SSRG International Journal of Electrical and Electronics Engineering
ISSN: 2348-8379/ https://doi.org/10.14445/23488379/IJEEE-V12I10P104

Volume 12 Issue 10, 34-54, October 2025

Original Article

Modeling and Control of a Drum-Boiler System with Shrink and Swell Effect Attenuation in MATLAB/Simulink

Adrian Moises Cascamayta Quispe[1], Ronald Gustavo Velasquez Choque[1], German Alberto Echaiz Espinoza[2], Pedro Alberto Mamani Apaza[3], Fernando Enrique Echaiz Espinoza[4]

[1]*Professional School of Electronic Engineering, National University of San Agustín de Arequipa, Arequipa, Perú.*
[2]*Department of Electronic Engineering, National University of San Agustín de Arequipa, Arequipa, Perú.*
[3]*Department of Mathematics, National University of San Agustín de Arequipa, Arequipa, Perú.*
[4]*Institute of Mathematics, Federal University of Alagoas, Maceio, Brazil.*

[2]*Corresponding Author: gechaiz@unsa.edu.pe*

Received: 03 August 2025 Revised: 05 September 2025 Accepted: 04 October 2025 Published: 30 October 2025

Abstract *- The control of drum-boiler systems is challenging due to their highly nonlinear dynamics and shrink-and-swell phenomena that complicate level regulation. To this end, a cascade control strategy with feedback action is designed to reduce the effects of contraction and expansion within the boiler, as well as to reduce the effect of disturbances under variable load conditions. The simulation results demonstrate the superiority and reliability of the three-element cascade controller compared to the conventional single-element scheme, achieving faster stability, reducing oscillations, and improving tracking performance. Unlike previous work, this study provides complete mathematical derivations of the fundamental equations and a complete and replicable implementation in MATLAB/Simulink, ensuring both transparency and industrial relevance. The developed simulator provides a robust platform for further research, operator training, and the design of advanced control strategies in thermal power plants.*

Keywords *- Cascade control, Drum boiler systems, Mathematical derivations, Matlab/Simulink, Shrink-swell effect.*

1. Introduction

In thermoelectric generation, the boiler system must maintain the water level within safe ranges while responding to the needs of the electrical load. Under stable conditions, control problems are usually not present, but frequent load fluctuations that cause sudden steam demands complicate this task. A poorly designed control can lead to costly plant shutdowns or accidents. The contraction and expansion effect causes the controller to respond incorrectly when the load changes, as the system has a non-minimal phase response.

The main objective of this research is to develop and simulate, in MATLAB/Simulink, the fourth-order nonlinear model proposed by Åström and Bell for boilers, incorporating initial value tables that allow for quicker configuration of simulation conditions. Likewise, the aim is to compare the performance of single-element and three-element controls in mitigating the shrink and swell effect to analyze their effectiveness in maintaining the stability of the water level in the dome. In drum boilers, a three-element control is used to correct the water level error using the difference between the steam and feedwater flows. In this scheme, an inner loop is responsible for quickly responding to load variations and disturbances, while an outer loop is responsible for process stabilization. This strategy is used in thermal power plants to control the water level in the boiler safely. [6]. This architecture has demonstrated its effectiveness under variable load conditions and has become a standard in industrial applications. However, it has limitations, especially with very low loads, where instabilities can occur, forcing a switch to single-element control based solely on water level measurement. Most electrical engineers agree that computational simulation is beneficial for designing control systems and analyzing plants. There are many mathematical models for boilers, from simple linear models to more complicated models using experimental data.

The Åström and Bell model [1] has high physical fidelity that is useful for controller design, as it provides useful information to power plant operators by capturing important dynamics, such as expansion and the shrink-swell effect. Figure 1 presents a simple PID diagram of the three-element control for a boiler. The system is based on a master-slave or cascade loop; the Level Transmitter (LT) measures the water level in the drum to compare it with the difference between the water and steam flow rate so that the Level Controller (LIC) generates a correction signal. The steam flow represents the plant's demand. This value is used to calculate the water flow necessary to maintain the mass balance between the incoming water and the steam generated. The actual feedwater flow, measured by its corresponding flow transmitter, is compared against the calculated demand, generating a quick corrective action in the Flow Controller (FIC). The output signal from The Flow Controller (FIC) adjusts the feedwater Control Valve (LY) through a positioner (LY) that acts on the valve actuator. Thus, it ensures that the actual flow of feedwater meets the process demand, keeping the drum level stable.

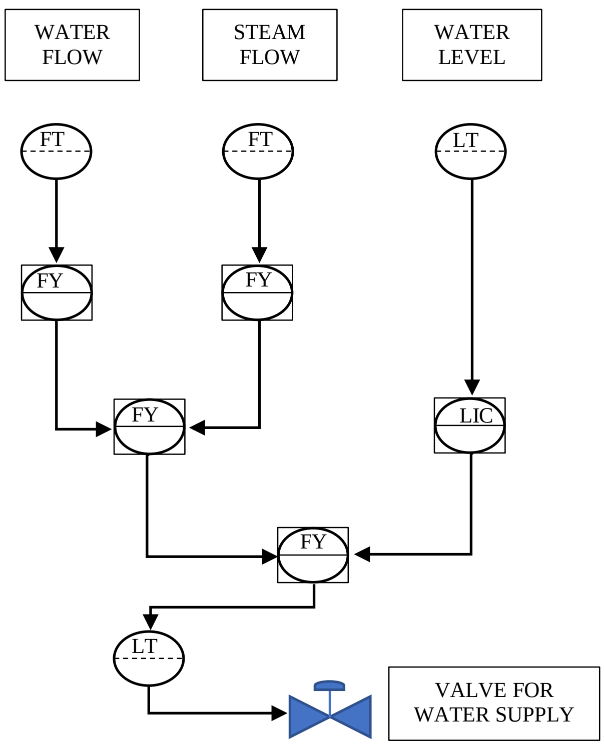

Fig. 1 PID diagram for the control of three elements for the drum-boiler

The design and simulation of a nonlinear drum boiler model together with cascade control are the main objectives of this work. Important elements such as superheater dynamics and actuator behavior are integrated into the system implementation. Compared to traditional single-loop control, the simulation results demonstrate improvements in stability and responsiveness in various operating scenarios, validating the effectiveness of the suggested approach.

This article presents a replica of the dynamic boiler model initially formulated by Åström and Bell [2]. This replica is an important scientific contribution, as it allows us to confirm the reliability of the results obtained previously and evaluate any possible errors or methodological deficiencies that may be found.

We have reimplemented the model in MATLAB/Simulink to verify the results, as the original article [2] did not include the complete code or the complete derivations of the equations. Our replication corroborates the main findings of [2], while allowing us to document the model in much greater detail. Our report is 24 pages long, while [2] is only 7 pages long. It includes the complete derivations of the fundamental equations and flowcharts. This comprehensive documentation not only facilitates the reproduction of the results but also reinforces and simplifies the basis for future research on boiler operation.

2. State of the Art

Drum boiler systems produce high-pressure steam, which is used to drive turbines that generate electricity. Their dynamics are highly nonlinear due to expansion and contraction.

The nonlinear dynamic model created by Åström and Bell [1], which is based on principles of physics, was one of the most significant modeling efforts. This model accurately describes the behavior of the drum, the downcomer, and the risers, including the shrink and swell effect that arises from pressure fluctuations and affects the water level. It has served as a solid basis for the development of advanced control systems and has been validated with real plant data.

On this basis, Iacob and Andreescu [2] extended the Åström-Bell model by applying it to a 16 MW thermal power unit and integrating subsystems such as actuators, valves, superheaters, and turbine dynamics. The proposed architecture was a programmed gain cascade control, combining the traditional three-element scheme with a pressure control loop driven by the system's heat input. In addition, they implemented their architecture in LabVIEW, including a graphical user interface that allows switching between manual and automatic modes, improving operational interaction.

Recent years have seen the development of new techniques that make use of predictive modeling and artificial intelligence. Transformer networks were used by Su et al. (2024) in [12] to predict boiler water levels, successfully reducing delays and nonlinearities to allow for more proactive control measures.

A validated dynamic model for a supercritical boiler was presented by Liu et al. (2023) in [13]. It incorporates disturbances in critical parameters such as fuel and water flow, improving water-fuel control through dynamic compensation. Ye et al. (2024) also proposed a hybrid model in [14] that predicts operational parameters in circulating fluidized bed boilers by combining CNN and Transformer. This model compensated for time delays and accurately predicted complex dynamics.

By applying improved optimization techniques to accurate dynamic models alongside predictive tools, these recent studies expand on traditional approaches. This is in line with the trend in modern thermoelectric power generation toward more flexible and effective controls.

The Åström and Bell drum boiler model [2] is based on basic ideas and has been tested with real plant data at portal.research.lu.se. We discovered that [2] employs distinct expressions for the coefficients e_{ij} based on the nature of the operating conditions, whether static or dynamic. So, we reviewed how these coefficients were derived from first principles to ensure they were all the same. While doing this, we found mistakes in some of the equations in [2] that would worsen the simulation results. To fix those mistakes, we did a long mathematical derivation of the state-space equations, which is something that has never been done before. Thanks to the corrected equations, our simulation now works as expected for the boiler system, just like it did in [2] under the same conditions.

In addition, [2] used LabVIEW to build the model, but we built it all in MATLAB/Simulink. This meant that we had to build Simulink blocks by hand and add steam tables to find the necessary partial derivatives. We also set each integrator to fixed values (based on Table 2 of our paper) to make it easier to run the same simulations over and over. We make the model available with regular industrial simulation tools by doing this. It is important to note that the LabVIEW simulation in [2] was hard to see or copy, but our Simulink implementation is completely clear. We have confirmed that its dynamic and static responses correspond with an actual industrial boiler at identical setpoints, thereby validating our model's precision and practical applicability.

This work builds on earlier studies and fixes their problems by creating a fully documented and efficient MATLAB/Simulink implementation of the nonlinear drum boiler model. It also includes corrected derivations of the basic equations and coefficient expressions. Also, a cascade control strategy with feedforward action is planned and tested under different operating conditions. This study creates a replicable and validated framework that broadens drum boiler modeling and control in academic research and industrial practice by integrating a stringent mathematical formulation with practical application on a commonly utilized industrial simulation platform.

3. Methodology

3.1. Description of the Model

A nonlinear model based on the work of Åström and Bell [1] describes the dynamic behavior of drum boilers under time-varying conditions. The model represents critical physical processes, such as the masses of water and steam in the drum, the dynamics of water and steam flow in the drum boiler, and the shrinkage and swelling caused by pressure changes. The model represents the physical processes that occur in the boiler, including the separation of water and steam inside the drum, the flow dynamics in the downcomer and riser tubes, and the shrinkage and swell effect.

This study used MATLAB/Simulink to run the model, using differential equations to illustrate the mass and energy balances in the drum, risers, and downcomers. The system inputs are feedwater input, steam demand, and heat input.

3.2. Strategy for Control

The stability of the system can be altered due to disturbances such as increased demand. To prevent this, a three-element cascade control architecture is used. It consists of:

- The internal control loop modifies the feedwater flow.
- The external control loop maintains the water level in the drum within the specified range.

PID controllers are used to implement both loops, and Cohen and Coon tuning was used for the controller gains and trial and error simulation to modify the parameters.

3.3. Simulation Environment

MATLAB/Simulink R2024a was used to implement the entire model using blocks to assemble the equations. The simulation was run with a fixed-step solver (e.g., ode4 Runge-Kutta) and a step size of 0.01 s.

To simulate a medium-capacity drum boiler operating under nominal conditions, the essential parameters were taken from the model in [1] and modified as necessary:

3.4. Validation and Comparison

The graphs generated after simulating the three-element cascade control and single-element control were interpreted and compared according to the system response speed, stabilization time, overshoot percentage, and steady-state error through disturbances in the inputs, such as variations in load demand over time, changes in heat input to the system, changes in steam output flow, and changes in the setpoint of the controllers.

4. Theoretical Foundations

4.1. Shrink-and-Swell Effect

Abnormal variations in the water level in the drum, especially when the boiler load varies, are the result of the shrinkage and swelling effect. The physical arrangement of the steam generator tubes that are installed in the boiler has a direct bearing on this effect [7]. During normal operation, the tubes exposed to the radiant heat of the flame continuously generate steam. As the steam rises through these tubes, it also carries water towards the steam drum.

The tubes are classified into two types: riser tubes, which carry steam and water upwards, and descender tubes, which carry water towards the mud drum. Depending on the combustion rate, the same tube can behave as an ascending or descending tube, except for those that are directly exposed to radiant heat.

The circulation of water within the steam generator, driven by the heat flow, is the main cause of the dynamic behavior of the shrinkage and swelling effect, as shown in Figure 2.

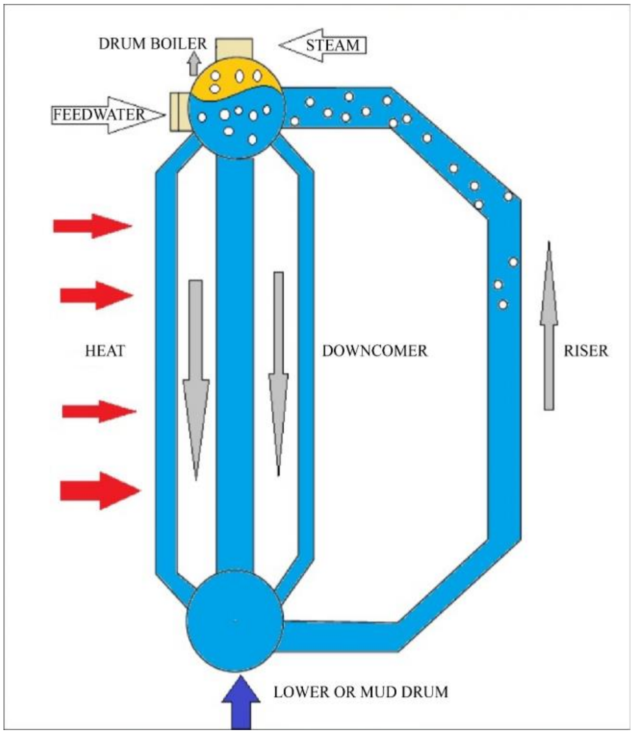

Fig. 2 Drum-Boiler schematic

Because the combustion rate cannot instantly adapt to the new demand level, this abrupt increase in load results in an instantaneous drop in steam drum pressure. A portion of the saturated water in the tubes spontaneously steamizes as a result of the pressure drop, greatly increasing the amount of steam generated in practically all of the tubes and momentarily transforming them into upward tubes. The liquid level in the steam drum rises as a result.

However, the level rise is a transient reaction to the load change. The increase in steam output causes a decrease in the water mass in the system, but the water level will continue to rise. The level controller reduces the feedwater flow rate upon detecting an increase, and this is dangerous.

This phenomenon is known as dynamic swelling. On the contrary, when a sudden load drop occurs, a dynamic contraction can be observed, although its impact on the natural circulation of the boiler is much less severe. In these cases, the decrease in drum level is usually of lesser magnitude and duration compared to the effect caused by swelling [8].

4.2. Cascade Control

Cascade control is a control strategy that is very common in the process industry. The main objective of this strategy is to improve the system's behavior when there are unmeasurable distractions and thus overcome the limitations that single-loop control strategies usually have. This control scheme has two nested control loops: the master controller, also called the leader, regulates the main process, while the slave controller, or follower, is responsible for subsection [3].

One of the main characteristics of cascade control is that it allows for the measurement of several process variables, even though only one manipulated variable can be acted upon. The slave loop, which is usually faster, is the first to act to correct the issues affecting the section. This means those same disturbances do not bother the main variable much, allowing the master controller to continue working well in its auxiliary regulation purpose.

The adjustment of these controllers follows a hierarchical approach: first, the secondary controller is adjusted to stabilize the inner loop, and then the primary controller is adjusted, considering the combined dynamics of the inner system. Given that many industrial processes exhibit nonlinear behavior and asymmetric dynamics, properly configuring the outer loop can be challenging. It is essential to take into account the response of the secondary loop to achieve an effective adjustment of the master controller [10].

4.3. Single-Element Control and Three-Element Control

Currently, most medium and high-pressure industrial boilers employ advanced control strategies to regulate the water level inside the steam drum. One of the most commonly used is the three-element control scheme, which is usually illustrated in boiler system diagrams.

To implement the 3-element control, three Process Variables (PV) are used, which are measured in real time to regulate the operation of the water valve. These variables are:

- The heat input to the system
- The flow of feedwater into the drum
- The flow of steam from the drum

The water level is kept within an ideal and constrained range, low enough to provide adequate steam decoupling volume and high enough to keep all of the boiler tubes submerged and operational.

A control system with two different configurations, a three-element loop and a single-element loop, is shown in:

Many boiler systems have simple control architectures, such as single-element control, where the water level controller only regulates the drum level and acts on the water supply valve. This method is often deficient when subject to sudden changes in feedwater pressure or steam demand, but it is useful in small boilers. The two-element control strategy emerged to overcome these restrictions. This control strategy uses the incoming water flow in addition to the boiler drum level. The drum level controller generates the control signal for the water flow controller, thus forming a cascade control structure. The drum level controller provides the setpoint to the water flow controller, thus forming a cascade control structure. In this way, the flow controller adjusts the water supply valve more quickly to compensate for pressure variations.

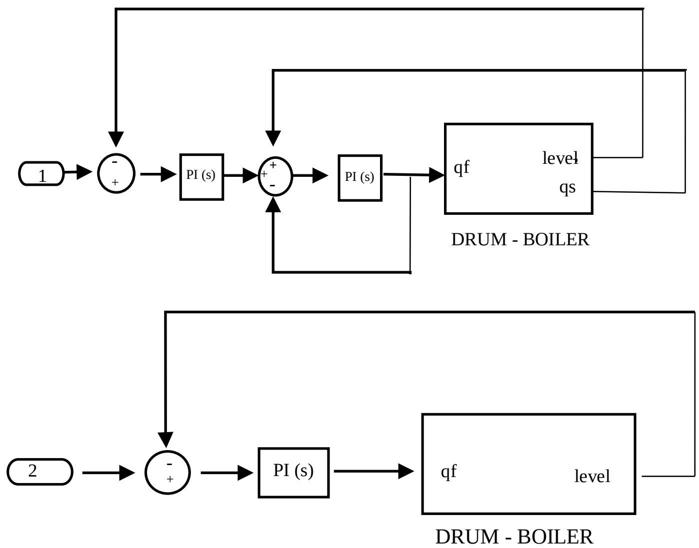

Fig. 3 Three-element control and single-element control

The three-element control strategy adds a third variable: the steam flow that exits the drum, which represents the main disturbance in most boiler systems-variations in steam demand cause changes in the drum's mass balance. By measuring the flow, the control system can implement a direct feedforward action, adjusting the target feedwater flow rate before the level varies significantly.

The system calculates the difference between the feedwater inflow and steam outflow rates to anticipate sudden changes in demand, rather than relying solely on the steam flow rate. This difference is related to the control point of the caudal of food and water. For example, if the turbine starts and the steam flow suddenly increases, the feedwater flow controller should increase the feedwater flow rate to maintain mass balance within the drum.

If both flow meters are correct in terms of the material balance, the flow correction will exactly match the steam loss, keeping the water level stable without adding more errors to the level control loop. If the demand for steam suddenly drops, like when a turbine shuts down, the flow of feedwater will also drop by the same amount, keeping the level stable without going over it. However, this direct feed strategy does not keep track of unmeasured losses like purging or steam venting through safety valves (which are located upstream of the steam flow meter). Moreover, direct feed cannot compensate for changes in the boiler's total water volume caused, for example, by the starting or stopping of the generating sections in forced circulation boilers. The level controller must compensate for discrepancies in these circumstances.

4.4. Matlab/Simulink

MATLAB and Simulink, developed by MathWorks, are among the most widely used platforms for controller design, especially in advanced model-based control methods, as they provide an interactive graphical environment within MATLAB for the simulation, analysis, and automation of complex dynamic systems [9]. A wide range of specialized toolboxes in MATLAB makes it easier to model, optimize, and design control systems. The System Identification Toolbox, Stateflow, Optimization Toolbox, and Control System Toolbox are some of these.

These tools make it possible to do everything from finding models to putting complicated control strategies into action. MATLAB also has functions for reducing model order, which is very important for making controllers based on reduced models. It also has Hardware-In-the-Loop (HIL) simulation features that let controllers built in very realistic virtual models of real plants be tested in real time.

5. Modeling and Simulation

The Åström-Bell model [1] describes nonlinear effects, such as the initial inverse response of the water level caused by the shrink and swell effect, which in turn is triggered by changes in steam demand. This model has many adjustable parameters for any boiler, as it includes volumes, metal masses, combustion, and other important parameters for a correct simulation.

5.1. Mass Balance in the Boiler Drum

The sum of the total volume of water V_{wt} and the total volume of steam V_{st} in the system, it is equal to the total volume V_t.

$$V_t = V_{st} + V_{wt} \quad (1)$$

The total mass of water and steam in the drum is:

$$m = \rho_w V_{wt} + \rho_s V_{st} \quad (2)$$

Where:

ρ_w : density of the water in the drum.
ρ_s : density of the steam in the drum.

The variation over time of the total mass of water and steam in the drum is equal to the difference between the mass flow rate of water entering the drum. q_f and the mass flow rate of steam exiting q_s.

$$\frac{d}{dt}[\rho_s V_{st} + \rho_v V_{wt}] = q_f - q_s \quad (3)$$

5.2. Energy Balance in the Boiler Drum

From equation (3), we obtain the energy balance, the internal energies, including the mass of the metal, are equal to the sum of the input heat flow and the difference between the energy that enters with the water flow and the energy that is carried by the generated steam.

$$\frac{d}{dt}\left[\rho_s V_{st} u_s + \rho_v V_{wt} u_w + m_t C_p t_m\right] = Q + q_f h_f - q_s h_s \quad (4)$$

Where:

u_s : Internal energy of the steam
u_w : Internal energy of the water
m_t : Mass of the drum metal
C_p : Specific heat of the metal
t_m : Average temperature of the metal (approximately equal to the saturation temperature of the water t_s) [5].
Q : Heat provided by the combustion
h_f : Enthalpy of water
h_s : Enthalpy of the steam

The first member of equation (4) represents the internal energy of the water, the steam, and the metal in the drum. The second member represents the energy flows, with $Q + q_f h_f$ being the energy entering the drum and $q_s h_s$ Being the energy leaving the drum.

5.3. Åström-Bell Model

From the mass and energy balances, it is possible to construct a second-order model. These balances result in a simple model that accurately represents the dynamics of the drum pressure.

However, this model does not simulate the water level, which is important for control purposes. To construct the Åström-Bell equations, it is necessary to start from mass and energy balances of water and steam in the drum.

By adding a circulation flow equation and an equation that describes the dynamics of the steam in the drum, the resulting equations can be manipulated to obtain a representation of the drum dynamics in a fourth-order state space [1], given by:

$$e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s \quad (5)$$

$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s \quad (6)$$

$$e_{32}\frac{dp}{dt} + e_{33}\frac{d\alpha_r}{dt} = Q - \alpha_r h_c q_{dc} \quad (7)$$

$$e_{42}\frac{dp}{dt} + e_{43}\frac{d\alpha_r}{dt} + e_{44}\frac{dV_{sd}}{dt} = \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd})\frac{h_f - h_w}{h_c} q_f \quad (8)$$

The previous equations represent a coupled system of differential equations that describe the nonlinear dynamics in the boiler drum.

Equation (5) represents the global mass balance in the system. Equation (6) represents the global energy balance in the system.

Equation (7) represents the energy balance in the riser tubes, and equation (8) represents the mass balance for steam below the liquid level, where:

p: pressure in the durm
α_r: steam quality (mass fraction of the generated steam)
h_c: condensation enthalpy
q_{dc}: recirculation flow through the downcomer.

Equation (8) is the dynamic equilibrium equation of the steam volume in the drum, where:

V_{sd}: Volume of water in the drum
V_{sd}^0: volume of steam in the drum when there is no condensation
T_d: residence time of the steam in the drum

The enthalpy of condensation is the heat released when a gas is converted to a liquid.

$$h_c = h_s - h_w \tag{9}$$

According to [1], the parameter q_{dc} The value of equation 7 can be calculated as follows:

$$q_{dc} = \sqrt{\frac{2\rho_w A_{dc}(\rho_w - \rho_s) g \bar{\alpha}_v V_r}{k}} \tag{10}$$

Where:

A_{dc}: cross-sectional area of the descending tubes
g: gravity
V_r: volume of the riser
$\bar{\alpha}_v$: average steam volume ratio
k: Friction coefficient of downcomer-riser loop

The coefficients e_{ij} are an important part of the Åström-Bell model, the deduction of each e_{ij} In Appendix 1, and their implementation in Matlab/Simulink is in Figure 4:

$$e_{11} = \rho_w - \rho_s \tag{11}$$

$$e_{12} = V_{wt}\frac{\partial \rho_w}{\partial p} + V_{st}\frac{\partial \rho_s}{\partial p} \tag{12}$$

$$e_{21} = \rho_w h_w - \rho_s h_s \tag{13}$$

$$e_{22} = V_{wt}\left(h_w \frac{\partial \rho_w}{\partial p} + \rho_w \frac{\partial h_w}{\partial p}\right) + V_{st}\left(h_s \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p}\right) - V_t + m_t C_p \frac{\partial t_s}{\partial p} \tag{14}$$

$$e_{32} = \left(\rho_w \frac{\partial h_w}{\partial p} - \alpha_r h_c \frac{\partial \rho_w}{\partial p}\right)(1-\bar{\alpha}_v)V_r + \left((1-\alpha_r)h_c \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p}\right)\bar{\alpha}_v V_r + (\rho_s + (\rho_w - \rho_s)\alpha_r)h_c V_r \frac{\partial \bar{\alpha}_v}{\partial p} - V_r + m_t C_p \frac{\partial t_s}{\partial p} \tag{15}$$

$$e_{33} = ((1-\alpha_r)\rho_s + \alpha_r \rho_w)h_c V_r \frac{\partial \bar{\alpha}_v}{\partial \alpha_r} \tag{16}$$

$$e_{42} = V_{sd}\frac{\partial \rho_s}{\partial p} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - V_{sd} - V_{wd} + m_d C_p \frac{\partial t_s}{\partial p}\right) + \alpha_r(1+\beta)V_r\left(\bar{\alpha}_v \frac{\partial \rho_s}{\partial p} + (1-\alpha_r)\frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w)\frac{\partial \bar{\alpha}_v}{\partial p}\right) \tag{17}$$

$$e_{43} = \alpha_r(1+\beta)(\rho_s - \rho_w)V_r \frac{\partial \bar{\alpha}_v}{\partial \alpha_r} \tag{18}$$

$$e_{44} = \rho_s \tag{19}$$

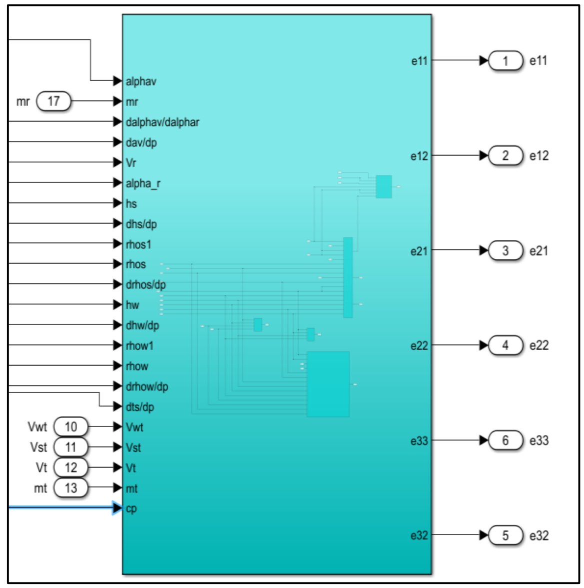

Fig. 4 Coefficients e_{ij} implemented in Simulink

The use of steam tables is necessary for the following parameters: $h_s, \frac{\partial h_s}{\partial p}, h_w, \frac{\partial h_w}{\partial p}, \rho_s, \frac{\partial \rho_s}{\partial p}, \rho_w, \frac{\partial \rho_w}{\partial p}, t_s, \frac{\partial t_s}{\partial p}$. It is possible to implement with lookup table blocks in Simulink.

In [4], approximations of these tables with quadratic equations, which depend on the pressure p of the boiler drum, can be found and will be implemented in Matlab/Simulink in Figure 5:

$$h_s = a_{01} + (a_{11} + a_{21} * (p-10)) * (p-10) \tag{20}$$

$$\frac{\partial h_s}{\partial p} = a_{11} + 2 * a_{21} * (p-10) \tag{21}$$

$$\rho_s = a_{02} + (a_{12} + a_{22} * (p-10)) * (p-10) \tag{22}$$

$$\frac{\partial \rho_s}{\partial p} = a_{12} + 2 * a_{22} * (p-10) \tag{23}$$

$$h_w = a_{03} + (a_{13} + a_{23} * (p-10)) * (p-10) \tag{24}$$

$$\frac{\partial h_w}{\partial p} = a_{13} + 2 * a_{23} * (p-10) \tag{25}$$

$$\rho_w = a_{04} + (a_{14} + a_{24} * (p-10)) * (p-10) \tag{26}$$

$$\frac{\partial \rho_w}{\partial p} = a_{14} + 2 * a_{24} * (p-10) \tag{27}$$

$$t_s = a_{05} + (a_{15} + a_{25} * (p-10)) * (p-10) \tag{28}$$

$$\frac{\partial t_s}{\partial p} = a_{15} + 2 * a_{25} * (p-10) \tag{29}$$

The coefficients a_{ij} The previous functions were extracted from [5] and are presented in Table 1:

Table 1. coefficients a_{ij} for the quadratic approximation of the steam properties table

Coefficients a_{ij}	Values
a_{01}	$2.73E+06$
a_{11}	$-1.90E+04$
a_{21}	-1160
a_{02}	53.1402
a_{12}	7.673
a_{22}	0.36
a_{03}	1.4035
a_{13}	$4.93E+04$
a_{23}	-880
a_{04}	691.35
a_{14}	-18.672
a_{24}	-0.0603
a_{05}	310.6
a_{15}	8.523
a_{25}	-0.33

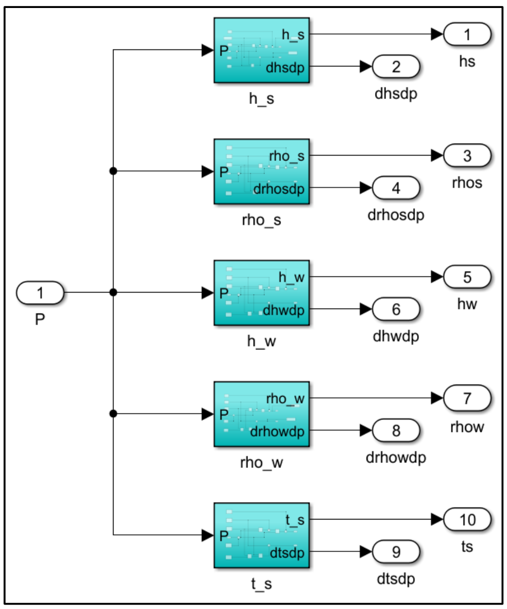

Fig. 5 Approximate steam tables with quadratic equations implemented in MATLAB/Simulink

The partial derivatives related to steam quality and average volume: $\frac{\partial\bar{\alpha}_v}{\partial p}, \frac{\partial\bar{\alpha}_v}{\partial\alpha_r}, \bar{\alpha}_v$ They were extracted from page 370 of [1]. Figure 6 is implemented in MATLAB/Simulink.

$$\frac{\partial\bar{\alpha}_v}{\partial p} = \frac{1}{(\rho_w-\rho_s)^2}\left(\rho_w\frac{\partial\rho_s}{\partial p} - \rho_s\frac{\partial\rho_w}{\partial p}\right)\left(1 + \frac{\rho_w}{\rho_s}\frac{1}{1+\eta} - \frac{\rho_s+\rho_w}{\eta\rho_s}\ln(1+\eta)\right) \quad (30)$$

$$\frac{\partial\bar{\alpha}_v}{\partial\alpha_r} = \frac{\rho_w}{\rho_s\eta}\left(\frac{1}{\eta}\ln(1+\eta) - \frac{1}{1+\eta}\right) \quad (31)$$

Where η:

$$\eta = \alpha_r\left(\frac{\rho_w-\rho_s}{\rho_s}\right) \quad (32)$$

$$\bar{\alpha}_v = \frac{\rho_w}{\rho_w-\rho_s}\left(1 - \frac{\rho_s}{(\rho_w-\rho_s)\alpha_r}\ln\left(1 + \frac{\rho_w-\rho_s}{\rho_s}\alpha_r\right)\right) \quad (33)$$

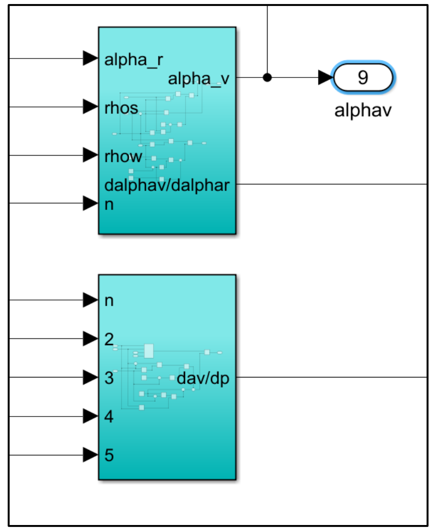

Fig. 6 $\frac{\partial\bar{\alpha}_v}{\partial p}, \frac{\partial\bar{\alpha}_v}{\partial\alpha_r}$ and $\bar{\alpha}_v$ implemented in Simulink

The equations of the Åström-Bell model must be converted to state-space equations to implement them in MATLAB/Simulink and assign an integrator to observe the graphs through the simulator's tools.

For equation (5), we solve for $\frac{dp}{dt}$

$$e_{12}\frac{dp}{dt} = q_f - q_s - e_{11}\frac{dV_{wt}}{dt} \quad (34)$$

$$\frac{dp}{dt} = \frac{q_f - q_s - e_{11}\frac{dV_{wt}}{dt}}{e_{12}} \quad (35)$$

Substitute $\frac{dp}{dt}$ in (6)

$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{q_f - q_s - e_{11}\frac{dV_{wt}}{dt}}{e_{12}} = Q + q_f h_f - q_s h_s \quad (36)$$

$$\frac{e_{12}e_{21}\frac{dV_{wt}}{dt}}{e_{12}} + e_{22}\left(\frac{q_f - q_s - e_{11}\frac{dV_{wt}}{dt}}{e_{12}}\right) = Q + q_f h_f - q_s h_s \quad (37)$$

$$e_{12}e_{21}\frac{dV_{wt}}{dt} + e_{22}q_f - e_{22}q_s - e_{22}e_{11}\frac{dV_{wt}}{dt} = e_{12}Q + e_{12}q_f h_f - e_{12}q_s h_s \quad (38)$$

$$\frac{dV_{wt}}{dt}(e_{12}e_{21} - e_{11}e_{22}) + e_{22}q_f - e_{22}q_s = e_{12}Q + e_{12}q_f h_f - e_{12}q_s h_s \quad (39)$$

$$\frac{dV_{wt}}{dt}(e_{12}e_{21} - e_{11}e_{22}) = e_{12}Q + e_{12}q_f h_f - e_{12}q_s h_s - e_{22}q_f + e_{22}q_s \quad (40)$$

$$\frac{dV_{wt}}{dt}(e_{12}e_{21} - e_{11}e_{22}) = e_{12}Q + q_f\left(e_{12}h_f - e_{22}\right) + q_s(e_{22} - h_s e_{12}) \quad (41)$$

$$\frac{dV_{wt}}{dt} = \frac{e_{12}Q + q_f(e_{12}h_f - e_{22}) + q_s(e_{22} - h_s e_{12})}{(e_{12}e_{21} - e_{11}e_{22})} \quad (42)$$

From equation (7), we obtain $\frac{d\alpha_r}{dt}$:

$$e_{33}\frac{d\alpha_r}{dt} = Q - \alpha_r h_c q_{dc} - e_{32}\frac{dp}{dt} \quad (43)$$

$$\frac{d\alpha_r}{dt} = \frac{Q - \alpha_r h_c q_{dc} - e_{32}\frac{dp}{dt}}{e_{33}} \quad (44)$$

From equation (8), we obtain $\frac{dV_{sd}}{dt}$:

$$e_{44}\frac{dV_{sd}}{dt} = \frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) + \frac{h_f - h_w}{h_c}q_f - e_{42}\frac{dp}{dt} - e_{43}\frac{d\alpha_r}{dt} \quad (45)$$

$$\frac{dV_{sd}}{dt} = \frac{\frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) + \frac{h_f - h_w}{h_c}q_f - e_{42}\frac{dp}{dt} - e_{43}\frac{d\alpha_r}{dt}}{e_{44}} \quad (46)$$

The state variables are: drum pressure p, total water volume V_{wt}, steam quality α_r and the steam volume in the drum V_{sd}.

The inputs are the water flow rate. q_f[2], the steam flow rate, and the heat flow rate Q. Figure 7 shows their implementation in MATLAB/Simulink.

$$\frac{dV_{wt}}{dt} = \frac{e_{12}Q + q_f(e_{12}h_f - e_{22}) + q_s(e_{22} - h_s e_{12})}{(e_{12}e_{21} - e_{11}e_{22})} \quad (47)$$

$$\frac{dp}{dt} = \frac{q_f - q_s - e_{11}\dot{V}_{wt}}{e_{12}} \quad (48)$$

$$\frac{d\alpha_r}{dt} = \frac{Q - \alpha_r h_c q_{dc} - e_{32}\dot{P}}{e_{33}} \quad (49)$$

$$\frac{dV_{sd}}{dt} = \frac{\frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) + \frac{h_f - h_w}{h_c}q_f - e_{42}\dot{P} - e_{43}\dot{\alpha_r}}{e_{44}} \quad (50)$$

The water level in the drum is:

$$l = \frac{V_{wd} + V_{sd}}{A_d} \quad (51)$$

Where V_{wd} is:

$$V_{wd} = V_{wt} - V_{dc} - (1 - \alpha_v)V_r \quad (52)$$

The water level is an output of the system as seen in Figure 10. Parameters such as water volume in the drum V_{wd}, h_f and q_{dc} They are implemented in Figure 8.

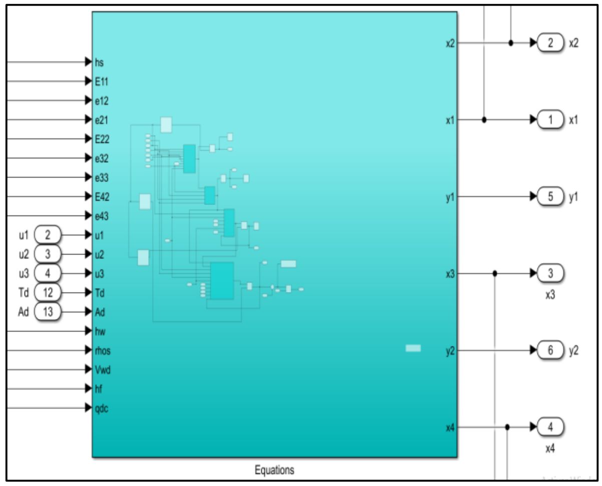

Fig. 7 Equation (47), (48), (49), and (50) implemented in MATLAB/Simulink

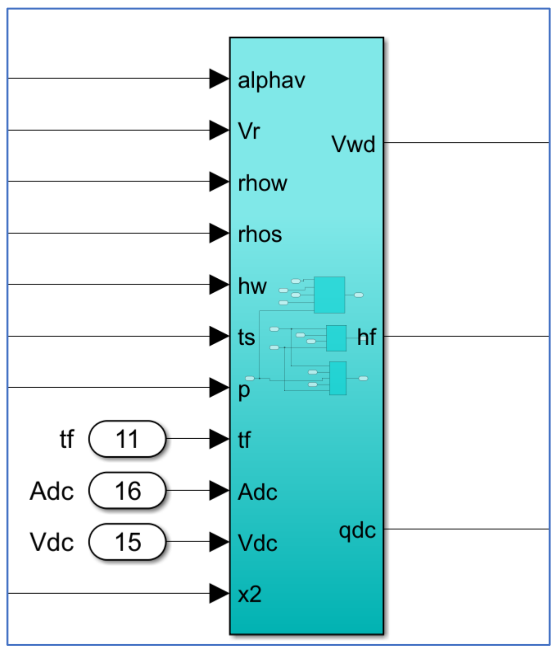

Fig. 8 Block that contains V_{wd}, h_f and q_{dc}

5.4. Plant Parameters

The operating parameters were taken from the Timisoara South thermal power plant [2]. $V_d = 31m^3$, $m_t = 18000kg$, $m_r = 16000kg$, $m_d = 2000kg$, $\beta = 0.3$, $k = 25$, $T_d = 12s$, $V_{sd}^0 = 8m^3$, $V_{dc} = 11.41m^3$, $V_r = 15m^3$, $A_d = 20m^2$, $A_{dc} = 1.36m^2$, $C_p = 650J/kg$ and $T_{fw} = 104\,°C$.

The authors mention that it was difficult to find some values, such as $T_d, k, \beta,$ and V_{sd}^0 which were taken from the original model [2].

Additionally, the superheater from [11] was used so that the steam flow is an output of the system in Figure 9.

$$q_s = K_v * \left[(PD - PT) * \frac{1}{K_{SH}}\right]^{0.66} \quad (53)$$

Where:

K_v: steam valve opening
PD: Drum pressure
PT: Turbine pressure
K_{SH}: friction coefficient

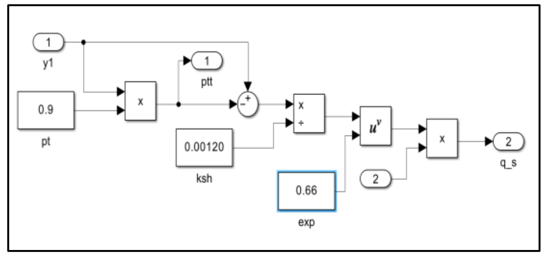

Fig. 9 Superheater in MATLAB/Simulink

In Figure 10, the fully implemented fourth-order Åström-Bell boiler model is divided into several blocks, as previously presented in the earlier figures.

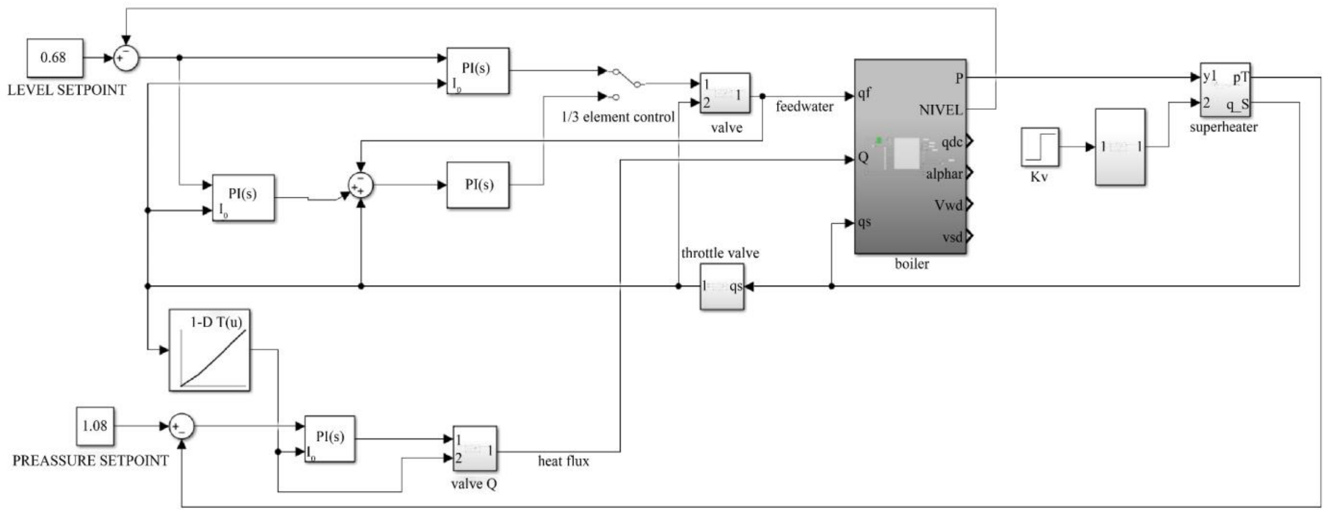

Fig. 10 Drum-boiler model implemented in Matlab/Simulink

6. Results and Discussion

To validate the model in MATLAB/Simulink, two open-loop and closed-loop tests must be performed. Table 2 contains the initial values obtained directly from the simulation. The initial values of the integrators were obtained directly from the simulation and are shown in Table 2.

This table provides uniform initial conditions for each scenario, eliminating the need to adjust the integrators in MATLAB/Simulink manually.

Table 2. Initial conditions for Åström–Bell nonlinear model integrators

Steam demand (kg/s)	$V_{wt_i}(m^3)$	α_{r_i}	$V_{sd_i}(m^3)$
6	31.1	0.0006	6.5
9	31.2	0.0073	6
11	31.3	0.0087	5.7
12.92	31.4	0.01	5
15	31.5	0.0112	4.7
17.22	31.74	0.0124	4.3

6.1. Open Loop

The system has no feedback. A step change is applied to the heat input, while the inlet and outlet flows remain fixed. In Figure 11, the heat flow was increased, as the water and steam flows are maintained. At the water level, a swell effect occurs before it drops from its normal value, and the water volume in the drum begins to decrease because more water starts to turn into steam. Consequently, the steam quality begins to increase, and despite a small drop, the steam volume in the drum also increases thanks to the increase in heat flow. The flow in the downcomer also begins to decrease due to the reduction of water in the system.

This test can not only be performed with the heat input, but it is also possible to do it by modifying the steam flow valve opening, keeping the other values fixed. Figure 12 reduces the flow from 23.7 kg/s to 13 kg/s. The pressure increases because the same steam continues to be produced, but not all of it is released, so it accumulates dangerously. The shrink effect is visible in the water level, but it begins to rise because the water does not turn into steam.

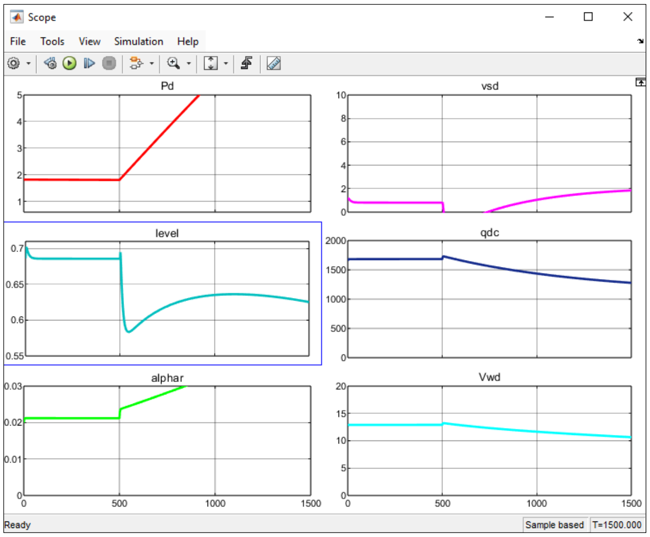

Fig. 11 System response to increased heat flow

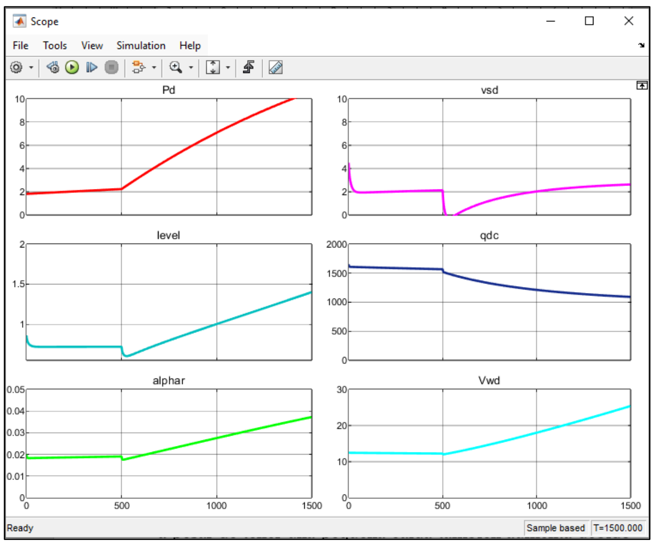

Fig. 12 System response to the reduction of steam outlet flow

6.2. Closed loop

The system uses feedback to compare the output with a setpoint and adjust the inputs. Two ways to test this mode are: One-element and three-element level control. In one-element control, only the water flow through the level is controlled. The Ziegler-Nichols tuning is very easy to apply to this control.

For the case of three elements, an internal control is added, which is the difference between the water and steam flow. This result is fed forward to the internal controller. A positive result indicates an increase in steam production; therefore, the controller must open the q_f Valve to mitigate the swell effect. Conversely, if the difference is negative, the controller must counteract the shrink effect by closing the water inlet valve as needed to prevent water accumulation in the drum.

A 50% load is used for the first test, opening the valve from 60 to 75%. Figure 13 shows the changes in the input flow caused by the variation in steam demand, as shown in Figure 14. Here, the swell effect occurs, the water level increases in Figure 15, the single-element control is slower and orders a decrease in the input flow, the three-element controller reacts faster by increasing the input flow, because despite the level rising, the water volume is actually decreasing since more water is being converted into steam than is entering. The pressure in Figure 16 decreases due to the steam output, but thanks to the heat flow controller, it stabilizes back to its setpoint.

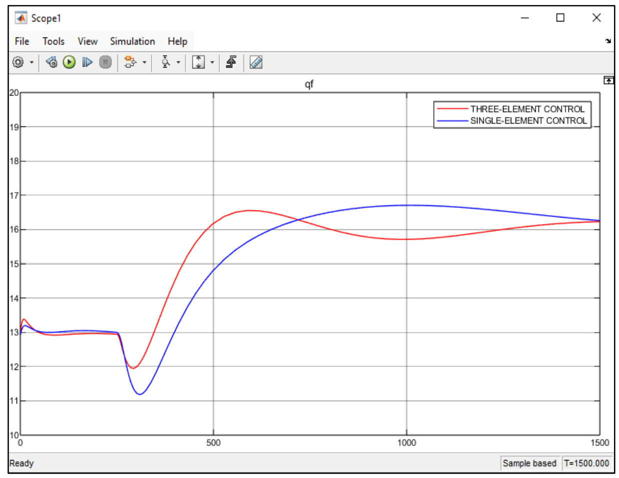

Fig. 13 Water inlet flow at an opening of 60% to 75% at a load of 50%

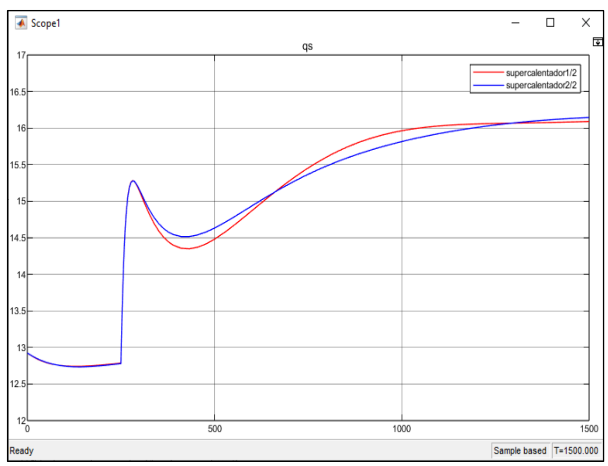

Fig. 14 Steam outlet flow at an opening of 60% to 75% at a load of 50%

In the second test, a load of 75% is used, opening the valve from 75% to 90%. Figure 17 shows the water inlet flow at a 15% opening. Figure 18 shows the swell effect on the water level in the drum for both controllers. The single-element controller takes longer to stabilize, while the three-element cascade control is more effective due to its quick action and faster stabilization. The pressure in Figure 19 experiences a drop due to the increase in q_s, but due to the effect of the heat flow controller, it returns to its setpoint.

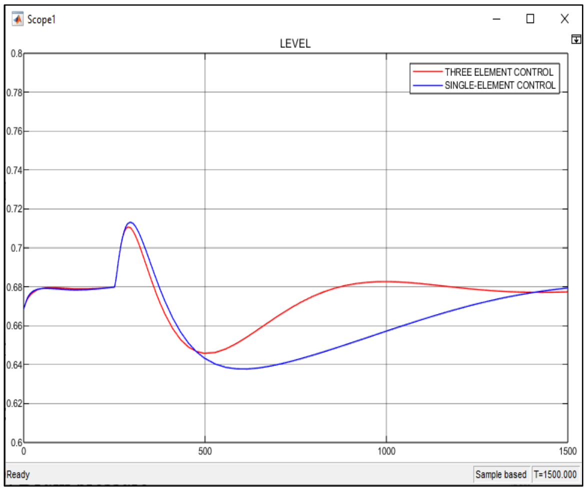

Fig. 15 Water level at an opening of 60% to 75% at a load of 50%

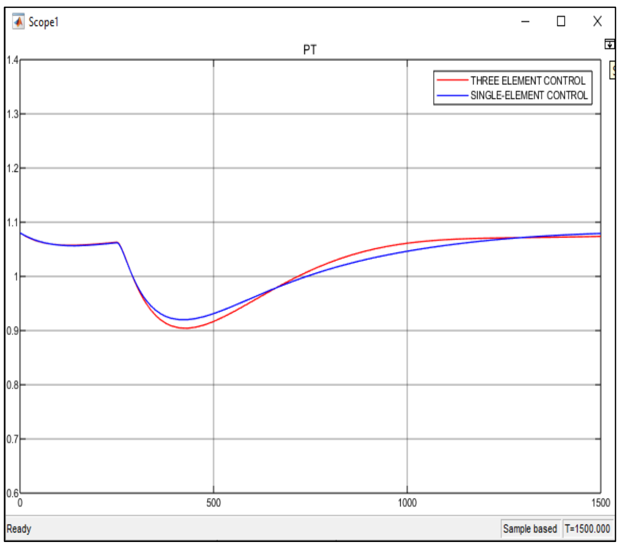

Fig. 16 Pressure at an opening of 60% to 75% at a load of 50%

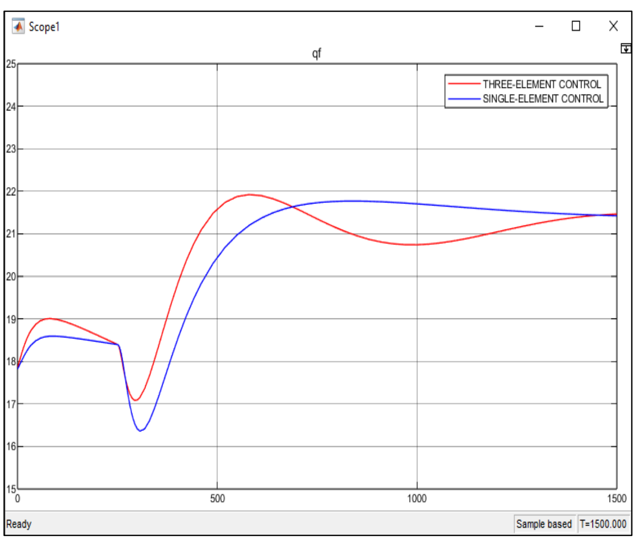

Fig. 17 Water inflow at an opening of 75% to 90% at a load of 75%

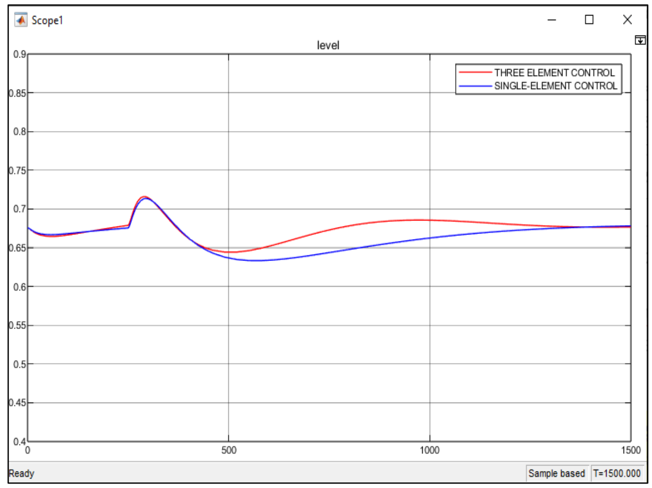

Fig. 18 Water level at an opening of 75% to 90% at a load of 75%

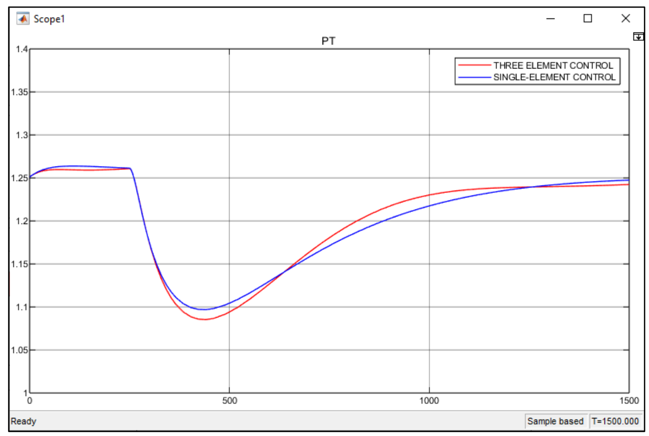

Fig. 19 Pressure at an opening of 75% to 90% at a load of 75%

7. Conclusion

This article has shown how to create and test an automatic control system for a drum boiler's water level and steam pressure using MATLAB/Simulink simulations. The work is based on Åström and Bell's nonlinear dynamic model, which is a fourth-order state-space representation of a natural-circulation drum boiler. This model was expanded and modified to facilitate a comprehensive case study that emphasizes distinctive dynamic phenomena, including shrink-and-swell. Open-loop simulations showed that the model could accurately reproduce important physical behaviors when there were disturbances. On the other hand, closed-loop studies tested the effectiveness of one-element and three-element control strategies. The results showed that the three-element cascade controller with feedforward action is better at rejecting disturbances, stabilizing faster, and compensating for shrink-and-swell effects than the standard one-element scheme. The simulator itself is also a big help because it gives people a way to study boiler dynamics, train operators, and teach technical skills. The current work makes sure that all equations, coefficients, and Simulink block structures are fully documented. This makes the work completely transparent and replicable, which provides a strong base for future research in this field. This study is different from previous ones because it provides the first fully documented and corrected MATLAB/Simulink implementation of the nonlinear drum-boiler model. This makes it possible to replicate, be open about, and use directly in industry. In conclusion, our replication study both confirms and builds on the original Åström–Bell model [2]. We have confirmed the original results and fixed mistakes that were not fixed before by fully re-deriving and documenting the boiler equations. The model is now in a format many people use (MATLAB/Simulink), and all equations and initial conditions are well-documented. Because of this, our work makes a strong, repeatable simulation of a natural-circulation boiler. These extensive efforts make the model clearer, make sure its predictions are accurate (in line with how boilers actually work), and lay a strong foundation for future research and real-world use in boiler control.

Acknowledgments

The authors express our gratitude to the National University of San Agustín (UNSA) for the academic training provided, which has been fundamental for the preparation of this article. Likewise, we extend our special recognition to Dr. Fernando, Dr. Pedro, and Dr. Germán, whose valuable observations and suggestions were key during the development of this research.

References

[1] K.J. Åström, and R.D. Bell, "Drum-Boiler Dynamics," *Automatica*, vol. 36, no. 3, pp. 363-378, 2000. [CrossRef] [Google Scholar] [Publisher Link]

[2] Mihai Iacob, and Gheorghe-Daniel Andreescu, "Drum-Boiler Control System Employing Shrink and Swell Effect Remission in Thermal Power Plants," *2011 3rd International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, Budapest, Hungary, pp. 1-8, 2011. [Google Scholar] [Publisher Link]

[3] G.F. (Jerry) Gilman, *Boiler Control Systems Engineering*, 2nd ed., International Society of Automation, 2010. [Google Scholar] [Publisher Link]

[4] K.J. Åström, and R.D. Bell, "Simple Drum-Boiler Models," *Power Systems: Modelling and Control Applications*, pp. 123-127, 1989. [CrossRef] [Google Scholar] [Publisher Link]

[5] Karl Johan Åström, and Rodney D. Bell, "*Drum-Boiler Dynamics*," Technical Reports TFRT-7577, Department of Automatic Control, Lund Institute of Technology (LTH), 1998. [Google Scholar] [Publisher Link]

[6] Technical Information ITSI-12, Automatic Level Control Systems in Steam Boilers, Industrial Boiler Systems, S.L., Sincal Industrial Boilers, 2018. [Online]. Available: https://sincal.es/descargas/ITSI-12-Alimentacion-de-agua.pdf

[7] Allen D. Houtz, Dynamic Shrink/Swell and Boiler Level Control, Cascade, Feed Forward and Three, Control Guru, 2015. [Online]. Available: https://controlguru.com/dynamic-shrinkswell-and-boiler-level-control/

[8] Nguyen Phuong Dong, Shrink and Swell, SlideShare, 2025. [Online]. Available: https://www.slideshare.net/slideshow/shrink-and-swell/54632166

[9] Engineering, MATLAB Simulink, ScienceDirect, 2023. [Online]. Available: https://www.sciencedirect.com/topics/engineering/matlab-simulink

[10] Engineering, Cascade Control, ScienceDirect, 2022. [Online]. Available: https://www.sciencedirect.com/topics/engineering/cascade-control

[11] C. Maffezzoni, "Boiler-Turbine Dynamics in Power-Plant Control," *Control Engineering Practice*, vol. 5, no. 3, pp. 301-312, 1997. [CrossRef] [Google Scholar] [Publisher Link]

[12] Gang Su et al., "Transformer-Based Drum-Level Prediction in a Boiler Plant with Delayed Relations among Multivariates," *2024 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)*, Dubrovnik, Croatia, pp. 1-5, 2024. [CrossRef] [Google Scholar] [Publisher Link]

[13] Kairui Liu et al., "Dynamic Performance Analysis and Control Strategy Optimization for a Supercritical Coal-Fired Boiler," *Energy*, vol. 282, 2023. [CrossRef] [Google Scholar] [Publisher Link]

[14] Yihua Ye, Haiqiang Lin, and Hua Zhou, "Modeling and Prediction of Key Parameters of Circulating Fluidized Bed Boiler Based on Transformer-CNN Fusion Model," *Journal of the Taiwan Institute of Chemical Engineers*, vol. 162, 2024. [CrossRef] [Google Scholar] [Publisher Link]

Appendix 1: Math Models

Deduction of Equation 5

The Mass Balance Equation, $\frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) = q_f - q_s$. Indeed,

$$\frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) = \rho_\omega \frac{dV_\omega}{dt} + V_\omega \frac{d\rho_\omega}{dt} + \rho_s \frac{dV_s}{dt} + V_s \frac{d\rho_s}{dt} \quad \text{(A1)}$$

On the other hand,

$$\left.\begin{aligned} \frac{d\rho_\omega}{dt} &= \frac{\partial\rho_\omega}{\partial p}\frac{dp}{dt} \\ \frac{d\rho_s}{dt} &= \frac{\partial\rho_s}{\partial p}\frac{dp}{dt} \end{aligned}\right\} \quad \text{(A2)}$$

Where p is the drum pressure. So, using (A2) in (A1) gives,

$$\frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) = \rho_\omega \frac{dV_\omega}{dt} + V_\omega \frac{\partial\rho_\omega}{\partial p}\frac{dp}{dt} + \rho_s \frac{dV_s}{dt} + V_s \frac{\partial\rho_s}{\partial p}\frac{dp}{dt}$$

Since the volume of the drum is the volume of steam plus the volume of water: $V_t = V_\omega + V_s$It follows that $\frac{d}{dt}V_s = \frac{d}{dt}V_t - \frac{d}{dt}V_\omega$. Then,

$$\begin{aligned} \frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) &= \rho_\omega \frac{dV_\omega}{dt} + V_\omega \frac{\partial\rho_\omega}{\partial p}\frac{dp}{dt} + \rho_s\left(\frac{dV_t}{dt} - \frac{dV_\omega}{dt}\right) + V_s \frac{\partial\rho_s}{\partial p}\frac{dp}{dt} \\ &= \rho_\omega \frac{dV_\omega}{dt} + V_\omega \frac{\partial\rho_\omega}{\partial p}\frac{dp}{dt} + \rho_s \frac{dV_t}{dt} - \rho_s \frac{dV_\omega}{dt} + V_s \frac{\partial\rho_s}{\partial p}\frac{dp}{dt} \\ &= (\rho_\omega - \rho_s)\frac{dV_\omega}{dt} + \left(V_\omega \frac{\partial\rho_\omega}{\partial p}\frac{dp}{dt} + V_s \frac{\partial\rho_s}{\partial p}\frac{dp}{dt}\right) + \rho_s \frac{dV_t}{dt} \\ &= (\rho_\omega - \rho_s)\frac{dV_\omega}{dt} + \left(V_\omega \frac{\partial\rho_\omega}{\partial p} + V_s \frac{\partial\rho_s}{\partial p}\right)\frac{dp}{dt} + \rho_s \frac{dV_t}{dt} \end{aligned}$$

Since V_t is constant

$$\frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) = \underbrace{(\rho_\omega - \rho_s)}_{e_{11}}\frac{dV_\omega}{dt} + \underbrace{\left(V_\omega \frac{\partial\rho_\omega}{\partial p} + V_s \frac{\partial\rho_s}{\partial p}\right)}_{e_{12}}\frac{dp}{dt}$$

$$\frac{d}{dt}(\rho_\omega V_\omega + \rho_s V_s) = e_{11}\frac{dV_\omega}{dt} + e_{12}\frac{dp}{dt}$$

From the mass balance equation, it is finally obtained.

$$e_{11}\frac{dV_\omega}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s$$

Deduction of Equation 6

From the global energy balance:

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = Q + q_f h_f - q_s h_s$$

where m_t The total mass of the system is the total mass of the metal tubes together with the mass of the drum, which is indeed constant. On the other hand, C_p It is the specific heat of the metal, which is also constant. V_{wt} and V_{st}, denote the total volume of water and steam, respectively; finally, t_m Is the metal temperature, which can be considered as a function of pressure. Indeed,

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = \frac{d}{dt}(\rho_w V_{wt} u_w) + \frac{d}{dt}(\rho_s V_{st} u_s) + \frac{d}{dt}\left(m_t C_p t_m\right)$$

Then,

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) =$$
$$\rho_w u_w \frac{dV_{wt}}{dt} + V_{wt}\frac{d}{dt}(\rho_w u_w) + \rho_s u_s \frac{dV_{st}}{dt} + V_{st}\frac{d}{dt}(\rho_s u_s) + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = \rho_w u_w \frac{dV_{wt}}{dt} + \rho_s u_s \frac{dV_{st}}{dt} +$$
$$+V_{st}\frac{d}{dt}(\rho_s u_s) + V_{wt}\frac{d}{dt}(\rho_w u_w) + m_t C_p \frac{dt_m}{dt}$$

Using that $V_{st} + V_{wt}$ It is constant; it follows that. $\frac{dV_{st}}{dt} = -\frac{dV_{wt}}{dt}$After substituting into the previous equality, the following is obtained:

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = \rho_w u_w \frac{dV_{wt}}{dt} - \rho_s u_s \frac{dV_{wt}}{dt} +$$
$$+V_{st}\frac{d}{dt}(\rho_s u_s) + V_{wt}\frac{d}{dt}(\rho_w u_w) + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w u_w - \rho_s u_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\frac{d}{dt}(\rho_s u_s) + V_{wt}\frac{d}{dt}(\rho_w u_w) + m_t C_p \frac{dt_m}{dt}$$

As the internal energy is given by

$$u = h - \frac{p}{\rho}$$

In particular, it holds that,

$$u_w = h_w - \frac{p}{\rho_w}$$
$$u_s = h_s - \frac{p}{\rho_s}$$

Substituting into the previous equality yields the following result.

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = \left(\rho_w\left(h_w - \frac{p}{\rho_w}\right) - \rho_s\left(h_s - \frac{p}{\rho_s}\right)\right)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\frac{d}{dt}\left(\rho_s\left(h_s - \frac{p}{\rho_s}\right)\right) + V_{wt}\frac{d}{dt}\left(\rho_w\left(h_w - \frac{p}{\rho_w}\right)\right) + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\frac{d}{dt}(\rho_s h_s - p) + V_{wt}\frac{d}{dt}(\rho_w h_w - p) + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\left(h_s\frac{d\rho_s}{dt} + \rho_s\frac{dh_s}{dt} - \frac{dp}{dt}\right) + V_{wt}\left(h_w\frac{d\rho_w}{dt} + \rho_w\frac{dh_w}{dt} - \frac{dp}{dt}\right) + m_t C_p \frac{dt_m}{dt}$$

Temperature, density, and enthalpy are dependent on pressure.

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\left(h_s\frac{\partial\rho_s}{\partial p}\frac{dp}{dt} + \rho_s\frac{\partial h_s}{\partial p}\frac{dp}{dt} - \frac{dp}{dt}\right) + V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p}\frac{dp}{dt} + \rho_w\frac{\partial h_w}{\partial p}\frac{dp}{dt} - \frac{dp}{dt}\right) + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+V_{st}\left(h_s\frac{\partial\rho_s}{\partial p} + \rho_s\frac{\partial h_s}{\partial p} - 1\right)\frac{dp}{dt} + V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p} + \rho_w\frac{\partial h_w}{\partial p} - 1\right)\frac{dp}{dt} + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+\left[V_{st}\left(h_s\frac{\partial\rho_s}{\partial p} + \rho_s\frac{\partial h_s}{\partial p}\right) - V_{st}\right]\frac{dp}{dt} + \left[V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p} + \rho_w\frac{\partial h_w}{\partial p}\right) - V_{wt}\right]\frac{dp}{dt} + m_t C_p \frac{dt_m}{dt}$$
$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = (\rho_w h_w - \rho_s h_s)\left(\frac{dV_{wt}}{dt}\right) +$$
$$+[V_{st}\left(h_s\frac{\partial\rho_s}{\partial p} + \rho_s\frac{\partial h_s}{\partial p}\right) + V_{wt}\left(h_w\frac{\partial\rho_w}{\partial p} + \rho_w\frac{\partial h_w}{\partial p}\right) - \underbrace{(V_{wt} + V_{st})}_{V_t}]\frac{dp}{dt} + m_t C_p \frac{dt_m}{dt}$$

Since $\frac{dt_m}{dt} = \frac{\partial t_m}{\partial p}\frac{dp}{dt}$It has been experimentally verified that $\frac{\partial t_m}{\partial p} = \frac{\partial t_s}{\partial p}$. Now, let us define,

$$e_{21} = \rho_w h_w - \rho_s h_s$$
$$e_{22} = V_{st}\left(h_s \frac{\partial \rho_s}{\partial p} + \rho_s \frac{\partial h_s}{\partial p}\right) + V_{wt}\left(h_w \frac{\partial \rho_w}{\partial p} + \rho_w \frac{\partial h_w}{\partial p}\right) - V_t + m_t C_p \frac{\partial t_s}{\partial p}$$

It follows that

$$\frac{d}{dt}\left(\rho_w V_{wt} u_w + \rho_s V_{st} u_s + m_t C_p t_m\right) = e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt}$$

therefore

$$e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s$$

Deduction of Equation 7

The mass in the risers is given by $M = \rho_s \bar{\alpha}_v V_r + \rho_w (1 - \bar{\alpha}_v) V_r$, where the first term represents the mass of steam in the risers and the second term the mass of water in the risers. Therefore, the mass balance in the risers is given by:

$$\frac{dM}{dt} = \frac{d}{dt}\left(\rho_s \bar{\alpha}_v V_r + \rho_w (1 - \bar{\alpha}_v) V_r\right) = q_{dc} - q_r$$

where,

ρ_s: steam density in the drum
ρ_w: water density in the drum
$\bar{\alpha}_v$: average steam volume fraction
V_r: Volume of the risers
q_{dc}: downcomer flow entering the risers
q_r: outlet flow from the risers

The total energy in the risers is $E = \rho_s h_s \bar{\alpha}_v V_r + \rho_w h_w (1 - \bar{\alpha}_v) V_r - pV_r + m_r C_p t_s$Therefore, the total energy balance in the risers is:

$$\frac{dE}{dt} = \frac{d}{dt}\left(\rho_s h_s \bar{\alpha}_v V_r + \rho_w h_w (1 - \bar{\alpha}_v) V_r - pV_r + m_r C_p t_s\right) = Q + q_{dc} h_w - q_r(\alpha_r h_c + h_w)$$

The expression $\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt}$ Will be considered. First, this expression will not be derived, but the corresponding parts of it will be simplified:

$$\begin{aligned}\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} &= Q + q_{dc} h_w - q_r(\alpha_r h_c + h_w) - (h_w + \alpha_r h_c)(q_{dc} - q_r) \\ &= Q + q_{dc} h_w - q_r(\alpha_r h_c + h_w) - h_w q_{dc} - \alpha_r h_c q_{dc} + (h_w + \alpha_r h_c) q_r\end{aligned}$$

Then,

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = Q - \alpha_r h_c q_{dc} \qquad \text{(A3)}$$

The derivatives $\frac{dE}{dt}$ y $\frac{dM}{dt}$, are computed to obtain an expression for $\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt}$

$$\begin{aligned}\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = \frac{d}{dt}\left(\rho_s h_s \bar{\alpha}_v V_r + \rho_w h_w (1 - \bar{\alpha}_v) V_r - pV_r + m_r C_p t_s\right) - \\ -(h_w + \alpha_r h_c)\frac{d}{dt}\left(\rho_s \bar{\alpha}_v V_r + \rho_w (1 - \bar{\alpha}_v) V_r\right)\end{aligned}$$

Taking V_r as constant,

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{\frac{d}{dt}(\rho_s h_s \bar{\alpha}_v + \rho_w h_w (1-\bar{\alpha}_v) - p) - \right.$$
$$\left. -(h_w + \alpha_r h_c)\frac{d}{dt}\big(\rho_s \bar{\alpha}_v + \rho_w(1-\bar{\alpha}_v)\big)\right\} + \frac{d}{dt}\big(m_r C_p t_s\big)$$
$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{h_s\frac{d}{dt}(\rho_s\bar{\alpha}_v) + \rho_s\bar{\alpha}_v\frac{dh_s}{dt} + h_w\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) + \right.$$
$$\left. +\rho_w(1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt} - (h_w+\alpha_r h_c)\frac{d}{dt}(\rho_s\bar{\alpha}_v) - (h_w+\alpha_r h_c)\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big)\right\} +$$
$$+\frac{d}{dt}\big(m_r C_p t_s\big)$$

Group the terms $\frac{d}{dt}(\rho_s\bar{\alpha}_v)$:

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{\big(h_s - (h_w+\alpha_r h_c)\big)\frac{d}{dt}(\rho_s\bar{\alpha}_v) + \rho_s\bar{\alpha}_v\frac{dh_s}{dt} + \right.$$
$$\left. +h_w\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) + \rho_w(1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt} - (h_w+\alpha_r h_c)\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big)\right\} +$$
$$+\frac{d}{dt}\big(m_r C_p t_s\big)$$

Group the terms $\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big)$:

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{\big(h_s - (h_w+\alpha_r h_c)\big)\frac{d}{dt}(\rho_s\bar{\alpha}_v) + \right.$$
$$\left. +\big(h_w - (h_w+\alpha_r h_c)\big)\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) + \rho_s\bar{\alpha}_v\frac{dh_s}{dt} + \rho_w(1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt}\right\} +$$
$$+\frac{d}{dt}\big(m_r C_p t_s\big)$$

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{\big((h_s-h_w) - \alpha_r h_c\big)\frac{d}{dt}(\rho_s\bar{\alpha}_v) - \alpha_r h_c\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) + \right.$$
$$\left. +\rho_s\bar{\alpha}_v\frac{dh_s}{dt} + \rho_w(1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt}\right\} + \frac{d}{dt}\big(m_r C_p t_s\big)$$

From $h_c = h_s - h_w$,

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} = V_r\left\{(h_c - \alpha_r h_c)\frac{d}{dt}(\rho_s\bar{\alpha}_v) - \alpha_r h_c\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) + \rho_s\bar{\alpha}_v\frac{dh_s}{st} + \rho_w(1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt}\right\} + \frac{d}{dt}\big(m_r C_p t_s\big) \quad \text{(A4)}$$

Furthermore,

$$\frac{d}{dt}(\rho_s\bar{\alpha}_v) = \bar{\alpha}_v\frac{d\rho_s}{dt} + \rho_s\frac{d\bar{\alpha}_v}{dt}$$
$$\frac{d}{dt}(\rho_s\bar{\alpha}_v) = \bar{\alpha}_v\frac{\partial\rho_s}{\partial p}\frac{dp}{dt} + \rho_s\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\frac{d\alpha_r}{dt} + \rho_s\frac{\partial\bar{\alpha}_v}{\partial p}\frac{dp}{dt} \quad \text{(A5)}$$
$$\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) = (1-\bar{\alpha}_v)\frac{d\rho_w}{dt} + \rho_w\frac{d(1-\bar{\alpha}_v)}{dt} \quad \text{(A5)}$$
$$= (1-\bar{\alpha}_v)\frac{d\rho_w}{dt} - \rho_w\frac{d\bar{\alpha}_v}{dt} \quad \text{(A5)}$$
$$\frac{d}{dt}\big(\rho_w(1-\bar{\alpha}_v)\big) = (1-\bar{\alpha}_v)\frac{\partial\rho_w}{\partial p}\frac{dp}{dt} - \rho_w\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\frac{d\alpha_r}{dt} - \rho_w\frac{\partial\bar{\alpha}_v}{\partial p}\frac{dp}{dt} \quad \text{(A6)}$$

Substituting (A5) and (A6) into (A4) leads to the following result:

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} =$$
$$V_r\left\{(h_c - \alpha_r h_c)\left(\bar{\alpha}_v \frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + \rho_s \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} + \rho_s \frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt}\right) -\right.$$
$$-\alpha_r h_c\left((1-\bar{\alpha}_v)\frac{\partial \rho_w}{\partial p}\frac{dp}{dt} - \rho_w \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} - \rho_w \frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt}\right) +$$
$$\left.+\rho_s \bar{\alpha}_v \frac{dh_s}{dt} + \rho_w (1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt}\right\} + \frac{d}{dt}\left(m_r C_p t_s\right)$$

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} =$$
$$V_r\left\{\bar{\alpha}_v h_c (1-\alpha_r)\frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} + \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt} -\right.$$
$$-\alpha_r h_c (1-\bar{\alpha}_v)\frac{\partial \rho_w}{\partial p}\frac{dp}{dt} + \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} + \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt} +$$
$$\left.+\rho_s \bar{\alpha}_v \frac{dh_s}{dt} + \rho_w (1-\bar{\alpha}_v)\frac{dh_w}{dt} - \frac{dp}{dt}\right\} + \frac{d}{dt}\left(m_r C_p t_s\right)$$

Since $m_r C_p$ is constant and

$$\frac{dh_w}{dt} = \frac{\partial h_w}{\partial p}\frac{dp}{dt}$$
$$\frac{dh_s}{dt} = \frac{\partial h_s}{\partial p}\frac{dp}{dt}$$
$$\frac{dt_s}{dt} = \frac{\partial t_s}{\partial p}\frac{dp}{dt}$$

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} =$$
$$V_r \bar{\alpha}_v h_c (1-\alpha_r)\frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + V_r \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} + V_r \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt} -$$
$$-V_r \alpha_r h_c (1-\bar{\alpha}_v)\frac{\partial \rho_w}{\partial p}\frac{dp}{dt} + V_r \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} + V_r \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial p}\frac{dp}{dt} +$$
$$+V_r \rho_s \bar{\alpha}_v \frac{\partial h_s}{\partial p}\frac{dp}{dt} + V_r \rho_w (1-\bar{\alpha}_v)\frac{\partial h_w}{\partial p}\frac{dp}{dt} - V_r \frac{dp}{dt} + m_r C_p \frac{\partial t_s}{\partial p}\frac{dp}{dt}$$

Group the terms $\frac{dp}{dt}$ y $\frac{d\alpha_r}{dt}$

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} =$$
$$\left(V_r \bar{\alpha}_v h_c (1-\alpha_r)\frac{\partial \rho_s}{\partial p} + V_r \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial p} - V_r \alpha_r h_c (1-\bar{\alpha}_v)\frac{\partial \rho_w}{\partial p} +\right.$$
$$\left.+V_r \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial p} + V_r \rho_s \bar{\alpha}_v \frac{\partial h_s}{\partial p} + V_r \rho_w (1-\bar{\alpha}_v)\frac{\partial h_w}{\partial p} - V_r + m_r C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} +$$
$$+\left(V_r \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial \alpha_r} + V_r \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\right)\frac{d\alpha_r}{dt}$$

In other words,

$$\frac{dE}{dt} - (h_w + \alpha_r h_c)\frac{dM}{dt} =$$
$$\left(V_r \bar{\alpha}_v h_c (1-\alpha_r)\frac{\partial \rho_s}{\partial p} + V_r \rho_s h_c (1-\alpha_r)\frac{\partial \bar{\alpha}_v}{\partial p} - V_r \alpha_r h_c (1-\bar{\alpha}_v)\frac{\partial \rho_w}{\partial p} +\right.$$
$$\left.+V_r \alpha_r h_c \rho_w \frac{\partial \bar{\alpha}_v}{\partial p} + V_r \rho_s \bar{\alpha}_v \frac{\partial h_s}{\partial p} + V_r \rho_w (1-\bar{\alpha}_v)\frac{\partial h_w}{\partial p} - V_r + m_r C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} +$$
$$+(\rho_s (1-\alpha_r) + \alpha_r \rho_w) V_r h_c \frac{\partial \bar{\alpha}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt}$$

Equivalently,

$$\frac{dE}{dt}-(h_w+\alpha_r h_c)\frac{dM}{dt}=\left\{\left(\rho_w\frac{\partial h_w}{\partial p}-\alpha_r h_c\frac{\partial \rho_w}{\partial p}\right)V_r(1-\bar{\alpha}_v)+\right.$$
$$+(\rho_s+(\rho_w-\rho_s)\alpha_r)h_c V_r\frac{\partial\bar{\alpha}_v}{\partial p}-V_r+m_t C_p\frac{\partial t_s}{\partial p}+$$
$$\left.+\left((1-\alpha_r)h_c\frac{\partial\rho_s}{\partial p}+\rho_s\frac{\partial h_s}{\partial p}\right)V_r\bar{\alpha}_v\right\}\frac{dp}{dt}+\left\{(\rho_s(1-\alpha_r)+\alpha_r\rho_w)V_r h_c\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\right\}\frac{d\alpha_r}{dt}$$

Defined

$$e_{32}=\left\{\left(\rho_w\frac{\partial h_w}{\partial p}-\alpha_r h_c\frac{\partial\rho_w}{\partial p}\right)V_r(1-\bar{\alpha}_v)+(\rho_s+(\rho_w-\rho_s)\alpha_r)h_c V_r\frac{\partial\bar{\alpha}_v}{\partial p}-\right.$$
$$\left.-V_r+m_t C_p\frac{\partial t_s}{\partial p}+\left((1-\alpha_r)h_c\frac{\partial\rho_s}{\partial p}+\rho_s\frac{\partial h_s}{\partial p}\right)V_r\bar{\alpha}_v\right\}$$

$$e_{33}=\left\{(\rho_s(1-\alpha_r)+\alpha_r\rho_w)V_r h_c\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\right\}$$

We finally have to

$$\frac{dE}{dt}-(h_w+\alpha_r h_c)\frac{dM}{dt}=e_{32}\frac{dp}{dt}+e_{33}\frac{d\alpha_r}{dt} \tag{A7}$$

The proposition is proven by equating the relations (A7) with (A3).

Deduction of Equation 8

According to [1], the mass balance for steam below the liquid level is important because the effects of shrinkage and swelling, as well as how the bubbles affect the water level, are noticeable.

$$\frac{d}{dt}(\rho_s V_{sd})=\alpha_r q_r-q_{sd}-q_{cd}$$

Where,

V_{sd} : steam volume in the drum

q_{sd} : steam flow through the liquid surface in the drum

q_{cd} : mass flow of steam condensed in the drum

q_r : outlet flow (water and steam) from the risers

$$\frac{d}{dt}(\rho_s V_{sd})=V_{sd}\frac{d\rho_s}{dt}+\rho_s\frac{dV_{sd}}{dt}$$

So,

$$\frac{d}{dt}(\rho_s V_{sd})=V_{sd}\frac{\partial\rho_s}{\partial p}\frac{dp}{dt}+\rho_s\frac{dV_{sd}}{dt} \tag{63}$$

From equation (58) q_r is obtained.

$$q_r=q_{dc}-\mathrm{Vr}\left[\left(\bar{\alpha}_v\frac{\partial\rho_s}{\partial p}+(1-\bar{\alpha}_v)\frac{\partial\rho_w}{\partial p}+(\rho_s-\rho_w)\frac{\partial\bar{\alpha}_v}{\partial p}\right)\frac{dp}{dt}+(\rho_s-\rho_w)\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\frac{d\alpha_r}{dt}\right]$$
$$X=\mathrm{Vr}\left[\left(\bar{\alpha}_v\frac{\partial\rho_s}{\partial p}+(1-\bar{\alpha}_v)\frac{\partial\rho_w}{\partial p}+(\rho_s-\rho_w)\frac{\partial\bar{\alpha}_v}{\partial p}\right)\frac{dp}{dt}+(\rho_s-\rho_w)\frac{\partial\bar{\alpha}_v}{\partial\alpha_r}\frac{d\alpha_r}{dt}\right] \tag{64}$$
$$q_r=q_{dc}-X$$

q_{sd} Is the steam flow crossing the water surface in the drum, obtained empirically in [1]

$$q_{sd}=\frac{\rho_s}{T_d}(V_{sd}-V_{sd}^0)+\alpha_r q_{dc}+\alpha_r(\beta)(q_{dc}-q_r) \tag{65}$$

From equation (58) $q_{dc} - q_r$ is obtained According to [1], page 368, the mass flow of steam condensed in the drum q_{cd} is given by:

$$q_{cd} = \frac{h_w - h_f}{h_c} q_f + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} \quad (66)$$

Equations (64), (65), and (66) are substituted into equation (63)

$$V_{sd}\frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt}$$
$$= \alpha_r(q_{dc} - X) - \left(\frac{\rho_s}{T_d}\left(V_{sd} - V_{sd}^0\right) + \alpha_r q_{dc} + \alpha_r(\beta)(X)\right)$$
$$-\left(\frac{h_w - h_f}{h_c} q_f + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}\right)$$
$$V_{sd}\frac{\partial \rho_s}{dp}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt}$$
$$= \alpha_r(q_{dc} - X) - \frac{\rho_s}{T_d}\left(V_{sd} - V_{sd}^0\right) - \alpha_r q_{dc} - \alpha_r(\beta)(M) - \frac{h_w - h_f}{h_c} q_f$$
$$-\frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}$$
$$V_{sd}\frac{\partial \rho_s}{dp}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}$$
$$= \alpha_r(q_{dc} - X) - \alpha_r q_{dc} - \alpha_r(\beta)(X) + \frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) - \frac{h_w - h_f}{h_c} q_f$$
$$V_{sd}\frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}$$
$$= \alpha_r q_{dc} - \alpha_r X - \alpha_r q_{dc} - \alpha_r(\beta)(X) + \frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) - \frac{h_w - h_f}{h_c} q_f$$
$$= -\alpha_r X - \alpha_r(\beta)(X) + \frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) - \frac{h_w - h_f}{h_c} q_f$$

Group the term α_r

$$V_{sd}\frac{\partial \rho_s}{\partial p}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt}$$
$$= -\alpha_r(1 + \beta)X + \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd}) - \frac{h_w - h_f}{h_c} q_f$$

After substituting X

$$V_{sd}\frac{d\rho_s}{dp}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} = \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd}) - \frac{h_w - h_f}{h_c} q_f - \alpha_r(1 + \beta)\left(Vr\left[\left(\bar{a}_v\frac{\partial \rho_s}{\partial p} + (1 - \bar{a}_v)\frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial p}\right)\frac{dp}{dt} - (\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt}\right]\right)$$

$$V_{sd}\frac{d\rho_s}{dp}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} = \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd}) - \frac{h_w - h_f}{h_c} q_f - \alpha_r(1 + \beta)(V_r)\left(\bar{a}_v\frac{\partial \rho_s}{\partial p} + (1 - \bar{a}_v)\frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial p}\right)\frac{dp}{dt} - \alpha_r(1 + \beta)(V_r)(\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt}$$

$$V_{sd}\frac{d\rho_s}{dp}\frac{dp}{dt} + \rho_s\frac{dV_{sd}}{dt} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right)\frac{dp}{dt} + \alpha_r(1 + \beta)(V_r)\left(\bar{a}_v\frac{\partial \rho_s}{\partial p} + (1 - \bar{a}_v)\frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial p}\right)\frac{dp}{dt} + \alpha_r(1 + \beta)(V_r)(\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial \alpha_r}\frac{d\alpha_r}{dt} = \frac{\rho_s}{T_d}(V_{sd}^0 - V_{sd}) - \frac{h_w - h_f}{h_c} q_f$$

The coefficients e_{42}, e_{43} e_{44} are:

$$e_{42} = \left(V_{sd}\frac{\partial \rho_s}{\partial p} + \frac{1}{h_c}\left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - (Vsd + Vwd) + m_d C_p \frac{\partial t_s}{\partial p}\right) + \alpha_r(1 + \beta)(V_r)\left(\bar{a}_v\frac{\partial \rho_s}{\partial p} + (1 - \bar{a}_v)\frac{\partial \rho_w}{\partial p} + (\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial p}\right)\right)$$

$e_{43} = \alpha_r(1+\beta)(V_r)(\rho_s - \rho_w)\frac{\partial \bar{a}_v}{\partial \alpha_r}$

$e_{44} = \rho_s$

Appendix 2: Simulation Files

GitHub Repository: Closed and Open loop simulation

https://github.com/ADRMOIS/Drum-Boiler