# Sliding Mode Controller Associates with Adaptive Load Mass and Friction 2D Overhead Crane Control

Ho Manh Tien

*Faculty of Electrical and Electrical Engineering, University of Transport and Communications, Vietnam*

**Abstract:** *A position control problem is presented in the paper. Different from other studies, the paper proposes a load mass and friction coefficient adaptive algorithm that can effectively replace these parameters which is hard to get accurately when the system is operating. The load's mass information is used for designing a control based on sliding mode control (SMC) that is better for robustness to system uncertainties and disturbances. The system performances with the designed control show good position tracking ability.*

**Keywords:** *Cranel system, load mass adaptive control, sliding mode control, Lyapunov's stability*

## I. INTRODUCTION

Overhead cranes are more and more applied commonly in many industrial factories, such as transporting and lifting goods or containers at seaports or factories. he activity of an overhead crane must be guaranteed definitely and safety. So as to limit accidents rate, the reliability and stability of control law are indispensable indicators in the design of the controller. However, the model of a crane belongs to a class of under actuated mechanical systems which means the number of control signals is less than the number of degrees of freedom. That leads to unexpected swing of payload in operation, which can endanger the lives of workers as well as destroying machineries and goods around. Therefore, designing appropriate controllers is essential.

In recent years, many studies for control of overhead crane have been investigated. For example, in paper [1], Fang et al. presented three different controllers comprising of proportional-derivative (PD) controller and two nonlinear controllers based on the square of the energy coupling and the gantry kinetic energy coupling control laws. Beside, the feedback and partial feedback linearization techniques are utilized in [2]. Some other researchers used fuzzy control [3] to deal with the parametric uncertainties of crane mode. Sling mode control which is regarded as a robust and effective technique was also widely used, as in papers [4-6]. The crane system stability was guaranteed even with external disturbances

In addition, because of the uncertain factors such as load changes after each transporting and lifting cycle, external disturbances, or other undetermined elements, many adaptive control laws have been proposed in which uncertain components are estimated [7-9]. In [10], the researchers used adaptive control law to estimate parameters in the system such as load volume, viscosity coefficient of viscosity. In [8], an adaptive fuzzy sliding mode control was applied to deal with nonlinearities caused by saturation actuators. However, the disadvantage of the all above method is that we need information of the the load mass, which changes constantly as well as difficult to measure in actual operation.

The paper introduces a new controller designed for 2D overhead crane system base on sliding mode control and a friction coefficient and load mass adaptive algorithm. Purpose of this controller is keepping the robust property under the uncertainty of friction and the changes of load mass. Comprehensive control design procedure and numerical simulations are given to support the proposed control.

## II. MODEL OF 2D OVERHEAD CRANE

Crane dynamics are constructed in the case of a simultaneous combination of trolley and cargo hoisting motions. The crane system presented in Figure 1 has two masses: $m_t$ of the trolley and $m_c$ of the cargo. Both of these masses are considered to have point masses concentrated at their centers. The generalized coordinates of the system include $x(t)$ and $\theta(t)$, which are trolley displacement, and cargo swing angle, respectively. Furthermore, friction of trolley moving are characterized by $b_t$. Forces of the driving motors of trolley travelling $u_t$ is designed to move the trolley from the starting points to their destinations as fast as possible under minimized cargo swing.

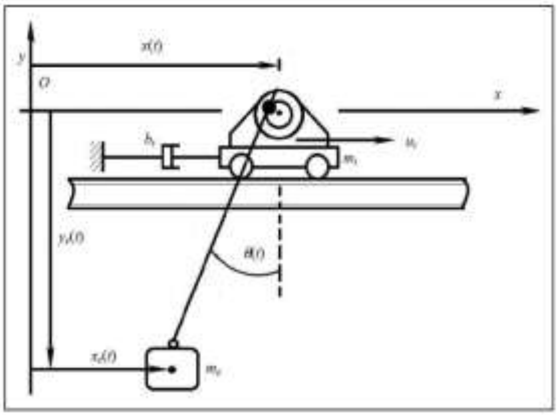

**Fig.1. Physical modelling of 2D overhead crane**

For convenience, the following assumptions are given: Mass and elastics of the wire rope are neglected; No disturbance is caused by the wind outside the factory floor because the overhead crane is usually operated indoors; Motions of all system components are considered in a plane. The kinetic energy of system is given by:

$$E = E_t + E_c \tag{1}$$

Where $E_t$, $E_c$ are the kinetic energys of the trolley and the cargo, respectively. The elements of kinetic energy are given by:

$$E_t = \frac{1}{2} m_t x^2 \tag{2}$$

$$\begin{aligned} E_c &= \frac{1}{2} m_c \left( x_c^2 + y_c^2 \right) \\ &= \frac{1}{2} m_c \left( x^2 + 2xl\sin\theta + l^2 \right) \end{aligned} \tag{3}$$

From (1), (2) and (3), we obtain the kinetic energy of the system as:

$$E = \frac{1}{2}(m_c + m_t) x^2 + m_c lx\sin\theta + \frac{1}{2} m_c l^2 \tag{4}$$

Potential energy of the system is shown as:

$$V = -m_c gl\cos\theta \tag{5}$$

There are two external forces that affect to the system that are $u_t$ and the friction which occurs with trolley moving. Using (4), (5) to apply in Lagrange's equation, the motion equations describing the system dynamics can be written as follows:

$$\begin{cases} (m_t + m_c)\ddot{x} + b_t\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u_t \\ l\ddot{\theta} + \ddot{x}\cos\theta + g\sin\theta = 0 \end{cases} \tag{6}$$

The mathematical model of the 2D overhead crane can be shown as:

$$M\ddot{q} + (C + B)\dot{q} + G = U \tag{7}$$

Where: $q = \begin{bmatrix} x \\ \theta \end{bmatrix}$ is state vector, $M = \begin{bmatrix} m_t + m_c & m_c l\cos\theta \\ \cos\theta & l \end{bmatrix}$ is the inertia matrix, $C = \begin{bmatrix} 0 & -m_c l\dot{\theta}\sin\theta \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b_t & 0 \\ 0 & 0 \end{bmatrix}$, $G = \begin{bmatrix} 0 \\ g\sin\theta \end{bmatrix}$, $U = \begin{bmatrix} u_t \\ 0 \end{bmatrix}$.

From (6), we obtain:

$$\ddot{x} = \frac{u_t + m_c l\dot{\theta}^2\sin\theta - m_c l\ddot{\theta}\cos\theta - b_t\dot{x}}{m_t + m_c} \tag{8}$$

$$\ddot{\theta} = \frac{-\ddot{x}\cos\theta - g\sin\theta}{l} \tag{9}$$

Replacing (9) in the first equation of (8), we have the formular:

$$\ddot{x} = \frac{m_c g\sin\theta\cos\theta + m_c l\dot{\theta}^2\sin\theta - b_t\dot{x}}{m_t + m_c\sin^2\theta} + \frac{u_t}{m_t + m_c\sin^2\theta} \tag{10}$$

## III . SILDING MODE CONTROL FOR 2D OVERHEAD CRANE

In the previous section, we considered about the mathematical model of the system. The system dynamics expressed the complex kinematic constraint among trolley motion and cargo swing angle. The are two variables that need to be controlled contains the position of the trolley $x$ and the cargo swing angle $\theta$. However, the system has only one control signal: the force that affects the trolley $u_t$. The purpose of the controller is driving the trolley from its initial position to the reference as fast and smooth as possible but the swing angle need to keep small and less fluctuation. The technique proposed is slidding mode control (SMC) because of the robust stable with uncertainties and external disturbances.This section will design SMC algoritm base on above modelling. Firstly, Define the error variables as:

$$\tilde{x} = x - x_d \tag{11}$$

$$\tilde{\theta} = \theta - \theta_d \tag{12}$$

Choose the Sliding surface as:

$$S = \dot{\tilde{x}} + \lambda\tilde{x} + \alpha\tilde{\theta} \tag{13}$$

Where $\lambda$ and $\alpha$ are the appropriate gains. Take time derivative (13) and using (10) and (11), we have:

$$\dot{S} = \frac{m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - b_t \dot{x}}{m_t + m_c \sin^2\theta} + \frac{u_t}{m_t + m_c \sin^2\theta} - \ddot{x}_d + \lambda\dot{\tilde{x}} + \alpha\dot{\tilde{\theta}} \quad (14)$$

In SMC controller, the control signal is the combination between two elements: the signal that drive the state of system approach the sliding surface and the signal that keep the state stable on the sliding surface. Suppose that the states of system are already on the sliding surface, so the condition to keep them stable is: $\dot{S} = 0$ , using (14), we obtain the control signal as:

$$u_{t1} = -\left(m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - b_t\dot{x}\right) + \left(m_t + m_c \sin^2\theta\right)\left(\ddot{x}_d - \lambda\dot{\tilde{x}} - \alpha\dot{\tilde{\theta}}\right) \quad (15)$$

In order to drive the states of system to the sliding surface, the condition that need to be guaranteed is $S\dot{S} < 0$ . From (14), we propose the signal control as:

$$u_{t2} = -k\left(m_t + m_c \sin^2\theta\right)\text{sign}(S) \quad (16)$$

Where k is a positive gain. The total control signal is:

$$\begin{aligned} u_t &= u_{t1} + u_{t2} \\ &= -\left(m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - b_t\dot{x}\right) \\ &\quad + \left(m_t + m_c \sin^2\theta\right)\left(\ddot{x}_d - \lambda\dot{\tilde{x}} - \alpha\dot{\tilde{\theta}} - k\text{sign}(S)\right) \end{aligned} \quad (17)$$

Let's prove that with the control signal (17) will make the system stable. Propose the Lyapunov candidate function as:

$$V = \frac{1}{2}S^2 \quad (18)$$

Take time derivative (18), use (10), (11) and repalce the control signal (17), we obtain:

$$V = -kS.\text{sign}(S) \le 0 \quad (19)$$

Control signals chosen above ensure rejecting bounded disturbance but the signum function will generate chattering that will reduce effect of controller. Thus, we propose replace signum element by a saturation linear function defined as:

$$y = sat(x) <=> \begin{cases} y = -1 \ \ if \ \ x \le -1 \\ y = x \ \ if \ \ -1 < x \\ y = 1 \ \ if \ \ x \ge -1 \end{cases} \quad (20)$$

The control signals are written as:

$$\begin{aligned} u_t &= u_{t1} + u_{t2} \\ &= -\left(m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - b_t\dot{x}\right) \\ &\quad + \left(m_t + m_c \sin^2\theta\right)\left(\ddot{x}_d - \lambda\dot{\tilde{x}} - \alpha\dot{\tilde{\theta}} - k\text{sat}(S)\right) \end{aligned} \quad (21)$$

## IV. CONTROLER ASSOCIATES WITH ADAPTIVE LOAD MASS AND FRICTION FOR 2D OVERHEAD CRANE

In fact, there are a lot of impacts that affect to crane when it is operating. The case that need to change load mass or the significant influence of friction are quite frequently. Unfortunately, these parameter affect significantly to the performance of the system. With a conventional controller, the responces will not ensure the desired purpose. So that, an algorithm that adapted these parameter is necessary. In this section, base on SMC controller designed in previous section, we will propose an adaptive algorithm that associate with SMC technique to reduce the unexpected effect of uncertain friction coefficient and changing load mass.

### *A. SMC with adaptive uncertain coefficient friction algoritm*

Assume that the friction coefficient is uncertain, we need a algorithm to caculated approximate value of this parameter and ensure that the system will still be stable. An adjust law will be proposed to deal this proplem. The approximate friction coefficient is $\hat{b}_t$ , this parameter will be updated online. Define the error between the real signal and the caculated one is:

$$\tilde{b}_t = b_t - \hat{b}_t \quad (22)$$

Propose the update law of approximate value of friction coefficient as:

$$\dot{\hat{b}}_t = -\frac{S.\dot{x}}{\kappa\left(m_t + m_c \sin^2\theta\right)} \quad (23)$$

Let us demonstrate the stability of the system with thi proposed adaptive law. Firstly, rewite the control signal (21) with the approximate value, we have:

$$\hat{u}_t = -\left(m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - \hat{b}_t\dot{x}\right) + \left(m_t + m_c \sin^2\theta\right)\left(\ddot{x}_d - \lambda\dot{\tilde{x}} - \alpha\dot{\tilde{\theta}} - k\text{sat}(S)\right) \quad (24)$$

Propose the Lyapunov candidate function as:

$$V_b = \frac{1}{2}S^2 + \frac{1}{2}\kappa\tilde{b}_t^2 \quad (25)$$

Take time derivative (25), we obtain:

$$\dot{V}_b = S\left(\frac{m_c g \sin\theta\cos\theta + m_c l\dot{\theta}^2 \sin\theta - b_t\dot{x}}{m_t + m_c \sin^2\theta} + \frac{\hat{u}_t}{m_t + m_c \sin^2\theta} - \ddot{x}_d + \lambda\dot{\tilde{x}} + \alpha\dot{\tilde{\theta}}\right) - \kappa\tilde{b}_t\dot{\hat{b}}_t \quad (26)$$

Replace the control signal (24) and the update law (23) in (26), we obtain:

$$\dot{V}_b = -kS.\text{sat}(S) \le 0$$

(27)

The derivative of the candidate Lyapunov function is negative, as the theory Lyapunov stability, the system will be stable with control law (24) and friction coefficient adaptive law (23). Next, we will consider about the changes of load mass

### *B. SMC with adaptive load mass algorithm*

When applying this algorithm, the approximate value of the cargo mass will be updated online. Define the approximate values of the mass is $\hat{m}_c$, then, the error between the real value of the mass and the approximate one is:

$$\tilde{m}_c = m_c - \hat{m}_c \tag{28}$$

The law to update online the approximate value of mass is proposed as:

$$\begin{aligned}\dot{\hat{m}}_c &= g\sin\theta\cos\theta + l\dot{\theta}^2\sin\theta \\ &+ \sin^2\theta\left(-\ddot{x}_d + \lambda\dot{x} + \alpha\dot{\theta}\right)\end{aligned} \tag{29}$$

Now we demonstrate that the system will be stale with the adaptive law of mass as (29). Firstly, with approximate mass, the signal control (21) will be rewritten as follows:

$$\begin{aligned}\hat{u}_t &= -\left(\hat{m}_c g\sin\theta\cos\theta + \hat{m}_c l\dot{\theta}^2\sin\theta\right) \\ &+ \left(m_t + \hat{m}_c\sin^2\theta\right)\left(\ddot{x}_d - \lambda\dot{x} - \alpha\dot{\theta} - k\mathrm{sat}(S)\right)\end{aligned} \tag{30}$$

Propose the Lyapunov candidate function as:

$$V_c = \frac{1}{2}S^2 + \frac{1}{2}\tilde{m}_c^2 \tag{31}$$

Take time derivative of (31), we obtain:

$$\begin{aligned}\dot{V}_c = S\Bigg(&\frac{m_c g\sin\theta\cos\theta + m_c l\dot{\theta}^2\sin\theta}{m_t + m_c\sin^2\theta} \\ &+ \frac{\hat{u}_t}{m_t + m_c\sin^2\theta} - \ddot{x}_d + \lambda\dot{x} + \alpha\dot{\theta}\Bigg) - \tilde{m}_c\dot{\hat{m}}_c\end{aligned} \tag{32}$$

Using control signal in (30) to repalce in (32), we obtain:

$$\begin{aligned}\dot{V}_c = S\frac{\tilde{m}_c}{m_t + m_c\sin^2\theta}&\left(\sin^2\theta\left(-\ddot{x}_d + \lambda\dot{x} + \alpha\dot{\theta}\right) + l\dot{\theta}^2\sin\theta\right. \\ &\left. + g\sin\theta\cos\theta\right) - S\frac{m_t + \hat{m}_c\sin^2\theta}{m_t + m_c\sin^2\theta}k\mathrm{sat}(S) - \tilde{m}_c\dot{\hat{m}}_c\end{aligned}$$

Replace adaptive law (29) in above equation, we obtain:

$$\dot{V}_c = -S\frac{m_t + \hat{m}_c\sin^2\theta}{m_t + m_c\sin^2\theta}k\mathrm{sat}(S) \le 0 \tag{33}$$

The derivative of the candidate Lyapunov function is negative, as the theory Lyapunov stability, the system (6)will be stable with the control signal (30) and adaptive law (29), or another way, the position of trolley will be drive to the desired values with swing angle will reduce to zero. The next section will epress some simulations of the proposed controller.

## V. SIMULATION RESULTS

In this section, the performance of the proposed controller will be evaluated through a numerical simulations. In order to test the quality of the designed controller under the changes of loas mass, the mass will be set at two different values. The parameters of the the overhead crane system are chosen as: $m_t = 20kg; m_{co} = 1kg; l = 1m$ . The parameters used in simulation for the proposed controller are selected as: $k = 3; \alpha = -2; \lambda = 1$.

In the first case, the load mass is 10kg. It means that the real load mass is 10 times heavier than the parameter that set in the controller. The following simulation results will compare the system responses between the conventional SMC controller and SMC associated with adaptive load mass algorithm.The results of the proposed controller are shown in these following figures:

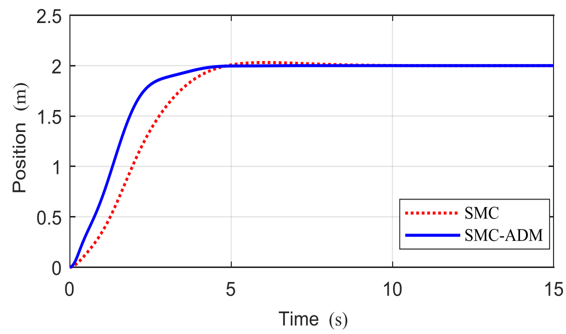


**Fig.2: The posision respones with load 10kg**

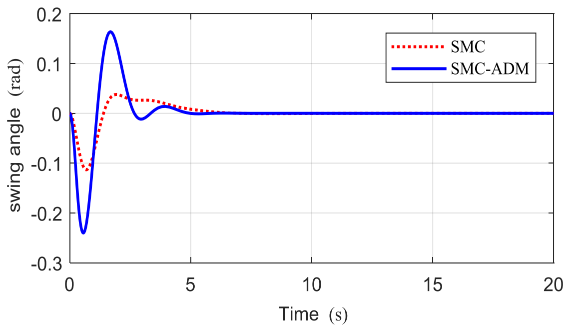


**Fig.3: The swing angle responses with load 10kg**

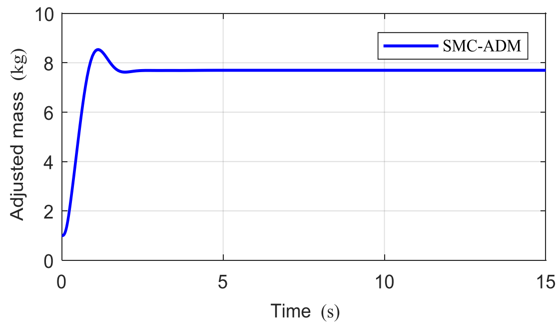


**Fig.4: The adjustation of mass though adaptive algorithm**

As the simulation results, the responses of the position and the swing angle are shown in figures 2;3, respectively. It can be seen that with the load mass is 10 times heavier than the initial, both the conventional controller and the mass adaptive controller will drive the system's states to the desired values. Because of SMC's anti-uncertainties property, the conventional SMC controller's responses still acceptable. But compare with the controller that associates to an mass adaptive algorithm, the performance with new controller is better distinctly. Both position and swing angle are stable in shorter time. However, the swing angle's amplitude fluctua-tion in case that apply adaptive controller is bigger than the conventional one. That is easy to understand because the settling time is reduced. This aspect is expessed clearly by figure 4. The computed mass increases from the initial and steady when the system was stable. These properties will be expessed more distinstly in case the load mass is 50 kg. The results of this case are shown in these following figures:

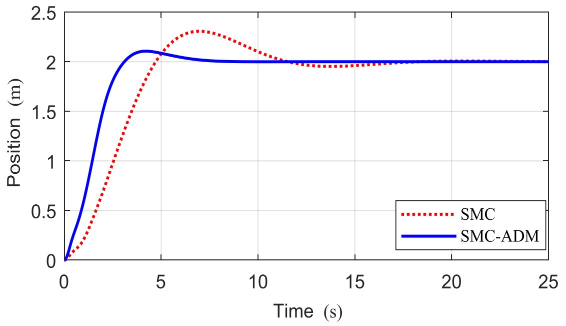


**Fig.5: The posision respones with load 50kg**

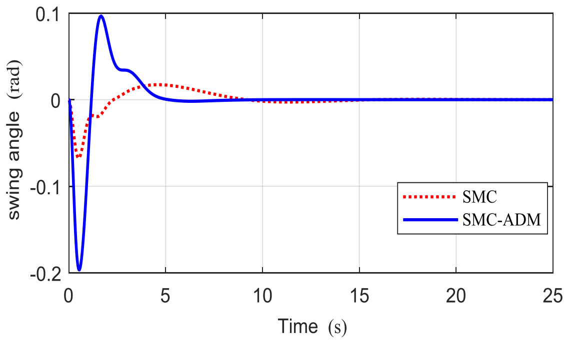


**Fig.6: The swing angle responses with load 50kg**

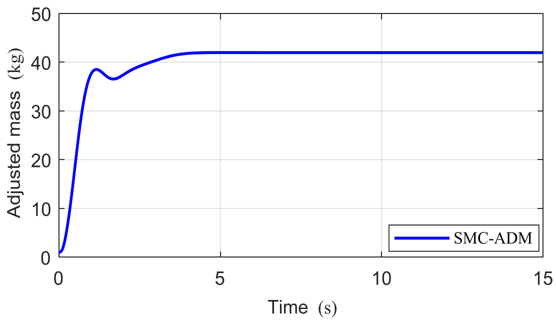


**Fig.7: The adjustation of mass though adaptive algorithm**

The advantages of new proposed controller are expressed distinstly in this case. When the mass is sinificantly big, the controller with mass adaptive algorithm had better performene (figure 5, 6). As previous comment, the adjustation of mass cause the different responces. Figure 7 showed the operation of load mass adaptive algorithm.

## VI. CONCLUSION

By using SMC technique and Lyapunov theory, the paper design an friction and load mass adaptive controller to adjust appropriately the load mass and the friction coefficient. The sliding mode control is employed for controller. Numerically, the control with adaptive algorithm is comparable with system's performance with conventional controller.

## ACKNOWLEDGEMENTS

Faculty of Electrical and Electrical Engineering, University of Transport and Communications, Vietnam, for supporting this work.

## REFERENCES

[1] Y. Fang, E. Zergeroglu, W. E. Dixon, and D. M. Dawson, "*Nonlinear coupling control laws for an overhead crane system*," in Proceedings of the 2001 IEEE International Conference on Control Applications (CCA'01)(Cat. No. 01CH37204), 2001, pp. 639–644.

[2] S.-G. Lee, V.-H. Dang, S. Moon, and B. Kim, "*Partial feedback linearization control of a three-dimensional overhead crane*," Int. J. Control. Autom. Syst., vol. 11, no. 4, pp. 718–727, 2013.

[3] M. Mahfouf, C. H. Kee, M. F. Abbod, and D. A. Linkens, "*Fuzzy logic-based anti-sway control design for overhead cranes*," Neural Comput. Appl., vol. 9, no. 1, pp. 38–43, 2000.

[4] K.-K. Shyu, C.-L. Jen, and L.-J. Shang, "*Design of sliding-mode controller for anti-swing control of overhead cranes*," in 31st Annual Conference of IEEE Industrial Electronics Society, 2005. IECON 2005., 2005, pp. 6-pp.

[5] T. A. Le, S.-G. Lee, and S.-C. Moon, "*Partial feedback linearization and sliding mode techniques for 2D crane control*," Trans. Inst. Meas. Control, vol. 36, no. 1, pp. 78–87, 2014.

[6] W. Wang, J. Yi, D. Zhao, and D. Liu, "*Design of a stable sliding-mode controller for a class of second-order underactuated systems*," IEE Proceedings-Control Theory Appl., vol. 151, no. 6, pp. 683–690, 2004.

[7] J. H. Yang and K. S. Yang, "*Adaptive coupling control for overhead crane systems*," Mechatronics, vol. 17, no. 2–3, pp. 143–152, 2007.

[8] M.-S. Park, D. Chwa, and S.-K. Hong, "*Antisway tracking control of overhead cranes with system uncertainty and actuator nonlinearity using an adaptive fuzzy sliding-mode control*," IEEE Trans. Ind. Electron., vol. 55, no. 11, pp. 3972–3984, 2008.

[9] J. A. Méndez, L. Acosta, L. Moreno, S. Torres, and G. N. Marichal, "*An application of a neural self-tuning controller to an overhead crane*," Neural Comput. Appl., vol. 8, no. 2, pp. 143–150, 1999.

[10] S.-G. Lee, L. C. Nho, and D. H. Kim, "*Model reference adaptive sliding mode control for three dimensional overhead cranes*," Int. J. Precis. Eng. Manuf., vol. 14, no. 8, pp. 1329–1338, 2013.

[11] Anumaka, M. C "*Development of Software for Power System and Cybercrime Monitoring*" International Journal of Engineering Trends and Technology 66.2 (2018)