SSRG International Journal of Electrical and Electronics Engineering
ISSN: 2348-8379/ https://doi.org/10.14445/23488379/IJEEE-V10I12P103

Volume 10 Issue 12, 18-26, December 2023

Original Article

Towards a Novel Hybrid Fuzzy Logic-Based Control Strategy to An Inverted Pendulum System

Quang-Hung Do[1], Ngoc-Khoat Nguyen[2]

[1]*Control, Automation in Production and Improvement of Technology Institute, Academy of Military Science and Technology, Hanoi, Vietnam.*
[2]*Faculty of Control and Automation, Electric Power University, Hanoi, Vietnam.*

[2]*Corresponding Author : khoatnn@epu.edu.vn*

Received: 03 October 2023 Revised: 05 November 2023 Accepted: 04 December 2023 Published: 23 December 2023

***Abstract** - The current study proposes a new solution with a successful control method to balance an inverted pendulum placed on a cart. The control plant is a nonlinear drive system consisting of a freely rotating rod and a small moving 4-wheel vehicle. A novel control strategy is created as a reasonable integration between a fuzzy logic structure built up depending on the Lyapunov stability theory and the Particle Swarm Optimization (PSO) algorithm. The Lyapunov theory has long been known for designing effectual control schemes. Meanwhile, the PSO mechanism, one of the most famous and efficient optimization methods, determines three scaling factors in the control diagram. Various simulation results of the proposed hybrid fuzzy logic controller in four scenarios have been successfully obtained using MATLAB/Simulink software, outperforming several existing counterparts, namely conventional PID, PD- and PI-like fuzzy logic regulators. Promising findings in this study verify the applicability of the proposed control methodology in both theoretical and practical aspects.*

***Keywords** - IPBCS, Fuzzy logic, Lyapunov theory, PSO, Hybrid controller.*

1. Introduction

The control of an inverted pendulum balancing system for various desired positions of the pendulum bar involves harmonizing two regulating aspects: the pendulum rod and the four-wheeled moving vehicle [1-5]. The control method proposed in this study proves valuable for managing diverse nonlinear systems, such as robots and rocket propulsion [6, 7]. Traditionally, the Inverted Pendulum Balancing Control System (IPBCS) has been frequently employed to validate numerous significant control methods [8-11].

In recent years, several increasingly efficient controllers have been built upon fuzzy logic techniques, relying on conventional regulators like Proportional-Integral (PI), Proportional–Derivative (PD), and Proportional-Integral-Derivative (PID). For variable-timing systems with multiple factors featuring nonlinearities and uncertainties, fuzzy logic controllers with an inference system based on operational engineers' or experts' knowledge and expertise are more suitable than traditional counterparts such as PI, PD, and PID. The set of fuzzy rules draws on the experience and understanding of operational experts and testers accumulated over time to manage system operations in alignment with desired goals. Understanding the system enables us to operate and control fuzzy logic-based systems most effectively [12-17].

This research applies fuzzy logic based on the Lyapunov stability theory and the Particle Swarm Optimization (PSO) algorithm to design an intelligent controller for balancing the inverted pendulum system. As the inverted pendulum system is a nonlinear Multiple-Input and Multiple-Output (MIMO) system, the balancing control problem must be tailored to the system, utilizing the Lyapunov stabilization method to determine the fuzzy logic controller parameters. Technically, the fuzzy logic set comprises two inputs: the output feedback response and the output feedback derivative. This set produces three scaling factors which directly influence the transient and stability capabilities of the system. Three meaningful coefficients are determined using an appropriate optimal method. Metaheuristic optimization techniques such as Genetic Algorithm (GA) and PSO mechanism can successfully determine these three parameters [10]. The current study selects the PSO mechanism for this task due to its simplicity and effectiveness.

The PSO algorithm operates by setting an initial value for the system and then assigning elements to search for the best parameters within a given interval. The search is repeated in several sets, updating the values until the set condition is satisfied. Finally, the best parameters that meet the terminating requirements can be obtained. These optimal factors can be used to design an efficient control methodology.

The subsequent sections of the current paper are organized as follows. After the Introduction section, section 2 introduces the model and formula of the driven inverted pendulum system. This is followed by Section 3, which discusses the controllers used and outlines how to construct a dimming controller based on Lyapunov's stability theory. A novel hybrid control strategy is also presented in this section. Section 4 introduces numerous scenarios of numerical simulation to validate the proposed controller. Finally, Section 5 provides conclusions and outlines future work inspired by this study.

2. Modelling the Inverted Pendulum System

The inverted pendulum model is mounted on a cart that can move along a rail, as shown in Figure 1. It is supposed to ignore friction between the cart, the road, and the cart and the pendulum. The parameters of the system, as indicated in Table 5 of the Appendix, include M - the mass of the vehicle; m - the group of the rod; l - length of the rod; x(t) - location of the vehicle; θ(t) - rotational angle joined by the rod to the vertical plane; u(t) - the driving force.

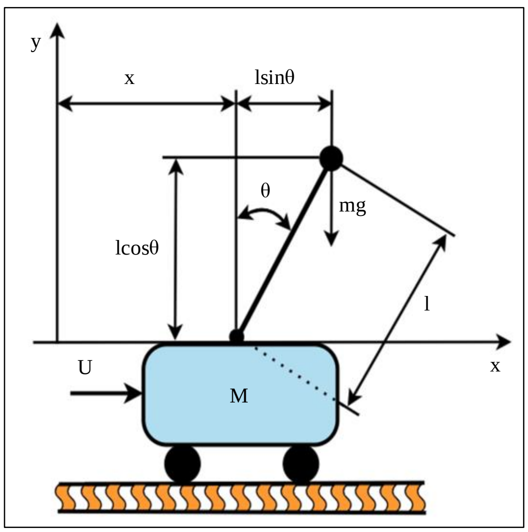


Fig. 1 A typical cart-mounted inverted pendulum model

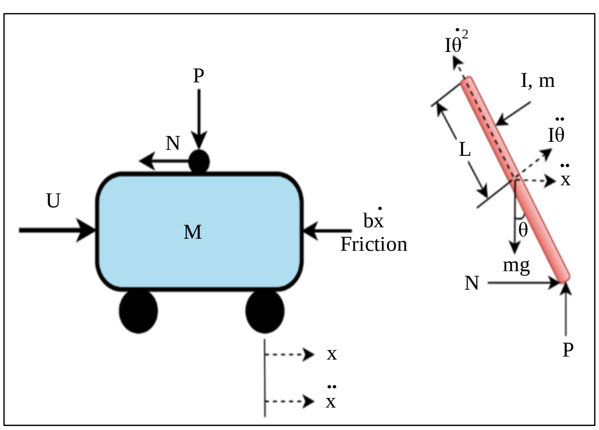


Fig. 2 A diagram of the forces acting on the inverted pendulum system

It is necessary to describe the dynamic characteristics of the inverted pendulum system based on Newton's laws of motion. Mechanical systems have two axes: the movement of the pendulum car above the x-axis and the rotation of the pendulum rod on the xy plane. Analyzing the dynamics of the inverted pendulum system, a diagram of the forces acting on the cart and the pendulum bar is plotted in Figure 2.

It is possible to sum up the forces vertically. Still, they are not helpful because the motion of the inverted pendulum system does not move in this direction, and the Earth's gravity is in balance with all the vertical forces. Hence, it is reasonable to establish a system of equations describing the nonlinear kinematic properties of the inverted pendulum system based on the *x*-axis as follows:

$$\left(M+m\right)\ddot{x}+b\dot{x}+ml\ddot{\theta}-ml\dot{\theta}^{2}\sin\theta=u \tag{1}$$

$$\left(J+ml^{2}\right)\ddot{\theta}+m\lg\sin\theta=-ml\ddot{x}\cos\theta \tag{2}$$

The above two equations can be transformed as:

$$\ddot{x}=\frac{u-b\dot{x}-ml\ddot{\theta}\cos\theta+ml\dot{\theta}^{2}\sin\theta}{M+m} \tag{3}$$

$$\ddot{\theta}=\frac{-ml\ddot{x}\cos\theta-m\lg\sin\theta}{J+ml^{2}} \tag{4}$$

From (1)-(4), the mathematical equations of the nonlinear inverted pendulum are obtained below:

$$\ddot{x}=\frac{\left(J+ml^{2}\right)\left(u-b\dot{x}-ml\dot{\theta}^{2}\sin\theta\cos\theta\right)}{\left(J+ml^{2}\right)\left(M+m\right)-m^{2}l^{2}(\cos\theta)^{2}}+\frac{m^{2}l^{2}g\cos\theta\sin\theta}{\left(J+ml^{2}\right)\left(M+m\right)-m^{2}l^{2}(\cos\theta)^{2}} \tag{5}$$

$$\ddot{\theta}=\frac{ml\left(b\dot{x}\cos\theta-u\cos\theta-ml(\cos\theta\sin\theta)\dot{\theta}^{2}\right)}{\left(J+ml^{2}\right)\left(M+m\right)-m^{2}l^{2}(\cos\theta)^{2}}+\frac{ml(M+m)g\sin\theta}{\left(J+ml^{2}\right)\left(M+m\right)-m^{2}l^{2}(\cos\theta)^{2}} \tag{6}$$

To simplify the system of ignoring the mass to be shaken, the linear mathematical model of the inverted pendulum subsystem is defined as follows:

$$\ddot{x}=\frac{u+ml\dot{\theta}^{2}\sin\theta-mg\cos\theta\sin\theta}{M+m-m(\cos\theta)^{2}} \tag{7}$$

$$\ddot{\theta}=\frac{u\cos\theta-(M+m)g(\sin\theta)+ml(\cos\theta\sin\theta)\dot{\theta}^{2}}{ml(\cos\theta)^{2}-(M+m)l} \tag{8}$$

Two equations expressed in (7) and (8) can be used to design control schemes of an IPBCS.

3. The Controllers Used for the IPBCS

3.1. Traditional PID Controller

It should not be denied that PID controllers have been widely used in control systems, especially in factories. The PID controller calculates the error value as the difference between the variable parameter and the desired setpoint. Such a PID controller aims to minimize this error by adjusting the input value. The PID controller's calculation relies on three separate parameters: Proportional P, Integral I and Derivative D. The typical formula representing the operation principle of the PID with three scaling factors P, I and D is as follows:

$$u(t) = \underbrace{P.e(t)}_{\text{Proportional}} + \underbrace{I.\int e(t)dt}_{\text{Integral}} + \underbrace{D.\frac{de(t)}{dt}}_{\text{Derivative}} \qquad (9)$$

3.2. Traditional Fuzzy Logic Controllers

Traditional PID controllers are gradually replaced by more advanced control strategies such as fuzzy logic and neural networks. In fact, intelligent controllers such as fuzzy logic have a dominant advantage in that they are designed relying almost exclusively on expert experiences and less on the exact mathematical model of the control object. This feature is highly significant in developing control schemes of nonlinear and uncertain systems, which are popular in reality. Theoretically, the structures of fuzzy controllers are highly diverse. Some fuzzy controllers are built based on the principles of traditional controllers like PI, PD, or PID. Figures 3 and 4 depict the structures of PI and PD fuzzy-based controllers, respectively.

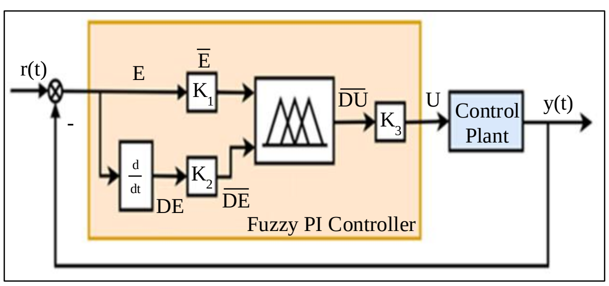


Fig. 3 Fuzzy PI controller

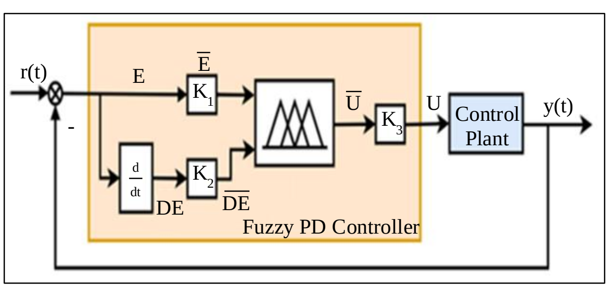


Fig. 4 Fuzzy PD controller

Table 1. Fuzzy logic law for PI/PD – like fuzzy logic controllers

	NB	**NM**	**NS**	**ZE**	**PS**	**PM**	**PB**
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NM	NM	NM	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PS
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	NS	NS	ZE	PS	PM	PM	PB
PM	NS	ZE	PS	PM	PM	PM	PB
PB	ZE	PS	PM	PB	PB	PB	PB

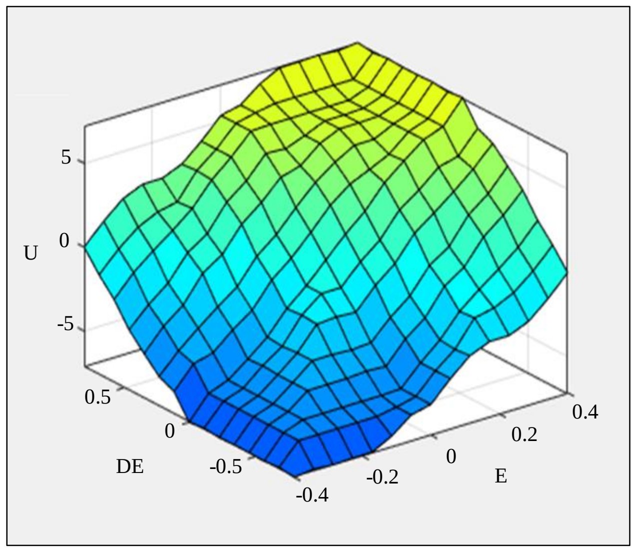


Fig. 5 A 3D illustration corresponding to the fuzzy rule base in Table 1

Theoretically, these controllers' input/output relationship adheres to the classic PI or PD control laws. This means that corresponding to traditional PI or PD controllers, the P and I or P and D coefficients will be adjusted by fuzzy logic without the need for the mathematical model of the control plant. Additionally, these tuning parameters will continuously change depending on the error e(t) rather than being fixed as in the corresponding classic controllers (see Figures 3 and 4).

Fuzzy control laws applied to the PI or PD controllers are also quite diverse, depending on each problem. Fuzzy rules can be constructed in 3x3, 5x5, 7x7 or more. This study will compare the last type of fuzzy rule (7x7) mentioned in Table 1 with the proposed fuzzy logic algorithm. The 3D-illustration corresponding to this rule base is depicted in Figure 5. It can be seen that the fuzzy rule and this 3D graph are entirely symmetrical, similar to the position of the inverted pendulum, which is symmetrical about the vertical axis.

3.3. Propose Lyapunov Theory-Based Fuzzy Logic Controllers

From the linear mathematical model of the inverted pendulum subsystem Equations (7) and (8).

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$ we get the equations of state:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{u\cos x_1 - (M+m)g(\sin x_1) + ml(\cos x_1 \sin x_1)(x_2)^2}{ml(\cos x_1)^2 - (M+m)l} \end{cases} \quad (10)$$

Select the semi-positively definite Lyapunov function as:

$$V = \frac{1}{2}\left(x_1^2 + x_2^2\right) \quad (11)$$

Take the derivative of the Lyapunov function given in (11), one can be obtained below:

$$\begin{aligned} \dot{V} &= x_1\dot{x}_1 + x_2\dot{x}_2 \\ &= x_1x_2 + x_2\frac{u\cos x_1 - (M+m)g(\sin x_1)}{ml(\cos x_1)^2 - (M+m)l} \\ &\quad + \frac{ml(\cos x_1 \sin x_1)(x_2)^2}{ml(\cos x_1)^2 - (M+m)l} \\ &= x_2\left[\frac{x_1ml(\cos x_1)^2 - x_1(M+m)l}{ml(\cos x_1)^2 - (M+m)l}\right] \\ &\quad + x_2\left[\frac{u\cos x_1 - (M+m)g(\sin x_1)}{ml(\cos x_1)^2 - (M+m)l}\right] \\ &\quad + x_2\left[\frac{ml(\cos x_1 \sin x_1)(x_2)^2}{ml(\cos x_1)^2 - (M+m)l}\right] \end{aligned} \quad (12)$$

According to Lyapunov's theory, for the system to be stable, it is mandatory to choose u(t) such that $\dot{V} \le 0$. Since the denominator is negative, it is recommended that to obtain $\dot{V} \le 0$ the control signal u(t) must be selected so that the numerator is greater than or equal to zero. It is easy to see that u(t) is chosen to satisfy the stability condition in three cases below:

- If $x_2 > 0$ then:

$$u > \frac{(M+m)\left[x_1 l + g(\sin x_1)\right] - x_1 ml(\cos x_1)^2}{\cos x_1} - \frac{ml(\cos x_1 \sin x_1)(x_2)^2}{\cos x_1} \quad (13)$$

- If $x_2 < 0$ then:

$$u < \frac{(M+m)\left[x_1 l + g(\sin x_1)\right] - x_1 ml(\cos x_1)^2}{\cos x_1} - \frac{ml(\cos x_1 \sin x_1)(x_2)^2}{\cos x_1} \quad (14)$$

- If $x_2 = 0$ then:

$$u = \frac{(M+m)\left[x_1 l + g(\sin x_1)\right] - x_1 ml(\cos x_1)^2}{\cos x_1} \quad (15)$$

Obviously, it is necessary to select the values of u(t) satisfying conditions (13), (14) and (15) at characteristic points (x_1, x_2), as shown in Table 2.

Table 2. Select control signal values u based on equations (13) – (15)

Characteristic Points		Stability Characteristics	Select the Control Signal u
x_1	x_2		
-0.4	-0.8	u < -6.0653	-8.0
-0.4	0.0	u = -5.9935	-6.0
-0.4	0.8	u > -5.9217	-4.2
0.0	-0.8	$u < 0$	-1.8
0.0	0.0	$u = 0$	0.0
0.0	0.8	$u > 0$	1.8
0.4	-0.8	u < 6.0653	4.2
0.4	0.0	u = 5.9935	6.0
0.4	0.8	u > 5.9217	8.0

From the selected values in (Table 2), a Sugeno fuzzy logic inference will be chosen in this study. The two inputs are x_1(t) and x_2(t). The output of the controller is u(t). The fuzzy structure is successfully proposed by choosing the SUM – PROD inference method and the weighted-mean fuzzy logic method. The membership functions and fuzzy logic rule base can be illustrated in Figure 6 and Table 3.

Table 3. A simplified fuzzy logic rule base applied for the proposed hybrid controller

u		x_2		
		NE	ZE	PO
x_1	NE	-8.00	-6.00	-4.20
	ZE	-1.80	0.00	1.80
	PO	4.20	6.00	8.00

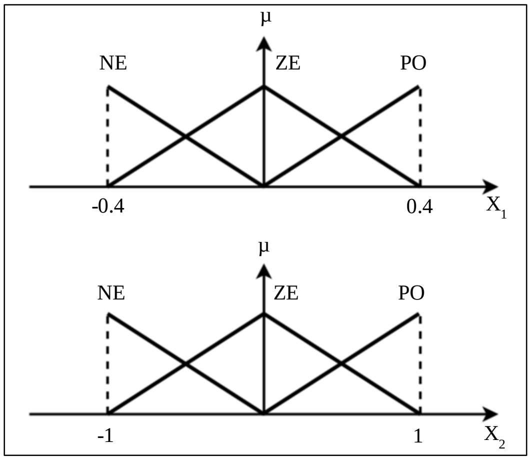


Fig. 6 Membership functions of the proposed fuzzy logic structure

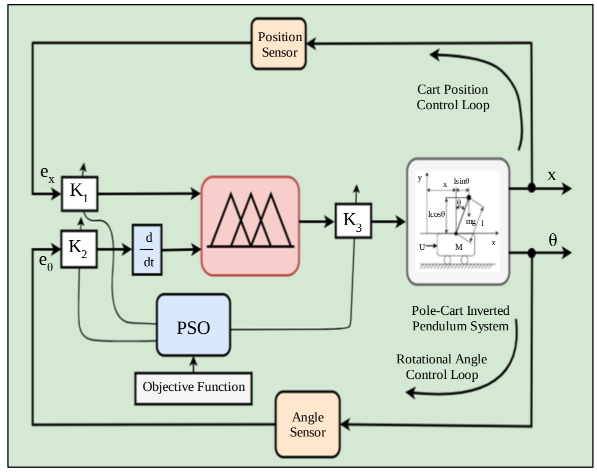


Fig. 7 The proposed control diagram for the IPBCS

Remember that these illustrations are highly simplified. There are only three levels for each membership function set. Meanwhile, this work applies nine fuzzy logic rules to balance the pendulum system.

4. Simulation Results and Discussions

4.1. Simulation Diagram

The control diagram is built up as shown in Figure 7. Remember that the core control idea is based on fuzzy logic and the Lyapunov stability law, as mentioned in Section 3. In Figure 7, the pole-cart inverted pendulum system model is built following the mathematical equation system expressed in Section 2. Furthermore, a modification has been added to the control system. Three scaling factors, namely K_1, K_2 and K_3, corresponding to three counterparts of a PID controller, have been embedded in the system to enhance the control quality. Significantly, they are determined by using the PSO algorithm, improving the convergence of the control solution. The objective here is formed by satisfying the sum of the absolute values of the position as well, and the rotational angle speed should be minimized. Simulation results are shown below, together with the comments to demonstrate the feasibility of the proposed controller in comparison with other counterparts.

4.2. Simulation Results and Comments

To verify the effectiveness of the proposed control scheme, the simulation processes are implemented in MATLAB/Simulink software for the following four types of controllers:

- The proposed hybrid fuzzy logic controller,
- The PD-like fuzzy logic model,
- The PI – like fuzzy logic controller and
- The conventional PID regulator.

The above four controllers are first determined using the PSO algorithm (see Figures 8-11). Table 4 indicates optimal values for these scaling factors. Then, they are applied to the IPBCS problem.

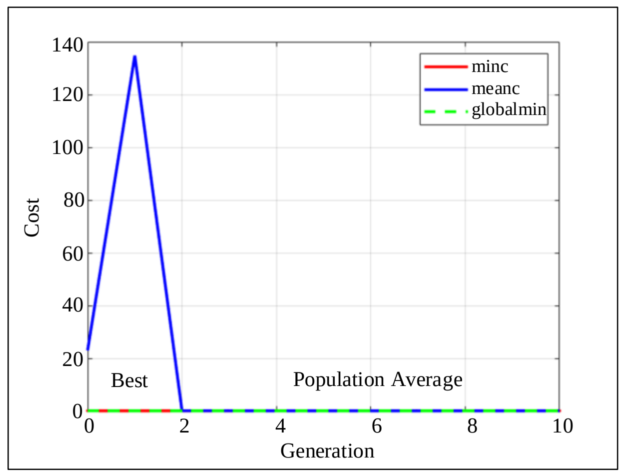


Fig. 8 Running the PSO algorithm to find three optimal scaling factors of the proposed hybrid fuzzy logic controller– the convergence

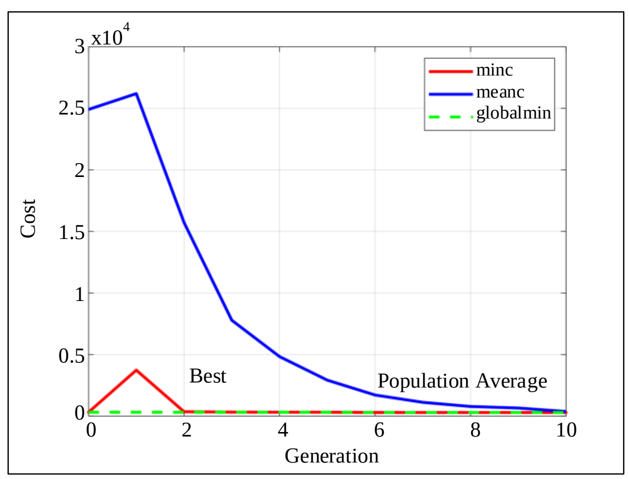


Fig. 9 Applying the PSO mechanism to find the parameter set of the traditional PID regulator– the convergence

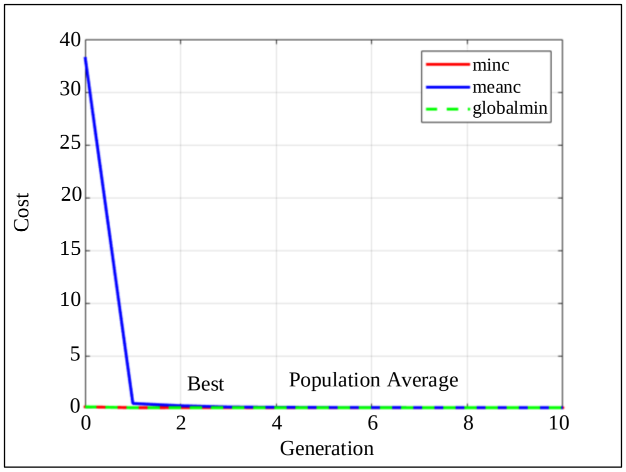


Fig. 10 Using the PSO algorithm to find the parameter set of the PD–based fuzzy logic controller – the convergence

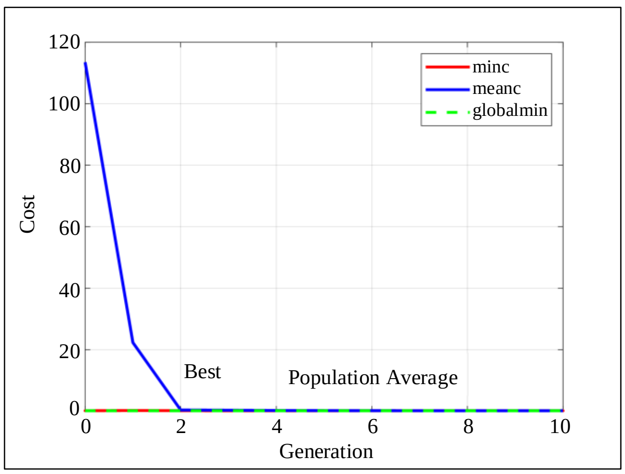


Fig. 11 Running the PSO to find the scaling factors of the fuzzy PI model– the convergence

Table 4. Parameters determined by the PSO algorithm

		Scaling Factors		
		K_1	**K_2**	**K_3**
Controllers	**Hybrid Fuzzy**	0.6556	1.0516	9.4443
	PD- Based Fuzzy	0.9708	0.2563	9.4033
	PI- Based Fuzzy	0.9342	0.2216	4.2646
	PID	0.0005	0.001	0.0009

Now, let us perform the simulation processes using the scaling factors given in Table 4. Four simulation scenarios regarding x(t) and θ(t) are also considered. It is noted that x_0 and θ_0 are two initial values of x(t) and θ(t), respectively.

(i) The first case: $x_0 = \frac{\pi}{6}; \theta_0 = 0.5(rad)$

Simulation results are plotted in Figure 12 and Figure 13. It is noted that subfigures (b) represent enlarged parts of the total figures (a).

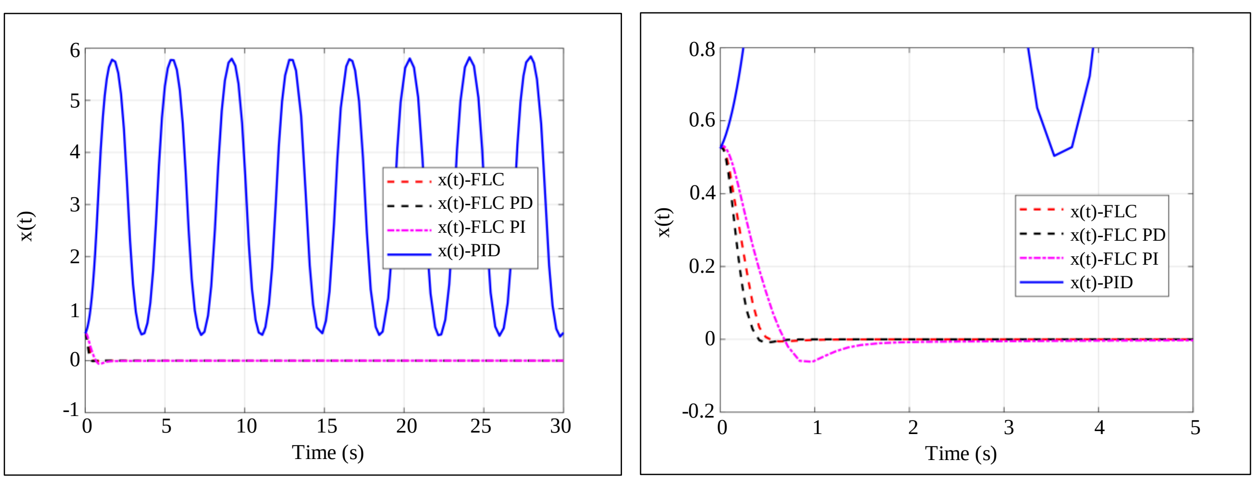
(ii) The second case: $x_0 = -\frac{\pi}{6}; \theta_0 = 0.8(rad)$

Simulation results are plotted in Figures 14 and 15.

(iii) The third simulation case with the following initial parameters:

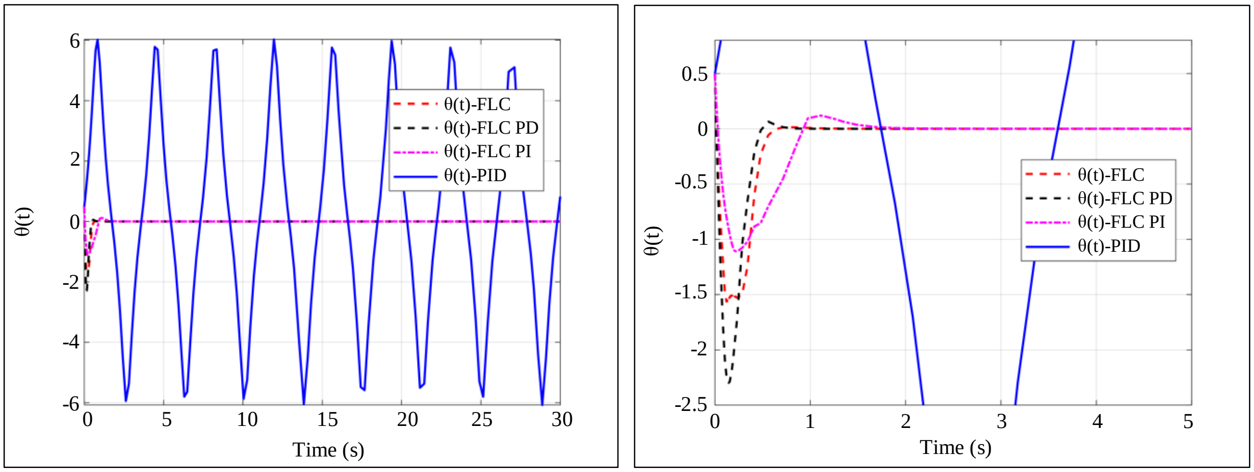
$$x_0 = -\frac{\pi}{8}; \theta_0 = -0.4(rad)$$

Simulation results in this case are plotted in the Figure 16 and Figure 17.



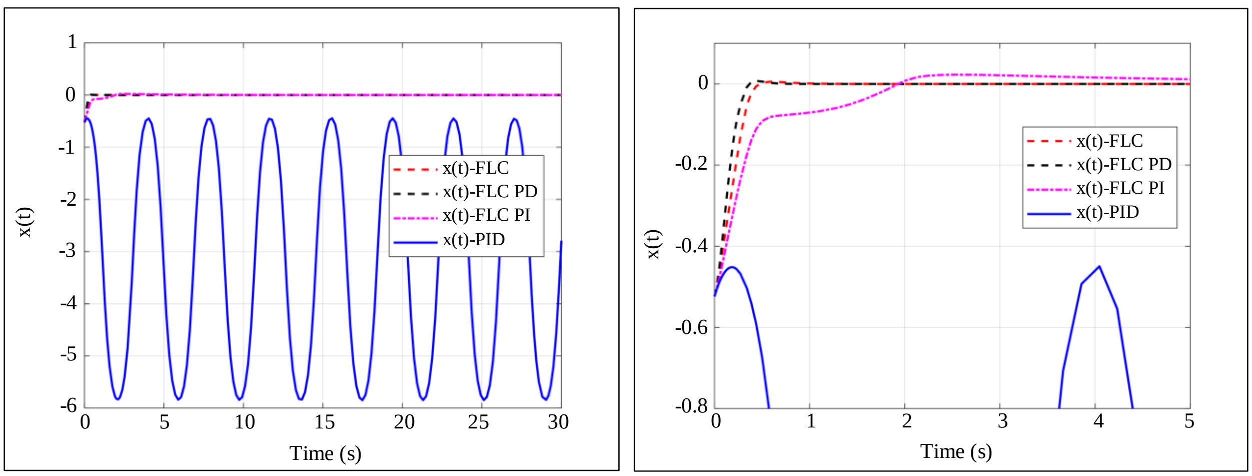
(a) **(b)**

Fig. 12 Simulation results of x(t) in the first case



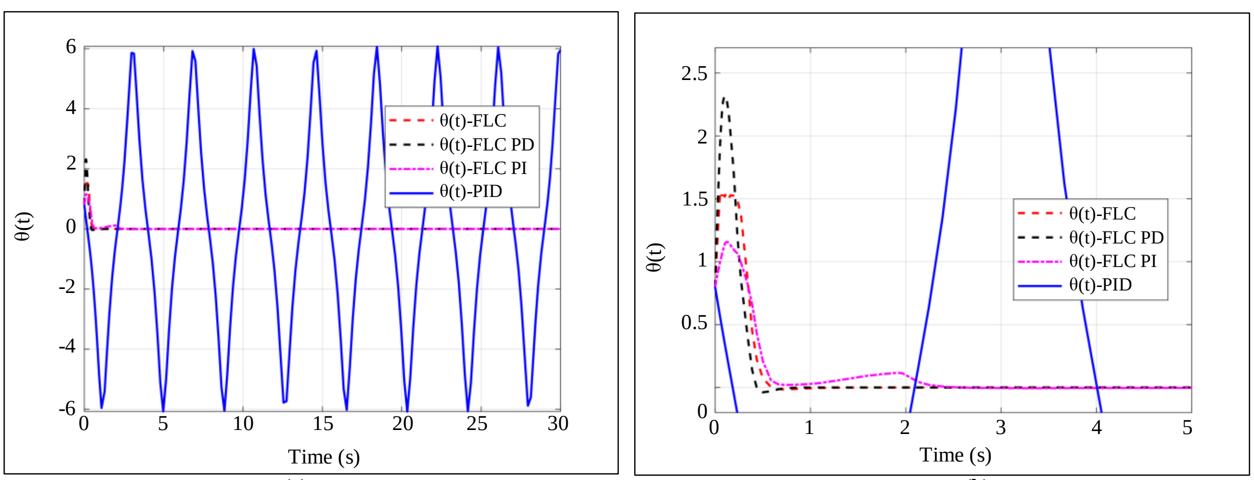
(a) **(b)**

Fig. 13 Simulation results of θ(t) in the first case



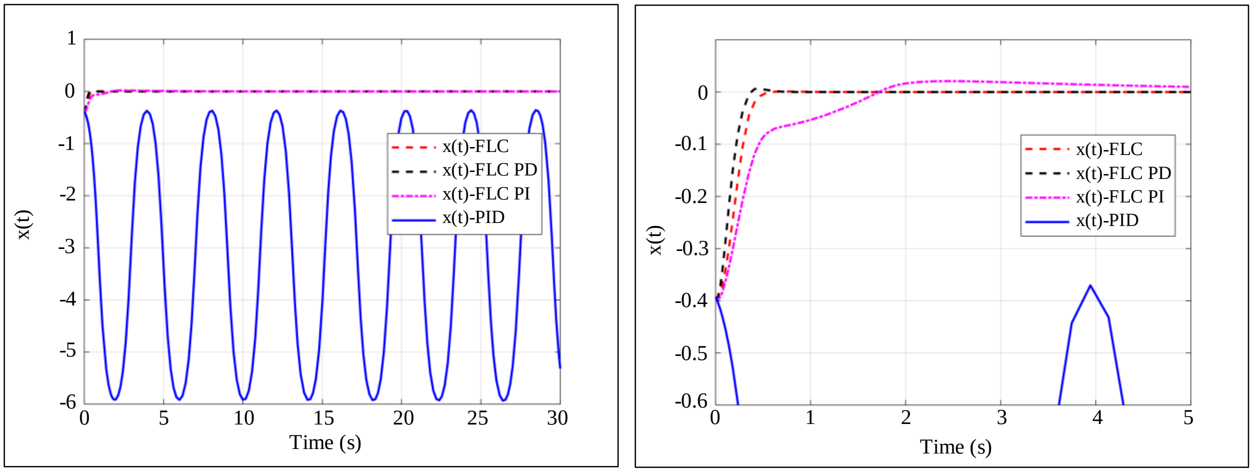
(a) **(b)**

Fig. 14 Simulation results of x(t) in the second case



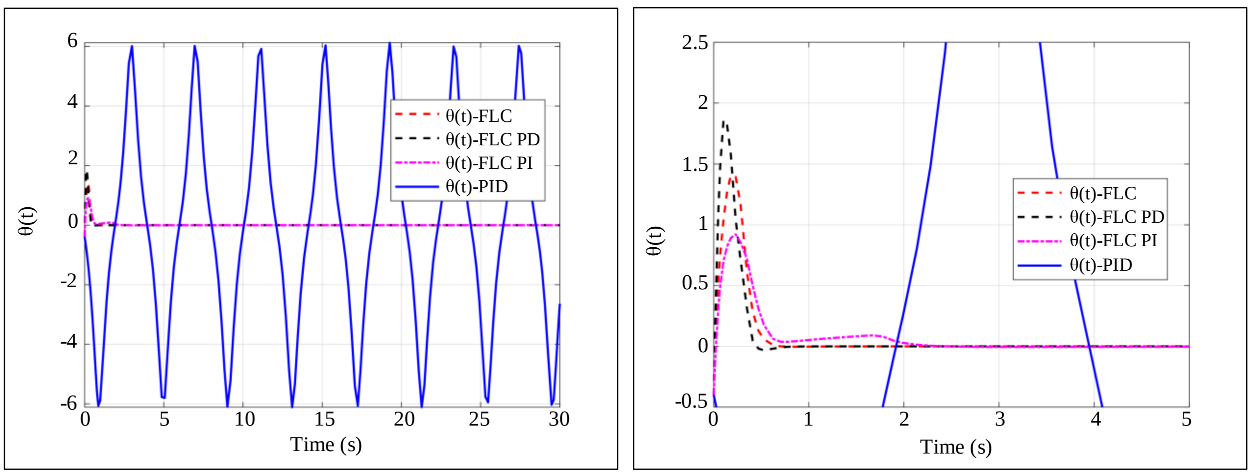
(a) **(b)**

Fig. 15 Simulation results θ(t) in the second case



(a) (b)

Fig. 16 Simulation results of x(t) in the third case



(a) (b)

Fig. 17 Simulation results of θ(t) in the third case

5. Conclusion and Future Work

This study has proposed a novel control strategy for dealing with the balance of an inverted pendulum system. The hybrid control scheme efficiently integrates the fuzzy logic technique, Lyapunov theory, and the PSO algorithm. Promising numerical simulation results compared with several existing controllers, including PID and PI/PD fuzzy logic counterparts, have demonstrated the dominant effectiveness and applicability of the proposed control approach. Besides, the fuzzy logic rules have been minimized, dramatically reducing the control system's executive time.

Future work will focus on designing a set of more adaptive fuzzy logic rules for various numbers of inverted pendulum systems. In this aspect, practical applications should also be considered to testify to the new control method.

References

[1] Sriram Shreedharan, Vignesh Ravikumar, and Senthil Kumaran Mahadevan, "Design and Control of Real-Time Inverted Pendulum System with Force-Voltage Parameter Correlation," *International Journal of Dynamics and Control*, vol. 9, pp. 1672-1680, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[2] Kent H. Lundberg, and Taylor W. Barton, "History of Inverted-Pendulum Systems," *IFAC Proceedings Volumes*, vol. 42, no. 24, pp. 131-135, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[3] Kavirayani Srikanth, and Gundavarapu V. Nagesh Kumar, "Novel Fuzzy Preview Controller for Rotary Inverted Pendulum under Time Delays," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 17, no. 4, pp. 257-263, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[4] S. Yasunobu, and M. Mori, "Swing up Fuzzy Controller for Inverted Pendulum Based on a Human Control Strategy," *Proceedings of the 6th IEEE International Conference on Fuzzy Systems*, Barcelona, Spain, vol. 3, pp. 1621-1625, 1997. [CrossRef] [Google Scholar] [Publisher Link]

[5] Wei Zhong, and H. Rock, "Energy and Passivity Based Control of the Double Inverted Pendulum on a Cart," *Proceedings of the 2001 IEEE International Conference on Control Applications*, Mexico, pp. 896-901, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[6] C. Sravan Bharadwaj, T. Sudhakar Babu, and N. Rajasekar, "Tuning PID Controller for Inverted Pendulum Using Genetic Algorithm," *Advances in Systems, Control and Automation*, pp. 395-404, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[7] Ahmad Ilyas Roose, Samer Yahya, and Hussain Al-Rizzo, "Fuzzy-Logic Control of an Inverted Pendulum on a Cart," *Computers & Electrical Engineering*, vol. 61, pp. 31-47, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[8] J. Yi, and N. Yubazaki, "Stabilization Fuzzy Control of Inverted Pendulum System," *Artificial Intelligence in Engineering*, vol. 14, no. 2, pp. 153-163, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[9] C.W. Tao et al., "Fuzzy Hierarchical Swing-Up and Sliding Position Controller for the Inverted Pendulum-Cart System," *Fuzzy Sets and Systems*, vol. 159, no. 20, pp. 2763-2784, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[10] Mahbubeh Moghaddas et al., "Design of Optimal PID Controller for Inverted Pendulum Using Genetic Algorithm," *International Journal of Innovation, Management and Technology*, vol. 3, no. 4, pp. 440-442, 2012. [Google Scholar] [Publisher Link]

[11] Zhijun Li, and Chunquan Xu, "Adaptive Fuzzy Logic Control of Dynamic Balance and Motion for Wheeled Inverted Pendulums," *Fuzzy Sets and Systems*, vol. 160, no. 12, pp. 1787-1803, 2009. [CrossRef] [Google Scholar] [Publisher Link]

[12] Bui Cong Cuong, and Nguyen Doan Phuoc, *Fuzzy Sets, Neural Networks and Applications*, Science and Technics Publishing House, 2001.

[13] Michael Voskoglou, *Fuzzy Sets, Fuzzy Logic and Their Applications*, MDPI, pp. 1-366, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[14] Phan Xuan Minh, and Nguyen Doan Phuoc, *The Theory of Fuzzy Logic*, Science and Technics Publishing House, 2016.

[15] Mahmoud Baklizi, "FLACC: Fuzzy Logic Approach for Congestion Control," *International Journal of Advanced Computer Science and Applications (IJACSA)*, vol. 10, no. 7, pp. 43-50, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[16] M. Madhusudhan, and H. Pradeepa, "Fuzzy Based Controller to Enhance Transient and Steady State Stability for Multimachine Power System," *International Journal of Engineering Trends and Technology*, vol. 70, no. 3, pp. 118-125, 2022. [CrossRef] [Publisher Link]

[17] Z. Bingul, G.E. Cook, and A.M. Strauss, "Application of Fuzzy Logic to Spatial Thermal Control in Fusion Welding," *IEEE Transactions on Industry Applications*, vol. 36, no. 6, pp. 1523-1530, 2000. [CrossRef] [Google Scholar] [Publisher Link]

Appendix

Table 5. Simulation parameters

Symbol	Meaning	Value, (unit)
M	Mass of the cart	1.2 (kg)
m	Mass of the rod	0.105 (kg)
l	Length of the rod (mass omitted)	0.11 (m)
x(t)	Position of the cart	(m)
$\dot{x}(t)$	The velocity of the cart	(m/s)
$\ddot{x}(t)$	Acceleration of the cart	(m^2/s)
θ(t)	Pendulum angle (upright position)	(rad), (degree)
$\dot{\theta}(t)$	Rotational velocity of the pole	(rad/s)
$\ddot{\theta}(t)$	Rotational acceleration of the pole	(rad^2/s)
u(t)	Variable force exerted on the cart	(N)
K_1, K_2, K_3	Three scaling factors of the fuzzy logic–based position controller	N / A