cogging torque will be less. Table I shows LCM and calculated



Fig 1. Winding overhang length comparison

fundamental winding factors for different pole and slot combinations. Pole and slot combination i.e. P=314 and Q=339 have highest LCM=106,446 with fundamental winding factor 0.94853.

Fig 2, shows, winding factors for the slots from 300 to 336 and for poles from 278 to 314. The combination of P=314 and Q =339 has been selected. The feasibility region is defined as "slots/pole/phase shall be within 0.25 to 0.50" [pg. 123, Ref 4]. The slots per pole per phase in this case is (339/314/3=0.36) which is well within the feasibility region.



Fig 2. Fundamental winding factors (z- axis) > 0.85 for various slot and pole combination

TABLE II POLE SLOT COMBINATION AND WINDING FACTORS

Inclose							
p Ns		$\mathbf{p}/N_s = \mathbf{q}$	LCM	Kw			
314	339	314/339	106446	0.94853			
314	315	314/315	95790	0.95492			
314	312	314/312	48984	0.9549			
302	300	302/300	45300	0.95489			
296	300	296/300	22200	0.98667			
312	306	312/306	15912	0.95463			
304	312	304/312	11856	0.95441			

#### **E.** WINDING LAYOUT AND WINDING FACTOR

In PMSM generator with non-overlapping concentrated windings, selection of number of winding layers and winding layout is important.

#### 1) NUMBER OF LAYERS

Single layer windings (Fig 3a.) have coils wound on the alternate teeth while double layer winding (Fig 3b.) have coils on each tooth. Double-layered concentrated windings have hotter end windings, more sinusoidal EMF, lower harmonic content of MMF and lower eddy current loss [3]. Therefore, double-layered concentrated winding configuration has been selected.



Fig 3a: Single layer concentrated winding



Fig 3b: Double layer concentrated winding

#### F. WINDING LAYOUT

The winding layout and winding factor depends on its combination of pole and slot numbers (calculation method is described in [3], [4] and [5]). Of the two methods "fractional slot windings' is chosen for its simplicity. In the present case, the number of slots (Ns) =339 and poles (p) =314. Total fundamental winding factor: 0.949.

The worked out winding layout full sequence is shown in Annexure 1.

#### G. MAGNET SELECTION AND DESIGN

Magnet type and its dimensions is important factor in design. While deciding magnet design, factors namely demagnetization curve, mass density, pole coverage, magnetization, magnet thickness are taken into account. For a selected magnet type, magnet dimensions are decided in optimization program.

Magnet Type: Nd2Fe14B, (N45) sintered has been selected. The demagnetization curve is shown in Fig 4.

 TABLE III

 MAGNET PROPERTIES AT ROOM TEMPERATURE

Parameter	Value
Permanence (B <sub>r</sub> ) (T) Normal coercively (Hcb)	1.35
(kA/m)	955
Mass energy product (Bhmax) (kJ/m3)	350

Maximum	operating	
temperature		80

#### H. MAGNET THICKNESS

Like other critical input parameters, magnet thickness is considered as one of the input parameters. Radial magnetization is considered.

Magnet width: The ratio of magnet width to pole pitch known as 'pole embrace' should be  $\sim 0.8$ . The lesser ratio will lead to higher harmonic content. It should also be noted that the larger ratio will lead to excessive leakage flux.

Magnet protection: During three phase short circuits, the short circuit currents should be limited to less than the magnets demagnetizing current ( $I_{dgmr}$ ). Short circuit limiting factors are primary leakage inductance and armature resistance. The magnets demagnetizing current is a function of Br, hm, Bd and radial air-gap. It is given by equation 1 [6].



Fig 4: Demagnetization curve of magnet N45

#### I. AIR-GAP INDUCTANCE

In rotor with surface mounted magnets, there is no saliency implying  $L_d = L_q$ . The fundamental component of air gap inductance [6] may be expressed as

$$L_{d1} = \frac{q}{2} \frac{4}{\pi} \frac{\mu_0 N_a^2 k_w^2 l R_s}{p^2 (g + h_m)}$$
(2)

#### 1) Air-gap flux density

The magnetization vectors of the permanent magnet for fundamental and harmonic are expressed as in [6] and [7].

$$M_1 = \frac{B_r}{\mu_0} \frac{4}{\pi} \sin \frac{p\theta_m}{2} \tag{3}$$

$$M_n = \frac{B_r}{\mu_0} \frac{4}{n\pi} \sin n \frac{\theta_{me}}{2} \sin n \frac{\pi}{2}$$
<sup>(4)</sup>

and the magnetic field for the fundamental and harmonic are given by

$$B_1 = \mu_0 M_1 k_g \tag{5}$$

$$B_n = \mu_0 M_n k_{gn} \tag{6}$$

$$B_r = \sqrt{4W\mu_0} \tag{7}$$

Where as

W= magnets energy product

$$\label{eq:Kg} \begin{split} K_g &= gap \; factor \; for \; the \; fundamental \\ K_{gn} &= gap \; factor \; for \; the \; harmonic \end{split}$$

$$k_{gn} = \frac{1}{1 - \left(\frac{R_i}{R_s}\right)^{2np}} \begin{pmatrix} \frac{np}{np+1} \left[ \left(\frac{R}{R_s}\right)^{np+1} - \left(\frac{R_i}{R_s}\right)^{np+1} \right] \\ + \frac{np}{np-1} \left(\frac{R_i}{R_s}\right)^{np+1} \left[ 1 - \left(\frac{R_i}{R}\right)^{np-1} \right] \end{pmatrix}$$
(8)

The fundamental component of back EMF can be calculated by

 $2\pi D$ 

$$E_{1} = 4.44 f N_{c} B_{1} k_{w1} \frac{2}{\pi} \frac{nD}{p} L$$
(9)
$$\int_{a}^{b} \frac{1}{p} \frac{1}{p}$$

#### J. WINDING INDUCTANCES

The inductances to be computed are

- a) d- and q- axis inductances
- b) air-gap fundamental inductance
- c) slot leakage inductance
- d) over- hang leakage inductance
- e) Differential leakage inductance

In the present case, since the magnets are surface mounted, there is no saliency,  $L_d = L_q$ . The fundamental part of inductance is [7]

$$L_{d1} = \frac{q}{2} \frac{4}{\pi} \frac{\mu_0 N_a^2 k_w^2 l R_s}{p^2 (g + h_w)}$$
(10)

The slot leakage and overhang leakage inductances for surface mounted concentrated winding machine is calculated as per ref [5]. The radial air-gap is large as compared to induction machines. Therefore, the differential leakage flux is negligible.

Total synchronous inductance is the sum of air-gap and leakage components.[7]

$$L_{d} = L_{d1} + L_{a}$$

$$L_{a} = L_{slot} + L_{overhang}$$

$$X_{d} = 2\pi f L_{d}$$
(11)

#### K. SHORT CIRCUIT CURRENT

The armature current under fault condition in general is given by [9]

The current consists of

- a) steady state ac component
- b) + transient ac component
- c) + sub-transient ac component
- d) + dc component
- e) + 2f component

$$i_{a} = \sqrt{2}E_{q1} \begin{bmatrix} \frac{1}{X_{d}} \\ +\left(\frac{1}{X_{d}'} - \frac{1}{X_{d}}\right)e^{-t/T_{d}'} \\ +\left(\frac{1}{X_{d}'} - \frac{1}{X_{d}'}\right)e^{-t/T_{d}'} \end{bmatrix} \cos\left(\omega t + \lambda\right)$$
$$-\sqrt{2}E_{q1}e^{-t/T_{a}} \begin{bmatrix} \frac{\cos\lambda}{x_{m}} \\ +\frac{\cos\left(2\omega t + \lambda\right)}{x_{n}} \end{bmatrix}$$
(12)

(12)

In PMSM machine, there is no field winding and hence transient part does not appear. After the decay of sub-transient term, current is limited solely by  $X_d$ . The amplitude of  $i_a$  can be approximated to the equation 13.



Fig 7: Short circuit current with speed

#### L. PERFORMANCE ESTIMATION

The losses considered are

- a) Stator winding copper loss
- b) Core loss (Hysteresis and eddy-current loss)
- c) Windage and friction loss

#### **III.PARTICLE SWARM OPTIMIZATION**

#### **A. OVERVIEW**

Particle Swarm Optimization (PSO) is an evolutionary technique to explore the search space to find parameters required to maximize or minimize a particular objective. Each candidate solution can be thought of as a particle "flying" finding maximize or minimize of the objective function. PSO algorithm consists of three steps and are repeated till stopping condition is met.

- 1. Evaluate the fitness of each particle
- 2. Update individual and global best fittness and position
- 3. Update velocity and position of each particle.

The flow chart for algorithm is shown in Fig 8.

The ability of PSO is decided by the velocity and postion update step of each particle. [10]

$$v_{i}(t+1) = wv_{i}(t) + c_{1}r_{1}\left[\hat{x}_{i}(t) - x_{i}(t)\right] + c_{2}r_{2}\left[g(t) - x_{i}(t)\right]$$
(14)

 $v_i(t)$  is the velocity of particle i at time t and xi (t) is the position of particle i at time t. The parameters w, c1 and c2 are user supplied coefficients. The values r1 and r2 random values generated for each velocity update. The value  $\hat{x}_i(t)$  is the individual best candidate solution for particle *i* at time t and g(t) is the swarm's global best candidate solution at time t.

#### **B. PSO IMPLEMENTATION**

Each particle is a four-dimensional vector. They are air gap diameter, active core length, magnet thickness and winding current density.

#### 1) Air gap diameter (D)

By reducing the diameter, weight decreases in square ratio, but space for number of poles and winding reduces. This leads to higher operating parameters like flux densities in air gap, teeth and core.

#### 2) Active core length (L)

The effects of core length reduction are similar to the air gap diameter reduction. While deciding the maximum permissible active core length, due consideration is to be given on mechanical aspects like centre of gravity of generator and structural design.

#### 3) Magnet thickness (hm)

The magnet thickness  $(h_m)$  together with radial air-gap controls the air-gap flux density. Wider air-gap length reduces the gap flux density and shorter air-gap is likely to have mechanical problems [11].

$$\frac{B_g}{B_r} = \frac{1}{1 + \mu_r \frac{l_g}{l_m}}$$
(15)

From the above equation it is clear that for deciding the air-gap flux density  $B_g$ , the magnet thickness is to be considered. The optimal ratio between magnet thickness and air-gap length is usually in the range of 4-6[11].

#### 4) Conductor current density (Alpha)

Conductor current density (A/mm2) is a design constraint. It determines the machines cooling system. The size of the machine, winding temperature rise, machine efficiency are decided by current density. The range for air cooled PMSM machines is within 2A/mm2 to 4.0 A/mm2.The upper limit for current density can be 10 A/mm2depending upon the cooling method.

Each particle represents a candidate solution. Each solution is evaluated by the objective function. 10 particles are considered by the PSO program to search for minimum value of object function.



Fig 8: Flow chart for PSO algorithm

#### **IV.RESULTS**

Two objective functions are considered.

Objective function 1: Active material weight is given maximum priority. The weight has to be minimized without violating the constraints

Objective function 2: Efficiency is given maximum priority. The efficiency has to be maximized without violating constraints.

In both the cases the number of particles are same and each particle is a four dimension vector same as mentioned above. 'Multi-objective Particle Swarm optimization over continuous functions" (MOPSO) based on Matlab program developed by Victor Martinez -Cagigal version 1.1.was used [12][13][14].



Particular	Value
Power output (MW)	10
Frequency	20
Speed (rpm)	7.64
Voltage induced per phase(volts)	6338
Number of slots	339
Phase current(amps)	525
Number of poles	314
Turns per phase	678
Winding factor (Fundamental)	0.948

Slot opening width (mm)	20.0
Slot opening height (mm)	0.5
Population size	20
Maximum generations	200
Inertia weight	0.4
Individual confidence factor (C1)	2
Swarm confidence factor (C2)	2
Maximum velocity in percentage	5
Uniform mutation percentage	0.5

The constraints selected are

- **1.** Flux density in teeth and core (Saturation constraint)
- 2. Demagnetizing effect.
- **3.** Minimum efficiency (When optimized for minimum weight)
- **4.** Maximum weight (When optimized for maximum efficiency)
- 5. Current density

When the constraints are violated, it triggers penalty function and a large value is added to objective function. This signals to all particles that, this is a poor candidate and drive away from solution space. The pareto diagram for both the objective functions i.e. weight in tons and the deficiency after 200 generations is shown in the Fig 9.

Two designs are selected from the set of designs in pareto diagram. They are high efficiency design and minimum weight design. The salient features of both designs are compared and shown in the table 4. The efficiency is improved from 96.3 to 96.83%, an increase of 0.53%. This is achieved mainly by copper loss reduction from 379.7 kW to 326.5 kW, a reduction of 53.2 kW. But the active material weight is increased from 58.46 tons to 66.53 tons. (an increase of 8.07 tons).



Fig 10. Pareto diagram for Weight and Deficiency objective functions

design features						
Particular	Efficiency (Maximum)	Weight (Minin				
Stator air-gap diameter (mm)	9990	9629				
Core length (mm)	1502	1500				
Magnet thickness (mm)	17.75	15				
Winding Current density (amp/mm <sup>2</sup> )	3.955	4.741				
Radial air-gap (mm)	4.36	3.48				
Losses						
Stator copper loss (kW)	326.5	379.7				
Core loss (kW)	0.16	0.16				
Efficiency (%)	96.83	96.3				
Weights						
Copper weight (tons)	7.01	5.94				
Magnet weight (tons)	5.28	4.3				
Core weight (tons)	54.24	48.2				
Active material weight (tons)	66.53	58.46				
Slot width (mm)	23.84	23.12				
Slot height (mm)	124.84	109.16				
Conductor area (mm2)	161.74	134.93				

### Table 9: Comparison of high efficiency and low weight



Fig 11. No load flux lines plot



Fig 12. No load flux density of one pitch in airgap

#### **Flux Plotting**

In order to view more clearly, a section consisting 4 poles is considered for flux plotting. The no-load flux plot is done using FEMM 4.2 software. The plotting is done considering M43 grade stator and rotor core material, N45 grade magnets. The figs below show 'mesh', flux density plots of air gap, stator teeth and stator and rotor core sections. It is observed that, the estimated values and FEMM plots are close.



Fig 10. Nodes and mesh plot



Fig 13. No load flux density in rotor core cross section



Fig 14. No load flux density in stator teeth cross section

#### **V.** CONCLUSION

The optimization is carried out with four critical variables namely air-gap diameter, active core length, permanent magnet thickness and winding current density within a given upper and lower limits. While the limits of air-gap diameter, core length and magnet thickness are decided by constraints like performance requirements, manufacturing, transportation constraints, however, the current density limits are decided mainly by cooling arrangements and generator

efficiency. In the present case, natural cooling is considered. So, the upper limit of current densities is limited to about 5 amp/mm<sup>2</sup>.

The permanent magnets characteristics are sensitive to its operating temperatures. Designing the armature windings for high thermal class like class H insulation, will not help to reduce the magnets operating temperature.

In the present case, temperature rise estimation by thermal analysis is not done. The estimation of temperature rise and inclusion in analysis program is essential.

In large industrial machines like induction or synchronous machines. the loss contribution is from copper, iron and friction and windage. But, in direct drive large wind generators, the speed is less and the frequency of flux in the core is also. So the hysteresis and eddy current (iron) loss which is dependent on frequency, is also less. The main contribution towards improvement in efficiency is from copper loss. Use of concentrated windings, as in this case, helps in reducing the copper loss. Further, reduction in copper means increasing the slot size and corresponding increase in stator diameter. The air-gap diameter has increased from 9629 mm to 9990mm for increase in efficiency from 96.3% to 96.83%.

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#### Acknowledgement

M Ramakrishna Rao thanks Ms. Mamidi Sri Rekha for her sincere help in preparing the paper and presentation.

#### VII. ANNEXURE

Number of slots (N<sub>s</sub>): 339

Number of poles (p): 314

Number of phases=3

Winding type: Double layer concentrated Non over lapping

winding.

The winding Layout is based on Cros' method [3] and is described in the following steps.

- a) Slots /pole/phase (q) = 0.36
- b) Slots /phase = 113 (b=113 and c= 314)
- c) So 113 ones and 201 (i.e. 334-113) zeros "1" are distributed in the sequence as regularly as possible.
- d) The sequence is repeated  $(Q_s/b= 330/113=3)$  times and compared to the layout of distributed winding with 3p slots and q=1.
- e) Conductors corresponding to "1" form one layer of concentrated winding. The second layer is the return conductor on the other side of tooth.
- f) A vector S corresponds to one layer and is used to calculate the winding factor.
- g) Using vector S, the corresponding EMF phasor Ei of a coil side I from phase A for the fundamental is

The fundamental winding factor  $k_{\rm w1}$  can be calculated as

$$k_{w1} = \frac{1}{n_l Q_s / 3} \left| \sum_{i=1}^{n_l Q_s / 3} \vec{E}_i \right|$$

$$\vec{E}_i = sign(S(i)) e^{j \frac{\pi p}{Q_s} |S(i)|}$$
(17)

Where i is an element of S and nl is the number of layers (nl=2)

Based on the above steps [5] the layout is worked out. Matlab M-script for Fractional Slot Concentrated Windings (FSCW) is followed [14]. The full sequence is given below

Slot	num	bers:
------	-----	-------

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	

Winding Layout:

Phases: Aa, Bb, Cc

Number of layers: Two

(A|a... 'A' represents conductor in top layer of phase A 'a' represents return conductor in bottom layer of phase A

Similarly for B|b and C|c))

A|aB|bb|BB|bb|Bc|CC|cc|CC|cc|Ca|AA|aa|AA|aB|bb|B B|bb|BB|bC|cc|CC|cc|Ca|AA|aa|AA|aa|Ab|BB|bb|BB|b C|cc|CC|cc|CC|cA|aa|AA|aa|Ab|BB|bb|BB|bb|Bc|CC|cc |CC|cA|aa|AA|aa|AA|aB|bb|BB|bb|Bc|CC|cc|CC|cc|Ca| AA|aa|AA|aB|bb|BB|bb|BB|bC|cc|CC|cc|Ca|AA|aa|AA| aa|Ab|BB|bb|BB|bC|cc|CC|cc|CC|cA|aa|AA|aa|Ab|BB| bb|BB|bb|Bc|CC|cc|CC|cA|aa|AA|aa|AA|aB|bb|BB|bb| BB|bC|cc|CC|cc|Ca|AA|aa|AA|aa|Ab|BB|bb|BB|bC|cc| CC|cc|CC|cA|aa|AA|aa|Ab|BB|bb|BB|bb|Bc|CC|cc|CC| cA|aa|AA|aa|AA|aB|bb|BB|bb|Bc|CC|cc|CC|cc|Ca|AA| aa|AA|aB|bb|BB|bb|BB|bC|cc|CC|cc|Ca|AA|aa|AA|aa| Ab|BB|bb|BB|bC|cc|CC|cc|CC|cA|aa|AA|aa|Ab|BB|bb| BB|bb|Bc|CC|cc|CC|cA|aa|AA|aa|AA|aB|bb|BB|bb|Bc| CC|cc|CC|cc|Ca|AA|aa|AA|aB|bb|BB|bb|BB|bC|cc|CC| cc|CC|cA|aa|AA|aa|Ab|BB|bb|BB|bb|Bc|CC|cc|CC|cA|aa|AA|aa|AA|aB|bb|BB|bb|Bc|CC|cc|CC|cc|Ca|AA|aa|AA|aB|bb|BB|bb|BB|bC|cc|CC|cc|Ca|AA|aa|AA|aa|Ab|BB|bb|BB|bC|cc|CC|cc|CC|cA|aa|AA|aa|Ab|BB|bb|BB|bb|Bc|CC|cc|CC|cA|aa|AA|aa|AA|aB|bb|BB|bb|Bc|CC| cc|CC|cc|Ca|AA|aa|AA|aB|bb|BB|bb|BB|bC|cc|CC|cc|C a|AA|aa|AA|aa|Ab|BB|bb|BB|bC|cc|CC|cc|CC|cA|aa|A A|aa|A

S1 is vector to describe the layout of one phase for calculating the winding factor.

The vector consists of the numbers corresponding to slots in phase A.

S2 and S3 are vectors for phase B and C respectively.

S1 vector: 1 -2 -11 12 12 -13 -13 14 14 -15 -24 25 25 -26 - 26 27 27 - 28 - 28 29 38 - 39 - 39 40 40 - 41 - 41 42 51 -52 -52 53 53 -54 -54 55 55 -56 -65 66 66 -67 -67 68 68 - 69 - 78 79 79 - 80 - 80 81 81 - 82 - 82 83 92 - 93 - 93 94 94 -95 -95 96 105 -106 -106 107 107 -108 -108 109 109 -110 -119 120 120 -121 -121 122 122 -123 -123 124 133 -134 -134 135 135 -136 -136 137 146 -147 -147 148 148 -149 -149 150 150 -151 -160 161 161 -162 -162 163 163 -164 -173 174 174 -175 -175 176 176 -177 -177 178 187 -188 -188 189 189 -190 -190 191 200 -201 -201 202 202 -203 -203 204 204 -205 -214 215 215 -216 -216 217 217 -218 228 -229 -229 230 230 -231 - 231 232 241 - 242 - 242 243 243 - 244 - 244 245 245 -246 -255 256 256 -257 -257 258 258 -259 -268 269 269 -270 -270 271 271 -272 -272 273 282 -283 -283 284 284 -285 -285 286 295 -296 -296 297 297 -298 -298 299 299 -300 -309 310 310 -311 -311 312 312 -313 -322 323 323 -324 -324 325 325 -326 -326 327 336 -337 -337 338 338 -339 -339 340

S2 vector: 2 -3 -3 4 4 -5 -5 6 15 -16 -16 17 17 -18 -18 19 19 -20 -29 30 30 -31 -31 32 32 -33 -42 43 43 -44 -44 45 45 -46 -46 47 56 -57 -57 58 58 -59 -59 60 69 -70 -70 71 71 -72 -72 73 73 -74 -83 84 84 -85 -85 86 86 -87 -96 97 97 -98 -98 99 99 -100 -100 101 110 -111 -111 112 112 -113 -113 114 114 -115 -124 125 125 -126 -126 127 127 -128 -137 138 138 -139 -139 140 140 -141 -141 142 151 -152 -152 153 153 -154 -154 155 164 -165 -165 166 166 -167 -167 168 168 -169 -178 179 179 -180 -180 181 181 -182 -191 192 192 -193 -193 194 194 -195 -195 196 205 -206 -206 207 207 -208 -208 209 218 -219 -219 220 220 -221 -221 222 222 -223 -232 233 233 -234 -234 235 235 -236 -236 237 246 -247 - 247 248 248 - 249 - 249 250 259 - 260 - 260 261 261 -262 -262 263 263 -264 -273 274 274 -275 -275 276 276 - 277 - 286 287 287 - 288 - 288 289 289 - 290 - 290 291 300 -301 -301 302 302 -303 -303 304 313 -314 -314 315 315 -316 -316 317 317 -318 -327 328 328 -329 -329 330 330 -331

S3 vector: -6 7 7 -8 -8 9 9 -10 -10 11 20 -21 -21 22 22 - 23 - 23 24 33 - 34 - 34 35 35 - 36 - 36 37 37 - 38 - 47 48 48 -49 -49 50 50 -51 -60 61 61 -62 -62 63 63 -64 -64 65 74 -75 -75 76 76 -77 -77 78 87 -88 -88 89 89 -90 -90 91 91 -92 -101 102 102 -103 -103 104 104 -105 115 -116 -116 117 117 -118 -118 119 128 -129 -129 130 130 -131 -131 132 132 -133 -142 143 143 -144 -144 145 145 -146 -155 156 156 -157 -157 158 158 -159 -159 160 169 -170 -170 171 171 -172 -172 173 182 -183 -183 184 184 -185 -185 186 186 -187 -196 197 197 -198 - 198 199 199 - 200 - 209 210 210 - 211 - 211 212 212 -213 -213 214 223 -224 -224 225 225 -226 -226 227 227 - 228 - 237 238 238 - 239 - 239 240 240 - 241 - 250 251 251 -252 -252 253 253 -254 -254 255 264 -265 -265 266 266 -267 -267 268 277 -278 -278 279 279 -280 -280 281 281 -282 -291 292 292 -293 -293 294 294 -295 - 304 305 305 - 306 - 306 307 307 - 308 - 308 309 318 -319 -319 320 320 -321 -321 322 331 -332 -332 333 333 -334 -334 335 335 -336