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*Original Article*

# Neumann Series based Precoding Matrix Generation for Next Generation High throughput Satellite

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***Abstract*** *- In next-generation high throughput satellite (NGHTS), precoding (a preprocessing technique at the gateway (GW) can mitigate the interbeam interference with no additional resource at the user terminal. However, the calculation of precoding requires matrix inversion, which involves huge complexity in terms of hardware resources with real-time calculation for near error-free results. This paper proposes a novel approach for precoding matrix calculation using the Neumann series. In this way, the algorithm does not directly compute the matrix inversion. Hence the computational complexity is highly reduced. The proposed method uses the 4$^{th}$-order iterative series with the initial guess as an inverse of the diagonal matrix of the input to construct the Neumann series. This leads to fast convergence of the matrix inversion process with fewer resources. The algorithm is claimed to be generic as it is seamlessly applied to any linear precoding scheme. The paper examines the resource complexity, convergence probability and Bit Error Rate (BER) performance for the proposed method with Zero Forcing (ZF) and Regularized Zero Forcing (RZF) based linear precoding scheme. Experiment results demonstrate that the proposed algorithm accomplishes superior performance with fewer resources compared to the Neumann series and Joint iterative Newton/Chebyshev-based Neumann series methods.*

***Keywords*** *- Precoding, Interbeam interference, Neumann series, Matrix Inversion Approximation (MIA), Next Generation High Throughput Satellite.*

## 1. Introduction

Communication satellites' design and utilization techniques are undergoing disruptive but positive paradigm transformations [1-2]. The NGHTS uses numerous spot beams with full or nearly full frequency reuse factors amongst the user beams. In these scenarios, NGHTS forward link behaves as an interference-limited link rather than a conventional thermal noise-constraint link. Despite all these developments, new methods still need to be investigated to solve this problem. Precoding is one of the multiple input, multiple outputs (MIMO) based interference mitigation strategies. [3-9]. The scientific community has recently become quite interested in using precoding techniques in SATCOM services [10-13]. Recently, the efficacy of precoding in SATCOM has been successfully demonstrated with EUTELSAT 7B and EUTELSAT 10A satellites [14].

Further, the publication of the digital video broadcast standard DVB-S2x provides the utilization strategy for precoding in multi-beam high throughput satellite (HTS) systems [15]. However, from the perspective of NGHTS, calculating the precoding matrix requires the inverse of the large matrix size. This is an important practical issue as matrix inversion operation requires a huge resource for real-time calculations and updates of the precoding matrix. A lot of work has been published in this domain by various authors. Direct inversion [16], Newton Raphson iteration [17], Chebyshev iteration [18], and Toutounian iteration [19] have recently been used in matrix inversion. The initial value guess and the number of iterations control the performance of these methods. Recently, the Neumann series has been proposed for the inverse of the matrix via an approximation approach [20]. The single-shot calculations in the Neumann series are thoroughly examined for matrix inversion approximation (MIA). However, the use of the Neumann series for precoding matrix calculation has been constrained by sluggish convergence or non-convergence in certain circumstances. The combined use of iterative series and Neumann series techniques have recently attracted a lot of attention in research activity.

The matrix inversion is performed by using Joint-Newton Iteration and Neumann series (Joint-NINS) in [21] and also through joint Chebyshev Iteration and Neumann series (Joint-CINS) in [22]. These approaches efficiently find the initial step of the Neumann series using either Newton or Chebyshev iteration method. However, all these researches are mainly focused on terrestrial communication, and less work is done on NGHTS. We suggest a unique approach using joint Toutounian iteration and Neumann Series (Joint-TINS) for the NGHTS scenario. In the Joint-

TINS, the Toutounian iteration provides the pivotal point for an efficient search direction for matrix inversion via the Neumann series. Furthermore, a convergence condition of the Joint-TINS is derived for the NGHTS scenario. The structure of the paper is as follows.

Section II defines the system architecture of NGHTS with full frequency reuse amongst receiving beams. Section III presents the Joint-TINS algorithm, its convergence condition and its complexity. Section IV provides details of the experiment results of the Joint-TINS method and a comparison with other methods for 16 Beams NGHTS design.

Notation: A capital letter denotes the matrix, while a small letter denotes the vector. B is a matrix, while b is a vector. $B^T$ is a transpose matrix, $B^{-1}$ is an inverse matrix, B† is a conjugate (Hermitian) transpose matrix, and ‖B‖ is a Forbenius norm of matrix B. The $O_N$ is N x N zero matrix, while $I_N$ N x N is the identity matrix.

## 2. Problem Formulation

Figure 1 shows the typical NGHTS architecture in bent pipe configuration. The satellite is equipped with N transmit antenna chains to provide the coverage area in K-shaped ground beams. The system is employed with full frequency reuse and uses precoding for interference mitigation. Further, it is assumed that GW has full knowledge of the channel state information matrix of the network and forward link is fully calibrated. We consider the precoded transmit signal as given in equation (1).

$$y_k = h_k^{\dagger} \sqrt[2]{p_k} s_k w_k + h_k^{\dagger} \sum_{k \neq j} \sqrt[2]{p_k} s_j w_j + n_k \quad (1)$$

Where $s_k$ denotes the unit power symbol, $p_k$ represents the gain factor, and $w_k$ is a normalized N x1 size precoding vector, respectively, for the user receiver in the $k^{th}$ beam. Further $h_k^{\dagger}$ denotes the conjugate transpose of the channel state vector $h_k$ for the corresponding $k^{th}$ beam. It is 1 × N vector composed of the channel state coefficients (mainly

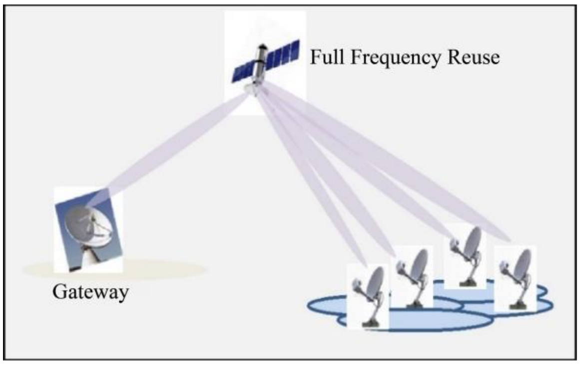


**Fig. 1 Forward link architecture of NGHTS**

consisting of propagation loss, feeder losses, antenna gains, and associated phase shift of local oscillators in the payload) between the user receiver in the $k^{th}$ beam and the satellite. Further, $n_k$ represents zero mean additive white Gaussian noise (AWGN) at the $k^{th}$ user receiver.

The signal-to-noise-and-interference ratio (SNIR) and achievable rate for the $k^{th}$ beam for the system are given by equations (2) and (3), respectively.

$$SINR_k = \frac{p_k \left| h_k^{\dagger} w_k \right|^2}{1 + \sum_{j \neq k} \; p_j \left| h_j^{\dagger} w_j \right|^2} \quad (2)$$

$$r = \ln\{1 + SINR_k\} \quad (3)$$

Further, for the performance validation of Joint-TINS, the paper considers the ZF and RZF precoding described by equation (4) and equation (5)

$$W_{ZF} = [H^{\dagger} P H]^{-1} H^{\dagger} \quad (4)$$

$$W_{RZF} = [\beta_{RZF} I_N + H^{\dagger} P H]^{-1} H^{\dagger} \quad (5)$$

Where $\beta_{RZF}$ is the SINR controlling parameter defined as [24].

## 3. Proposed Algorithm

We can see from equations (4) and (5) that the calculation of the precoding matrix involves a step of finding the inverse of the matrix. This matrix in the NGHTS scenario is huge; thus, the operation requires overwhelming resources. In this regard, we propose the Neumann series based fourth-order iterative series method (based on the Toutounian series) to calculate the inverse with reduced complexity.

The Neumann series is described in equation (6). Equation (7) is a prerequisite for equation (6).

$$Z^{-1} = \sum_{n=0}^{\infty} (I_k - \theta Z)^n \theta \quad (6)$$

$$lim_{n \to \propto} (I_k - \theta Z)^n = 0_k \quad (7)$$

Where Z is the input matrix whose inverse is to be calculated, the θ is the initial guess for the Neumann series. The initial guess (θ) controls the convergence and speed of convergence of the process. Thus θ should be intelligently selected to resolve sluggish convergence and non-convergence of the Neumann series. The Joint-TINS algorithm first finds a suitable θ and uses it to approximate the matrix inversion via equation (6).

Let the equation for the inverse function is defined as (8)

$$f(x) = x - A^{-1} \quad (8)$$

For the solution $x = A^{-1}$ or f(x) = 0, we use a three-step method defined in equation (9) to equation (11)

$$v_i = x_i - \frac{f(x)}{f'(x)} \quad (9)$$

$$u_i = x_i - \frac{f(x_i)}{2}\left\{\frac{1}{f'(x_i)} + \frac{1}{f'(v_i)}\right\} \quad (10)$$

$$x_{i+1} = v_i - \frac{f(v_i)}{v_i - x_i}\{f(v_i) - f(x_i)\} \quad (11)$$

If we apply equation (8) in equations (9) to (11), then the following iterative series is obtained [12].

$$Z_{i+1} = \frac{Z_i}{2}\left[9I - AZ_i\left\{16I - AZ_i\left(14I - AZ_i(6I - AZ_i)\right)\right\}\right], \ i = 0,1,2, \quad (12)$$

$Z_0$ is the initial value at the start of the calculation of equation (12). Additionally, [23] demonstrated that setting $Z_0$ to $D^{-1}$ (inverse of a matrix consisting only of diagonal elements of the input matrix) meets the convergence requirement of equation (7). As equation (12) is of four order, it accelerates the convergence speed for the Neumann series. This inspired us to use the first iteration output $Z_1$ as an initial guess for the Neumann series. This performs well in practice and only requires a few iterations. Thus, the Joint-TINS scheme can be obtained as per table 1.

**Table 1. The proposed algorithm (Joint-TINS)**

| **Algorithm** |
|---|
| 1. Calculate $Z_1 = \frac{D^{-1}}{2}\left[9I - AD^{-1}\left\{16I - AD^{-1}\left(14I - AD^{-1}(6I - AD^{-1})\right)\right\}\right]$ <br> 2. Construct the Neumann series using the $Z_1$ to find out the $A^{-1}$ as $A^{-1} \approx \sum_{n=0}^{m}(I_k - AZ_1)^n Z_1$ for initial m terms. <br> 3. Calculate the precoded signal as illustrated by equation (4) and equation (5) <br> 4. Repeat the process from step 2 to step 3 until the performance index (SINR/Bit error rate performance/Residual Error etc.) is achieved in the desired threshold range |

The suggested Joint-TINS precoding technique requires an additional 1-time series iteration to solve $Z_1$ in comparison to the Neumann series precoding technique. However, this makes the overall convergence faster. Compared with Joint-NINS or Joint-CINS, the Joint-TINS method is faster as Joint-NINS or Joint-CINS are two-order and three-order series equations, respectively. Additionally, when the number of terms in the Neumann series is less, equation (6) may not converge with any general initial value (θ). However, using $Z_1$ to build the Joint-TINS precoding produces a more suited output and enables faster convergence with fewer iterations than the Neumann series. As a result, the performance is enhanced, and resource usage is decreased.

### *3.1. Convergence Analysis*

The law of matrix power series states that the product of $B^N$ will only converge for a square matrix B of size K x K when its spectral radius of B, i.e. $\rho(B)$ is smaller ss than 1. Now the Joint-TINS algorithm converges when

$$=> lim_{n\to\propto}(I_k - Z_1A)^n = 0_k \quad (13)$$

$$=> \text{spectral line of } \rho(I_k - Z_1A) < 1 \quad (14)$$

$$=> \text{the eigenvalue o } \lambda(I_k - Z_1A) < 1 \quad (15)$$

Further, Note that $lim_{n\to\propto}(I_k - Z_1A)^n Z_1$ converges when

$$=> |B(\lambda)| < 1 \ where\ \lambda(A) is\ the\ eigen\ value\ of\ the\ B\ where\ B = I_k - Z_1A$$

Now for B convergence condition can be proved as per below

i.e. $\rho(B) < 1$

$$\rho(I_K - D^{-1}A) < 1, for\ B = I_K - D^{-1}A$$

Now, $B = I_k - Z_1A$, replacing $Z_1$ by the equation (12) with $Z_i$ by $D^{-1}$ as suggested in [23]

$$=> B = I_k - \frac{D^{-1}}{2}\left[9I_k - AD^{-1}\left\{16I_k - AD^{-1}\left(14I_k - AD^{-1}(6I_k - AD^{-1})\right)\right\}\right]A$$

$$=> B = \frac{1}{2}\left(I_k - \frac{D^{-1}}{2}\left[9I_k - AD^{-1}\left\{16I_k - AD^{-1}\left(14I_k - AD^{-1}(6I_k - AD^{-1})\right)\right\}\right]A\right)$$

$$=> B = \frac{1}{2}(I_k + R)(R)^4, \ where\ R = I_k - AD^{-1}$$

$$=> B = \frac{1}{2}(R^4 + R^5)$$

From the subordinate matrix norm, we can have

$$||B|| \le \left\|\frac{1}{2}(R^4 + R^5)\right\| \le \frac{1}{2}\left(||R^4|| + ||R^5||\right)$$

$$=> By\ preconditioning\ \ of\ R\ as\ ||I_k - AD^{-1}|| < 1$$

$$=> ||B|| \le \frac{1}{2}||R|| < 1$$

Further, based on [25], the convergence condition for $\sum_{n=0}^{\infty} (I_k - Z_1A)^n Z_1$ can be derived as

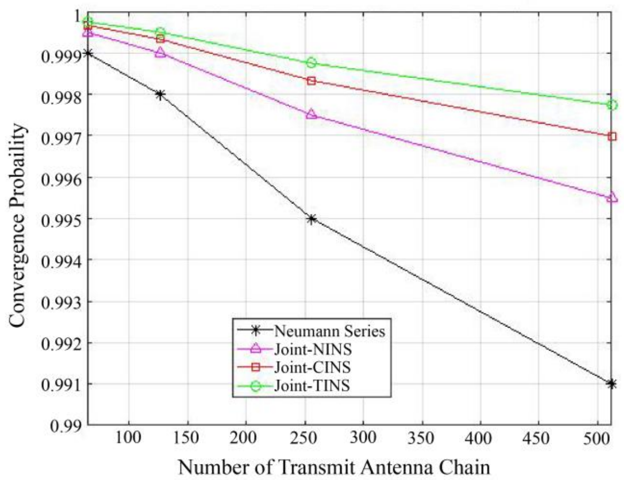


**Fig. 2 Convergence probability vs Number of transmit antenna**

$$\frac{N}{K} > \frac{1}{\left(\sqrt[2]{2}-1\right)^2} > 5.8284 \qquad (16)$$

Hence, the Joint-TINS method satisfies the convergence condition. Table 2 shows the maximum allowable number of Beams (K) to the number of transmit antennas (N) in the NGHTS network design for Joint-TINS. Figure 2 displays the convergence probability vs transmit the number of the antenna (minimum value of N as per equation (13)). It is clearly visible that the algorithm is achieving a very high value of convergence with a probability as high as 0.9995. Furthermore, the convergence probability of Joint-TINS increases as the ratio of N/K increases. Additionally, the Joint-TINS algorithm performs better for a lower value of N.

### *3.2. Estimation of Complexity Analysis*

The Joint-TINS scheme's computational complexity can be divided into three sections. The first step of the Joint-TINS yields the first component of the computation. This step has a complexity of 4NK + K. The second section comes from the computation of the Neumann series. It is evident that the computing cost of this step is (i − 2) $K^3$, where i denotes the total number of terms in the Neumann Series.

The last section of the estimation originates from the computation of matrix multiplication of the calculated matrix $[H^{\dagger}PH]^{-1}/[\ \beta_{RZF}I_N + H^{\dagger}PH]$ with $H$. The assumption in these steps is that once the system configuration has been completed, the values of $\beta_{RZF}$, P and H are known and the inner matrix in equation (4) and equation (5) is available at the start of the algorithm. Thus, the number of complex multiplications require are M + MK for both ZF and RZF.

**Table 2. Maximum $K$ values vs transmit antenna (N) chains under the condition of (16)**

| **N** | 64 | 128 | 256 | 512 |
|---|---|---|---|---|
| **K** | 10 | 21 | 43 | 87 |

**Table 3. The complexity of different algorithms for calculation of ZF and RZF precoding matrix**

| Algorithm | Complexity |
|---|---|
| Direct Method | $N+NK+2NK^2+K^3$ |
| Neumann Series | $N+NK+(i\text{-}2)K^3$ |
| Joint-NINS | $N+K+3NK+(i\text{-}2)K^3$ |
| Joint-CINS | $N+K+4NK+(i\text{-}2)K^3$ |
| Joint-TINS | $N+K+6NK+(i\text{-}2)K^3$ |

Table 3 compares the complexity of calculating the precoding matrix via different algorithms. Table 3 illustrates that the computational complexity of Joint-TINS is comparable to that of Joint CINS and Joint NINS. On the other hand, since the Neumann series in Joint-TINS has fewer iterations, this precoding scheme shows a higher convergence probability and faster convergence rate. In contrast to other algorithms, the Joint-TINS technique achieves a superior balance between performance and complexity.

## 4. Simulation Results

We demonstrate the Joint-TINS algorithm's performance in this section through a number of experiments. The Joint-NINS, Joint-CINS, and Neumann series-based ZF and RZF precoding methods are compared to the Joint-TINS algorithm. The benchmarking is done through classical ZF and RZF precoding with exact matrix inversion. A typical downlink NGHTS, with uniform 32 APSK modulation, N×K = 128×16 and flat fading with AWGN channel, is considered. Further, GW knows the channel state information matrix of all users, and the forward link is fully calibrated.

The experiments are BER measurements for ZF and RZF precoding schemes, comparing convergence speed and residual error amongst the different methods.

### *4.1. Experiment 1 (ZF BER) and 2 (RZF BER)*

Figures 3 and 4 show the BER performance of the Joint-TINS, Neumann series, Joint-CINS, and Joint-NINS for ZF and RZF precoding, respectively. The stopping criteria are that BER performance is better than $10^{-5}$. As can be observed, all techniques perform better in terms of BER with the increase in the number of terms in the Neumann series. It is clear that the Joint-TINS-based precoding technique can obtain enhanced performance than all other precoding techniques. For illustration, Joint-TINS achieves the stopping criteria at 30 dB, while the Joint-NINS, Joint-CINS and benchmarked approach (Direct Inversion) stops at SNR value 36 dB, 32 dB and 28 dB, respectively. In the case of RZF, Joint-TINS terminates at SNR value 24 dB against 22, 28 and 26 dB for direct, Joint-NINS and Joint-CINS. Furthermore, the Joint-TINS precoding requires fewer iterations to reach a similar performance. The Joint-TINS precoding only requires 2, while Joint-CINS and Joint-NINS series require 3, 4, and 6 Neumann series terms, respectively, for both ZF and RZF precoding. As a result, the Joint-TINS precoding scheme can produce nearly ideal results with fewer iterations, resulting in a lower resource requirement.

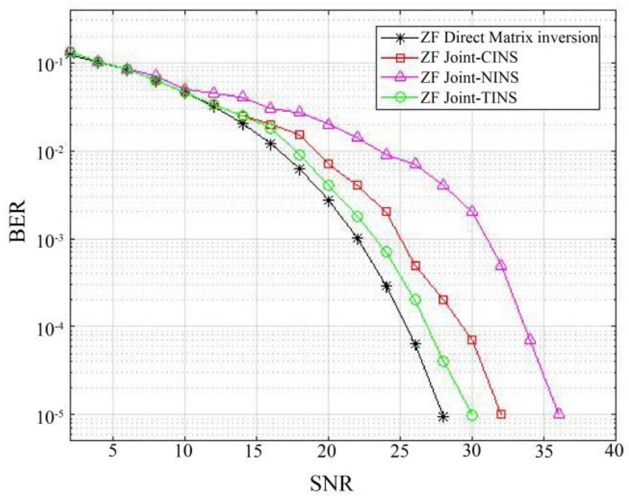


**Fig. 3 BER performance comparison for ZF precoding**

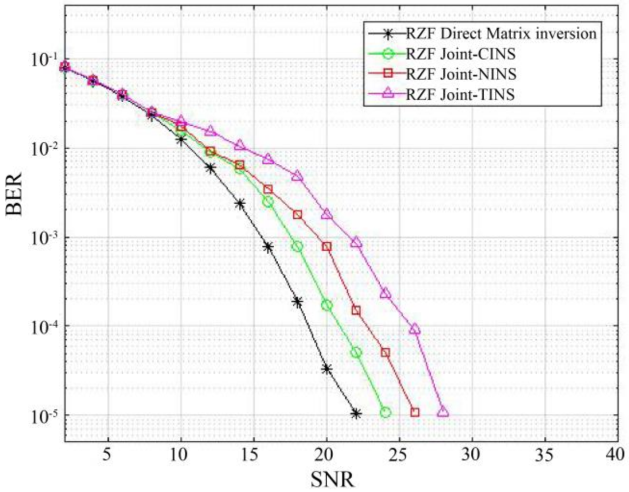


**Fig. 4 BER performance comparison for RZF precoding**

It is evident that the Joint-TINS scheme outperforms the Joint-CINS, Joint-NINS and Neumann series to achieve the BER performance. Therefore, the Joint-TINS precoding approach efficiently reduces the interference amongst the beam.

### *4.2. Experiment 3 (Convergence speed) and 4 (Residual error)*

In this experiment, we compare the convergence speeds of the Joint-TINS method with Joint-CINS, Joint-NINS and Neumann series. The convergence speeds are measured in terms of iteration required to meet the stopping criteria. In this experiment, we use the three types of random matrix with the size 100 x 16, 400 x 64 and 900 x 144. These matrices are chosen as per the condition of equation (16). The stopping criteria for the experiment is

$$\left\|\frac{R_k}{R_0}\right\| < 10^{-5} \quad , \text{ where, } \quad R_K = I - {A_0}^{-1}H \quad , \quad R_0 = I - {A_k}^{-1}H,$$

where ${A_0}^{-1}$ is initial guess and ${A_k}^{-1}$ is the k[th] iteration output. Table 4 shows the number of iterations required for the Neumann series, Joint-NINS, Joint-CINS and Joint-TINS algorithm.

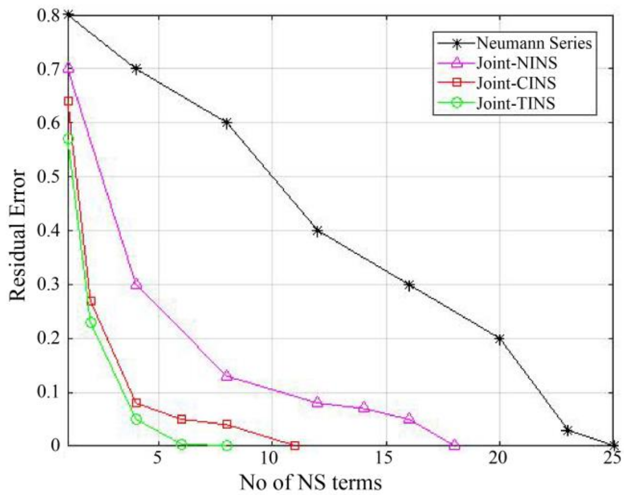


**Fig. 5 Residual error comparison**

**Table 4. The complexity of various algorithms with different input matrix sizes**

| Matrix Size/ Algorithm | 100x16 | 400x64 | 900x144 |
|---|---|---|---|
| | Average Neumann series term | | |
| Neumann | 25 | 48 | 90 |
| Joint-NINS | 18 | 36 | 54 |
| Joint-CINS | 11 | 20 | 30 |
| Joint-TINS | 8 | 16 | 20 |

It can be seen that Joint-TINS converges almost three times faster than the Neumann series, twice compared to Joint-NINS and 1.2 to 1.3 times fast compared to Joint-CINS. Figure -5 shows the residual error data for the Neumann series, Joint-NINS, Joint CINS and Joint-TINS algorithm for a random matrix of size 100 x 16. The residue of the Joint-TINS algorithm is less than the Neumann series, Joint-NINS and Joint-CINS scheme due to the preconditioning of the initial guess through steps 1 and step 2 of the Joint-TINS. This is one of the novelties of the algorithm.

## 5. Conclusion

In the paper, we put forth a novel approach for calculating the precoding matrix to achieve the near-ideal downlink performance for NGHTS's. The Joint-TINS algorithm's methodological steps are derived in this paper. The paper establishes the convergence condition in the NGHTS scenario. Even if the Joint-TINS algorithm performs better, the research asserts and demonstrates that its complexity is comparable to that of the Joint-NINS and Joint CINS algorithms. The algorithm's efficacy is proven through characterization in ZF and RZF scenarios. The experiment results validate that the Joint-TINS method outperforms the existing Neumann series-based Joint iterative precoding schemes. Further, the algorithm converges fast due to the preconditioning applied for the Neumann series.

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