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Original Article

Linear Programming Based Optimal Power Flow Optimization of DCOPF for an IEEE 5 and IEEE 14 Bus System

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Abstract - *In times of increasing industrialization and domestic utility requiring electricity, deregulated power or electricity concepts have been gaining widespread significance in recent times. Deregulated power signifies and reflects power transmission sectors/companies defining their own set of rules and regulations in an attempt to get relieved from a centralized control pattern, consequently leading to the improvisation of efficiency. However, deregulated power system experiences power instability issues at times of varying demands at the load side, which may be attributed to several reasons. Under such circumstances, power congestion along the transmission lines is observed, which is quite challenging. These further causes stress on the management of the deregulated power system concerning power distribution in the current competitive environment amongst various other transmission stakeholders. It has been taken as the problem formulation in this paper, and a Local Marginal Pricing (LMP) mechanism using a Linear Programming (LP) methodology has been proposed in this work. DC optimal power flow (DCOPF) concept has been taken as the base platform for the proposed LMP formulation. LMP is effectively used for assessing the pricing scheme of the different buses, while DCOPF aids in reducing the congestion effect due to varying peak loads. IEEE 5 and 14 bus system has been used in the proposed power flow analysis and pricing scheme. Superior performance is observed in the experimentation, justifying the validity of LP.*

Keywords - *Deregulated power system, Pricing schemes, DCOPF, Marginal Pricing, Linear programming.*

1. Introduction

Research in power generation and distribution has recently gained widespread significance. Concepts of Grids and micro-grids have added much-needed flavour to the hot research topic of power distribution and load balancing. Power Marked De-regulation is a crucial principle that dictates and binds together various entities involved right from the generation of power at the source, transmission through grid lines and right up to the point of power delivery to the end user [1]. Major stakeholders are involved in the process of buying and selling electricity from the centralized power generation system. Attractive pricing schemes of individual market players in the buying and selling electricity play a major role in improving their profits at the cost of providing better quality power to their consumers [2]. However, despite its numerous benefits, deregulated systems suffer from congestion along transmission lines due to a lack of coordination between various entities mentioned above from generation to distribution stages. In addition, congestion along the lines mainly occurs due to the restrictions and constraints placed on the transmission lines at the cost of pricing of the market players.

As a consequence, congestion of power leads to power instability, outages and instability of cost per unit consumption of the power [3]. Other factors leading to power congestion may be the imbalance of transmission line elements at the generation and delivery side of the power transmission unit. Sudden and unexpected patterns of load variations and their demand may pose significant overhead on power generation, causing congestion [4]. Mechanical failures in the form of equipment failure, technical faults and maintenance issues form the rest of the causes. A typical scenario of the deregulated power market is depicted in figure 1, shown below.

Figure 1 shows that cost and power flow go hand-in-hand in the deregulated power system market scenario. Quality power generated from the source and delivered to the customer ensures attractive cost benefits to the power generation company. Conventional methods to minimize power congestion would be to provide a limitation or constraint on the load demand [5]. However, an increasing amount of restriction would lead to losses incurred on the generation side as most of the power demand would not be serviced to despite power being abundant in supply. Another method is to utilize a flexible alternating current system (FACTS) which helps power conditioning and improves the transmission capability [26]. However, FACTS devices require huge initial costs and are quite expensive in the overall scenario, which may not best suit the deregulated power market.

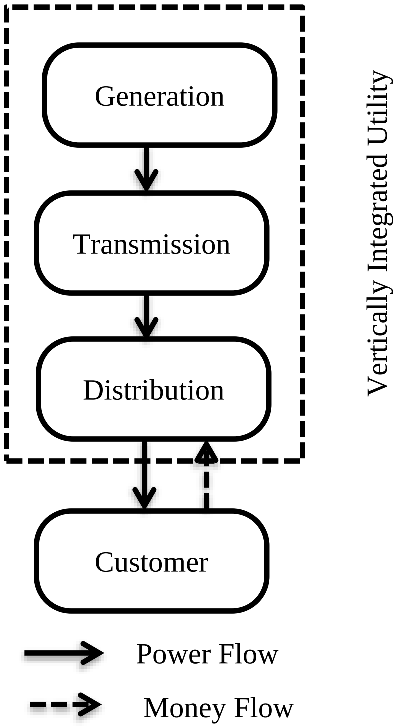

Fig. 1 A typical cost-power flow concept in the deregulated power market

On the other hand, an important attribute closely related to congestion and the market scenario of power distribution is Locational Marginal Pricing (LMP) [7] which projects the real-time electricity cost that varies region-wise. This region-wise variation is largely attributed to the fact that different regions may be characterized by different load patterns and their associated demands. It causes fluctuations in the congestion of power and the type of infrastructure used in the particular region's interest. The factors mentioned above determine the actual time price of electricity available for buying or selling [8]. Locational Marginal Pricing is a benchmark cost or pricing mechanism in the wholesale electricity market where different individual players stake their claim of buying at their own deregulated schemes. Internally, locational marginal pricing is formulated based on three attributes, namely Cost of Generation (CoG), Cost of Congestion (CoC) and Cost of Losses (CoL). They are formulated based on power flow analysis, either AC or DC, and accordingly termed AC optimal power flow (ACOPF) or DC optimal power flow (DCOPF) [9] [10]. The term optimal is introduced in the nomenclature to indicate the concept of optimization being done to accommodate various parameters and, at the same to provide a nominal cost for peak power transfer. On a comparative scale, ACOPF methods record high accuracy but at the cost of increased execution time, which is not cost-wise effective [11]. Hence, most locational marginal pricing methods employ the latter power flow analysis model for pricing computation.

In general, Locational Marginal Pricing is formulated using the optimal solution problem to find the best possible solution for power transfer from source to destination, considering the losses and congestion factors. A literature review also indicates obtaining solutions for a smooth power flow without using optimal solution-based approaches. This has been done by considering a classic 'copper plate' based approach where current flows from one end to another without any obstruction or consideration of any losses. However, this becomes entirely unsuitable for real-time implementation in distribution networks as considerable and significant losses and congestion, as mentioned above, problems exist. Hence, congestion-related losses and system parameters are considered critical attributes when formulating the optimal solution problem [12]. Amongst the two approaches, DCOPF registers reduced accuracy levels due to inadequacy in reactive power compensation in the system. In such cases, optimal flow solutions provide the optimal balance in power transfer from generation to consumption. Using this solution, the reactive power, which cannot compensate, is considered an imaginary factor in the load-balancing process. Many load-balancing problems are observed in the literature, of which many notable methods are iterative-based, which is indicative of the absence of instant solutions. One of the prominent methods is the Gauss-Seidel approach. This method is effectively used for load-balancing large power systems and is quite simple in construction and operation. However, a slow convergence rate is an observed limitation. In order to formulate the power flow analysis problem, fundamental insight into essential terminologies like Bus, Node etc. is required, which is provided in the sub-section shown below

1.1. Insight into Terminologies

Bus is the basic terminology for any power system or network. A typical bus system is depicted in figure 2, shown below.

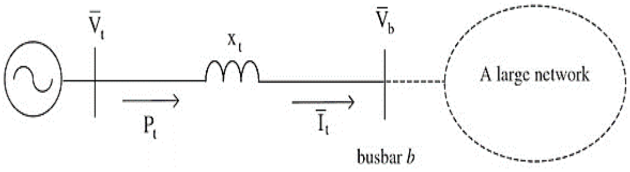

Fig. 2 A typical Bus scenario in power systems

The bus can be alternately defined as a node or junction where many sources (generators) or sinks (loads) are connected. A typical bus system is characterized by an equivalent network with four primary elements, namely Real Power (P), Reactive Power (Q), Bus Voltage (V) and Bus Voltage Angle (γ). The latter two terms constitute the bus voltage profile of a bus network. Based on their profile, they are classified into three major categories: Load Bus, Voltage Bus and Slack Bus, as shown in figure 3. The classification is based mainly on the point that, at any given point of time, not all of the four parameters, namely {P, Q, V, } will be specified in the network under consideration. Any two profiles may be specified, and the remainder must be computed accordingly.

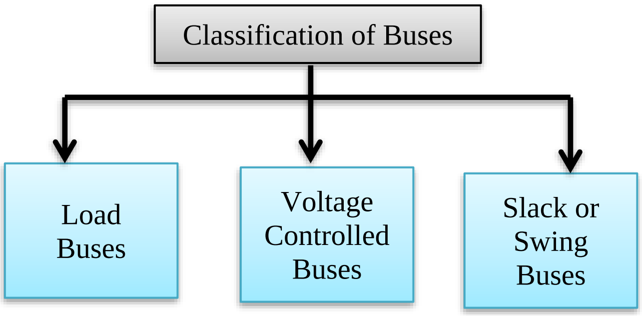

Fig. 3 Classification of a typical Bus system

In the first category, namely, the load bus, values of P and Q are given while the voltage profile is to be computed for the given power flow analysis. In the second class, namely, voltage-controlled bus, the voltage magnitude (V) and the real power magnitude (P) are specified while one needs to compute γ, and Q. In the last case, namely, Slack or alternately termed as Swing Bus, the voltage profile is specified, and the objective lies in computing P & Q. It could be termed as opposite of the load bus.

2. Related Work

An exhaustive survey of literature related to various methodologies adopted by researchers has been carried out, and essential findings are projected in this section.

Uncertainty of network topology has been taken as the primary constraint in the computation of LMP [13]. PJM – 5 networks have been taken as the study environment, and the reliability of networks has been computed and used as an essential attribute in applying optimal flow problem formulation. Experimentation has been done on a DC-based power flow system using LP methodology. A 95MW generation without uncertainty consideration is observed against an improved 120MW generation after considering the uncertainty factor. Reliability is computed using uncertainty modelled using the unit and line outages, and the cost incurred during these outages has been taken as target objectives in the optimization problem. This work is an important finding to ascertain that network uncertainty is an important parameter and has to be an integral part of the LMP computation process. A Fictitious Nodal Demand (FND) approach has been proposed in the literature [14], where applying an iterative procedure minimizes the non-linear marginal loss. Limitations in previous methods involving a large mismatch at the reference end are eliminated in this work by application of the proposed nodal demand model. The FND model reduces the accumulation of marginal losses at the reference end by distributing the losses amongst the several individual lines in the network.

Minimizing generation cost is taken as the objective function in the works of [15] observed in the literature. The Indian power sector has been taken as the case study. Generation failure is taken as the crucial parameter in the LMP scheme. A distribution-based LMP computation has been proposed in the literature [16] using three specialized power analysis tools: linear power flow distribution (LPF), tools for loss factor in the distribution network and optimized LPF. High yield compared to ACOPF and high accuracy results tested over a wide range of bus systems are significant findings of this research work. A variant of the distribution mechanism-based LMP has been proposed in the literature [17] to address the voltage profile management and the conventionally existing congestion problem in the transmission line. Optimization models are formulated in the proposed distributed framework derived from the Lagrangian function to quote the pricing based on optimal congestion management. Loss impact on the pricing [18] has been taken as a crucial parameter in the formulation of optimal power flow methodology for LMP computation by working on the Karush- Kuhn – Tucker (KKT) condition existing in the conventionally existing OPFs.

Since the computation of LMP is based on OPF, conventional existing optimization algorithms and nature inspired algorithms are quite apt for obtaining the convergence quickly. A genetic algorithm-based secure economic despatch method [27] has been proposed in the literature to minimize the total fuel cost. The experimentations have been conducted on well-known IEEE 14 and 75 bus systems. A comparative analysis is done by including and excluding the losses. Utilization of well-known and powerful GA-based LMP improves accuracy and aids in quick convergence towards the optimal solution. At the same time, LMPS computation based on decomposition methodology [20] has been investigated to compare the convergence rates and accuracy analysis in quoting the economic despatch or pricing based on constraints on the generation mechanism. Unlike the conventional and simplified formulation adopted for DCOPF, the problem formulation in ACOPF is quite sophisticated as it involves establishing and correlating a strong dependence between the congestion constraints and the pricing variation of individual bus lines [21]. This decomposition-based LMP computation method strongly depends on Financial Transmission Rights (FTR) and requires its hourly values for predicting the price.

Sensitivity [22] has been taken as yet another essential attribute in the computation of LMP and is reflected as perturbations nearing optimal solutions [23]. In such cases, a marginal unit generation methodology has been applied for sensitivity-based LMP computation and prediction of congestion in order to take precautionary measures. Predictors find quite a use in such algorithms [24]. Whilst the least research contributions are made in literature for LMP computation for slack bus category, [Wu] has provided formulations for LMP computations, which are proving to be an eye-opener in the proposed work. Nodal loss-based sensitivity parameters [25] have also been found to be critical constraints in LMP formulation.

The above literature survey has been quite useful in formulating the OPF for the proposed LMP calculation of the IEEE 5 and 14 bus system. The following findings have been observed from the survey.

- Linear programming models are best suited for LMP computation due to their adaptability to the problem formulation of cost prediction based on resource constraints
- DCOPF methods are more apt for LMP formulations of bus systems as their convergence times are quick and mostly adopted in literature due to their simplicity
- OPF methods attract the utility of nature-inspired optimization methods for predicting economic despatch.
- Multiple parameters influence the LMP process, out of which losses, outages, and sensitivity are found to be essential parameters of interest.

3. Methodology

In this research paper, the computation of LMP for an IEEE 5 and 14 bus system is taken as the primary objective. Based on literature studies, the optimal power flow method is selected over the copper plate methodology since the network under study is not ideal. In the OPF model, the problem narrows down to determine the pricing of the individual lines of the proposed IEEE 5 and 14 bus model.

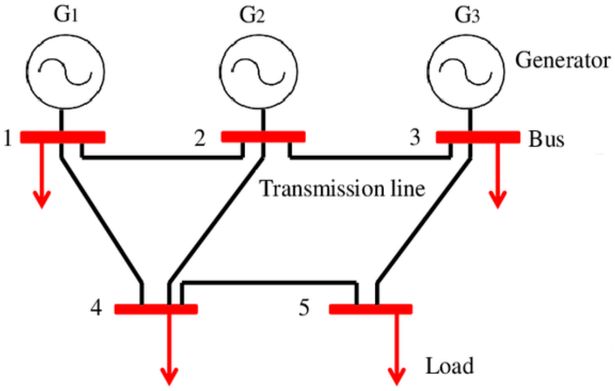

Fig. 4 A typical IEEE 5 bus power system (Zawani *et al.*)

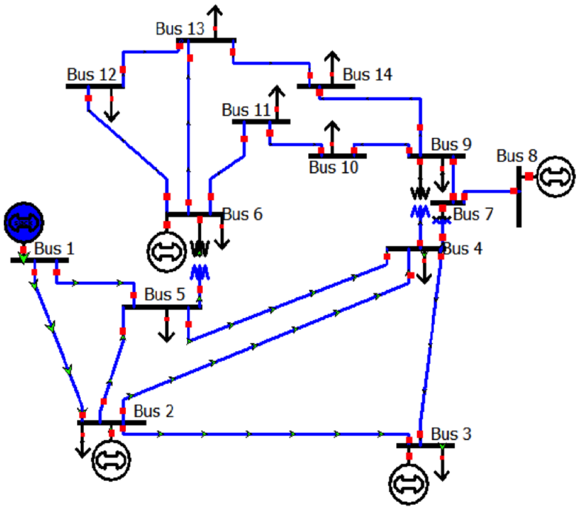

Fig. 5 A typical IEEE 14 bus power system (Courtesy: ICSEG)

The pricing depends on multiple parameters out of which congestion and losses are identified as key parameters of interest. A DCOPF model is selected for the proposed computation due to its merits mentioned in previous studies. The terminology of *per unit* (p.u) is utilized, and the following assumptions are made before initializing the OPF.

- V (magnitude of voltage) of the bus lines are considered to be 1pu.
- Losses are ignored.
- Reactive power (Q) is ignored.

Conventionally framed OPF models based on the above three conditions equate to an ideal transmission line as both the losses and Q factors are neglected. However, in the real-time scenario, such a condition of ignoring loss is impossible, which is the research challenge in the proposed OPF. If, for the given network, C_i is taken to be the cost of ith Generator unit (g) and 'n' being the total number of generators in the bus system under study, the OPF is formulated as a minimization function presented as

$$Min\ \sum_{i=1}^{n} C_i\, P_{gi} \tag{1}$$

Subject to the generation limit constraint given by

$$P_{gi}{}^{min} \le P_{gi} \le P_{gi}{}^{max} \tag{2}$$

and the transmission line limit is given by

$$I_f{}^{min} \le I_f \le I_f{}^{max} \tag{3}$$

In (3), I_f denotes the current flow with associated minimum and maximum values, namely 'min' and 'max'. Generation shift factor (GSF) is a crucial metric which is the ratio of incremental change in power flow on the transmission line to the change in the generation on ith line of the bus. If B_t and B_r are considered to be the sending and receiving end bus of the kth transmission line, then GSF could be formulated as

$$GSFM_{k-i} = ({B_{t,i}}^{-1} \text{-} {B_{s,i}}^{-1}) / X_k \tag{4}$$

From the above definition of influence of B, Power injection at a node and voltage angles are given as

$$P_i = \sum_{j=1}^{n} X_{ij}(\ \theta_i - \theta_j) \tag{5}$$

Xij – Reactance between bus i and bus j
Power flow on the transmission line is given by the equation

$$(6P_{Li} = \frac{1}{X_{Li}}(\theta_s - \theta_r) \tag{6}$$

Another important component to be considered in LMP formulation is the accountability of system loss which is given as

$$Total\ LMP = E_{marg} + C_{marg} + L_{marg} \tag{7}$$

In equation (7), E_{marg} denotes the marginal energy cost value, C_{marg} reflects the congestion cost and L_{marg} denotes the loss cost. LMP computation is projected in figure 6.

4. Results and Analysis

OPF is taken as the primary objective of this research work. Optimization has been applied through the Linear Programming approach, which best suits the minimization criteria of the proposed problem formulation. The implementation has been carried out as per figure 6 flow process in a MATLAB environment on an IEEE 5 and 14 bus system. The processor configuration is Intel Core – i3 with 2GB RAM capacity running at a speed of 2.20GHz. The experimentations for the optimization of the bus system have been carried out individually on the two networks, and observations are listed categorically. The projected information sequence involves the line data and generator data, which are inputs for the formulation of admittance matrices followed by GSF computation. From GSF formulation, LP is applied to obtain the desired LMP values for the proposed bus system. IEEE 14 bus system has been treated as case 1, and IEEE 5 bus system has been treated as case 2.

4.1. Case 1. IEEE 14 Bus System

IEEE 14 bus system consists of 20 lines and 2 generators. Line and generator data "s are used for the simulation work. Generator data for IEEE 14 bus system is given in the Table.

Table 2 represents the Line data of the IEEE 14 Bus system. A case study has been performed on the IEEE 14 bus system to assess the benefits of the loss distribution matrix.

LMP is calculated using DCOPF for the IEEE 14 bus system and presented in Table 3. LMP is calculated without a loss for the IEEE 14 bus system but with congestion. It is created by reducing the line power flow upper limit.

4.2. Case 2. IEEE 5 Bus System

IEEE 5 bus system consists of 7 lines and 2 generators. Table 4 shows the line data for IEEE 5 bus system. Table 5 shows the LMP values for IEEE 5 bus system.

5. Conclusion

Locational Marginal Pricing (LMP) is one of the emerging trends of research in recent times, with ever-increasing competitiveness in the wholesale electricity market. With de-regulation schemes in place, LMP plays a critical role in pricing while balancing the congestion and power instability issues in conventional systems. An LMP computation has been proposed in this research work and implemented on an IEEE 5 and 14 bus power network using an optimal power flow approach. Linear programming has been applied to optimization problems to achieve the required convergence. The experimentation has been tested in the presence of congestion taken as a constraint on the power distribution system.

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Annexure

Table 1. Generator data

Generator	$P_i min$ MW	$P_i max$ MW	a_i	b_i	c_i
1	10	600	1	0	600
2	20	200	3	0	200

Table 2. Line data

Sending end bus	Receiving end bus	Resistance p.u	Reactance p.u	Half Susceptance p.u	Tab transformer	Line limits
1	2	0.19	0.59	0.026	1	200
2	3	0.047	0.2	0.02	1	100
2	4	0.06	0.18	0.019	1	100
1	5	0.054	0.22	0.025	1	100
2	5	0.057	0.17	0.017	1	100
3	4	0.07	0.17	0.0173	1	50
4	5	0.013	0.042	0.0064	1	100
5	6	0	0.252	0	0.932	50
4	7	0	0.21	0	0.978	50
7	8	0	0.18	0	1	100
4	9	0	0.56	0	0.969	50
7	9	0	0.11	0	1	20
9	10	0.32	0.085	0	1	50
6	11	0.095	0.199	0	1	50
6	12	0.123	0.256	0	1	50
6	13	0.07	0.13	0	1	20
9	14	0.13	0.27	0	1	20
10	11	0.082	0.192	0	1	20
12	13	0.22	0.199	0	1	20
13	14	0.171	0.348	0	1	20

Table 3. LMP values for IEEE 14 bus system

Bus No.	LMP ($/M Whr
1	10
2	9.914321
3	9.995891
4	10.06522
5	10.03195
6	10.04325
7	10.05923
8	10.05923
9	10.05609
10	10.0538
11	10.04862
12	10.04426
13	10.04505
14	10.05127

Table 4. Line data for IEEE 5 bus system

Sending end bus	Receiving end bus	Resistance p.u	Reactance p.u	half Susceptance p.u	Tab transformer	line limits
1	2	0.02	0.06	0.026	1	200
1	3	0.08	0.24	0.02	1	100
2	3	0.06	0.25	0.019	1	100
2	4	0.06	0.18	0.025	1	100
2	5	0.04	0.12	0.017	1	100
3	4	0.01	0.03	0.0173	1	50
4	5	0.08	0.24	0.0064	1	100

Table 5. LMP values for IEEE 5 bus system

Bus No.	LMP ($/MWhr)
1	10
2	9.938776
3	10.2449
4	10.18367
5	10.02041