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Original Article

Performance Enhancement of Fuzzy Logic Controllers via Novel GWO-ABC-Based Optimization

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Abstract *- Achieving an optimal balance between convergence speed and solution quality remains a critical challenge in complex optimization problems. This work proposes an enhanced hybrid metaheuristic that synergistically combines a typical Grey Wolf Optimization (GWO) algorithm together with an Artificial Bee Colony (ABC) mechanism. The key innovation lies in utilizing GWO's exploratory strength to generate an advantageous initial food source distribution for the ABC mechanism, leading to accelerated convergence and improved solution quality. To evaluate the algorithm's performance, it is applied to the parameter tuning of a Fuzzy Logic Controller (FLC) for an inverted pendulum on the cart system. Comparative performance evaluations, executed via MATLAB/Simulink simulations, demonstrate that the developed hybrid algorithm significantly surpasses the capabilities of standard ABC and existing GWO-ABC hybrid implementations. This highlights the proposed method's effectiveness in addressing a broad spectrum of optimization tasks.*

Keywords *- Inverted pendulum, Fuzzy Logic Control, Grey Wolf Optimizer, Artificial Bee Colony, Balance control.*

1. Introduction

With the rapid advancement of both economic and social domains, complex optimization problems have become increasingly ubiquitous [1]. Contemporary optimization challenges are distinguished by their multi-faceted nature, involving high-dimensional variables, multiple objectives, diverse constraints, and varied optimization metrics. The objective of any optimization problem is to locate the best possible solution within the set of admissible solutions.

Optimization algorithms are designed to tackle a wide range of global optimization problems, which may be categorized as single or multi-objective, constrained or unconstrained, and combinatorial [2]. These problems arise across diverse sectors, including ICT, engineering disciplines, resource extraction, and logistical scheduling [2]. The primary objective of any optimization algorithm is to determine the most favorable solution in the search space. While finding a true global optimum is computationally expensive, the best acceptable solution can be selected from a set of candidates. The complexity of optimization problems continues to increase in the modern world. Therefore, optimization algorithms must evolve efficiently to handle a growing set of complex problems while mitigating computational costs associated with global optimization [3]. Compared to traditional optimization algorithms, nature-inspired optimization techniques have dominated the field of optimization. These techniques, known as metaheuristic algorithms, are primarily categorized into three main groups: Evolutionary Algorithms (EA), Physics-based Algorithms, and Swarm Intelligence-based (SI) Algorithms [4]. Swarm Intelligence (SI) algorithms are inspired by the collective behavior and natural rules of social organisms, such as flocks of birds or colonies of bees. Examples of SI algorithms include the Moth-Flame Optimization Algorithm (MFO) [5, 6] White Shark Optimizer (WSO) [7], Sparrow Search Algorithm (SSA) [8], Bat Algorithm (BA) [17]. Among SI algorithms, the Particle Swarm Optimization (PSO) [9, 18, 19] and Ant Colony Optimization (ACO) [21] method is one of the most widely adopted. It models a population of particles that iteratively update their positions based on the best solutions found, mimicking the behavior of a swarm searching for food.

The Grey Wolf Optimizer (GWO) [10], a contemporary nature-inspired algorithm, mimics the structured predatory behavior exhibited by grey wolves in the wild. GWO has been widely applied in optimization problems due to its advantages, such as having fewer parameters and fast convergence speed. However, despite these strengths, GWO struggles with global exploration and is prone to getting trapped in local optima.

The Artificial Bee Colony (ABC) method, a population-based metaheuristic within the swarm intelligence paradigm, emulates the collective foraging behavior observed in honeybee colonies. It is widely recognized for its potent global exploration capabilities and straightforward implementation [11]. However, ABC often suffers from slow convergence in later iterations and inefficient exploitation of promising solutions, leading to suboptimal performance in highly complex optimization tasks. Therefore, integrating GWO and ABC can potentially improve the trade-off between global exploration and local exploitation, thereby increasing optimization performance.

The inverted pendulum system is one of the fundamental theoretical and practical models in control theory and engineering, known for its instability and nonlinear dynamics [12-14]. The main challenge lies in maintaining the pendulum in an upright position while the cart moves along the horizontal axis. This problem has significant applications in robotics, transportation systems, and aerospace engineering, making it a crucial benchmark for testing advanced control strategies [15].

While the inverted pendulum system is employed as the application domain for this investigation, the primary research objective transcends mere system stabilization. It functions as a controlled experimental platform designed to facilitate the examination and refinement of advanced optimization methodologies. Specifically, this study aims to evaluate the efficacy of hybridizing the GWO and the ABC algorithm, with the intent of augmenting their performance in addressing the challenges inherent in complex, high-dimensional optimization problems. The inverted pendulum on the cart system using an FLC controller, therefore, serves as a rigorous benchmark, enabling the systematic assessment of the proposed hybrid algorithm's capacity to navigate intricate search landscapes and deliver superior optimization outcomes. This approach allows for the abstraction of findings beyond the specific control application, enabling the extrapolation of observed performance enhancements to a wider range of complex optimization tasks encountered in diverse scientific and engineering contexts.

2. Methodology

2.1. Mathematical Modeling of an Inverted Pendulum System

As shown in Figure 1, the dynamic configuration of the inverted pendulum system includes two fundamental parts: the horizontally mobile cart and the vertically attached pole. The cart is typically driven by a motor to provide translational motion. The second component, the pole, is a rod with one end attached to a mass *m*, while the other end is connected to the cart via a rotational joint, allowing the rod to rotate freely. In this mechanical setup, the system is influenced by a single external force, *F*, which is generated by the torque output of the electric motor driving the cart's motion.

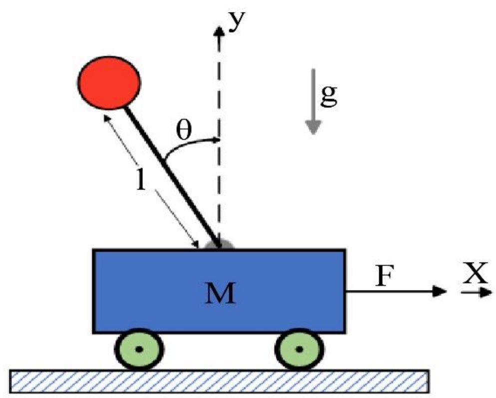

Fig. 1 Configuration of the inverted pendulum mechanism

The equations for the dynamic behavior of the inverted pendulum can be obtained through the Euler-Lagrange method, leading to the following results:

$$\begin{cases}(m_c + m_p)\ddot{x} + m_p l\cos(\theta)\,\ddot{\theta} - m_p l\sin(\theta)\,\dot{\theta}^2 = F \\ m_p l\cos(\theta)\,\ddot{x} + m_p l^2\ddot{\theta} - m_p g l\sin(\theta) = 0\end{cases} \quad (1)$$

To maintain the desired angle between the pendulum and the Y-axis, a Fuzzy Logic Controller (FLC) is developed to generate the control force F for the cart. This approach will be presented in the following section.

$$\begin{cases}\ddot{x} = \dfrac{(F + m_p l\sin(\theta)\dot{\theta}^2) - m_p g\cos(\theta)\sin(\theta)}{(m_c + m_p) - m_p\cos^2(\theta)} \\ \ddot{\theta} = \dfrac{(m_c + m_p) g\sin(\theta) - F\cos(\theta) - m_p l\cos(\theta)\sin(\theta)\dot{\theta}^2}{(m_c + m_p) l - m_p l\cos^2(\theta)}\end{cases} \quad (2)$$

Equation (2) is used to represent the mathematical formulation of the inverted pendulum system, capturing the dynamics of the system based on the applied forces and the motion of both the cart and the Pole.

2.2. Control Strategy

The control strategy integrates the Fuzzy Logic Controller (FLC) with the proposed mABC algorithm, where mABC is employed to optimize the FLC's processing parameters [20]. The main objective is to stabilize the system within an acceptable range, ensuring that both the cart's position and the pendulum's angular deviation are appropriately controlled. A hybrid control structure is plotted in Figure 2. The optimization of mABC is guided by the ITAE criterion, which involves integrating the product of time and the absolute error, and is defined as follows (3) [16]:

$$J = \int (e_x(t) + \rho . e_\theta(t)) . t dt \quad (3)$$

Here, e_x and $e_\theta$represent the deviation errors in the cart-pendulum system, corresponding to the cart's displacement and the pendulum's angular variation, respectively.

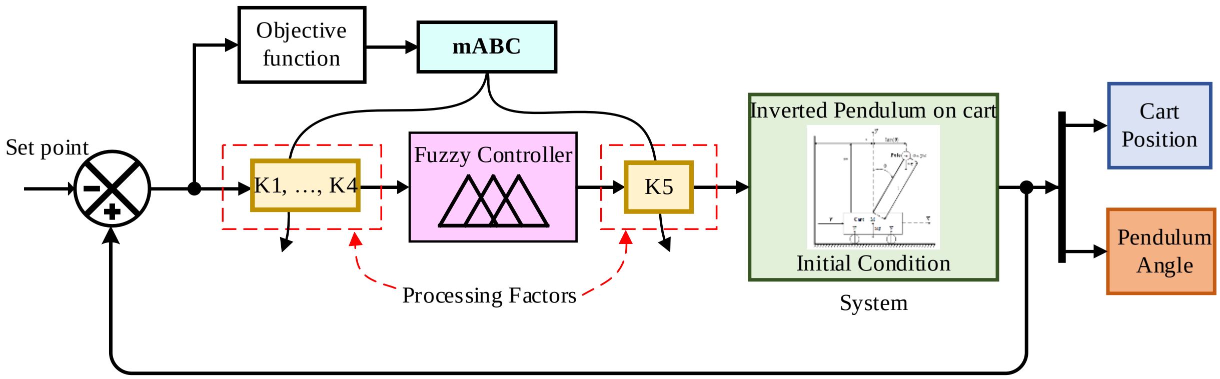

Fig. 2 Control strategy for balancing the inverted pendulum system

The ITAE criterion prioritizes long-term errors by assigning greater weight to errors that persist over time. Consequently, systems optimized using ITAE tend to reduce prolonged deviations, resulting in faster stabilization. Moreover, ITAE facilitates the design of controllers that exhibit smoother responses and fewer oscillations compared to other performance indices such as IAE or ITSE. For the fuzzy controller, each input variable is assigned three fuzzy sets, while the output variable is represented by seven fuzzy sets. This configuration results in a total of 81 fuzzy rule sets.

3. Overview of the Adopted Metaheuristic Techniques

3.1. Grey Wolf Optimizer Algorithm (GWO)

The Grey Wolf Optimizer (GWO) is a meta-heuristic algorithm modeled after the social behavior of grey wolves. Developed by Mirjalili et al. in 2014, the efficacy of this approach has been demonstrated across a diverse spectrum of optimization problems. The GWO stands out from other population-based optimization techniques due to its unique hunting strategy and the distinct way its operations are mathematically modeled. In its hunting strategy, GWO mimics the social hierarchy and cooperative hunting tactics observed in grey wolves. The wolf pack is organized into four distinct ranks: alpha (α), beta (β), delta (δ), and omega (ω). Each level has distinct responsibilities within the pack. The alpha wolves (α) serve as leaders, making decisions and guiding the entire pack. The beta wolves (β) hold the second-highest rank, assisting the alpha wolves and potentially replacing them when necessary. The delta wolves (δ) occupy the third hierarchical level, following the directions of the alpha and beta wolves. The omega wolves (ω) are at the lowest rank, representing the majority of the pack.

The GWO algorithm operates through three core stages: initialization, exploration, and exploitation, which guide the search for optimal solutions-an approach commonly found in Swarm Intelligence (SI)-based methods [10]. The mathematical representation of the algorithm is outlined as follows.

3.1.1. Grey Wolf Encircling the Prey Phase

During the initial stage of the hunting process, grey wolves approach their target by gradually reducing the distance between themselves and the prey. This behavior is mathematically modeled by adjusting each wolf's position according to its relative distance from the prey.

$$D = \left|C.X_{prey} - X(t)\right| \tag{4}$$

$$X(t+1) = X_{prey} - A.D \tag{5}$$

The position of the prey is represented by X_{prey}, while $X(t)$ denotes the current position of a grey wolf at iteration t, and $X(t+1)$ indicates its updated position in the subsequent iteration. The distance between the grey wolf and the prey is expressed as D. The encircling mechanism is controlled by two primary coefficients.

- A: the convergence coefficient, which determines the coordination between wide-ranging exploration and focused local search. When $|A| > 1$, the wolves explore globally, while $|A| < 1$ leads to local exploitation.
- C: the oscillation coefficient, which affects the direction and movement toward the prey.

The mathematical representation of this behavior is as follows:

$$A = 2.\alpha.r_1 - \alpha \tag{6}$$

$$C = 2.r_2 \tag{7}$$

$$\alpha = 2 - 2.\frac{iter}{max\_iter} \tag{8}$$

As part of this formulation, $r_1, r_2 \in [0,1]$ represents random variables while α serving as a control parameter that decreases linearly from 2 to 0 throughout the iterations. This mechanism enables the wolves to iteratively adjust their positions relative to the prey, promoting a gradual convergence toward the optimal solution.

3.1.2. Grey Wolf Hunting Phase

The hunting phase in the GWO is directed by the top three wolves in the hierarchy: alpha (α), beta (β), and delta (δ), who are assumed to possess the most accurate information about the prey's location. The remaining wolves in the pack adjust their positions accordingly, following the guidance provided by these leading individuals. Mathematically, the position of each wolf is updated by considering the average influence of α, β, and δ. This process is represented by the following equations:

$$\begin{cases} D_\alpha = |C_\alpha . X_\alpha - X| \\ D_\beta = |C_\beta . X_\beta - X| \\ D_\delta = |C_\delta . X_\delta - X| \end{cases} \tag{9}$$

$$\begin{cases} X_1 = |X_\alpha - A_1 . D_\alpha| \\ X_2 = |X_\beta - A_2 . D_\beta| \\ X_3 = |X_\delta - A_3 . D_\delta| \end{cases} \tag{10}$$

$$X(t+1) = \frac{X_1 + X_2 + X_3}{3} \tag{11}$$

Where:

- $X_\alpha, X_\beta, X_\delta$ represent the positions of the top three wolves. Wolf in the population.
- $C_\alpha, C_\beta, C_\delta$ and A_1, A_2, A_3 are computed using the same equations as in the encircling phase.
- $X(t+1)$is the updated position of the wolf in the next iteration.

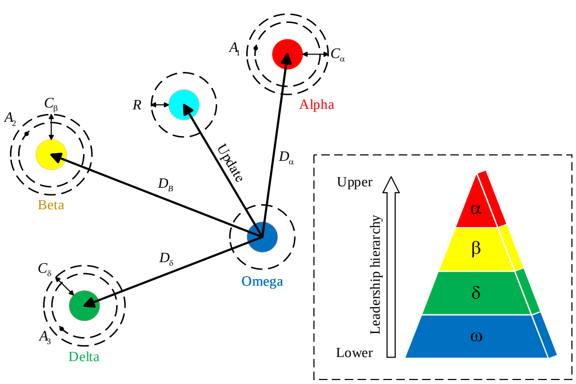

Fig. 3 The mechanism of the GWO algorithm

Through this strategy, the grey wolves are guided toward favorable areas within the search domain, thereby enhancing the algorithm's ability to effectively exploit regions that are likely to contain optimal solutions. GWO is characterized by its simplicity in design and the limited number of parameters needed for configuration. Its hierarchical leadership system allows for rapid convergence toward optimal solutions. This makes GWO particularly suitable for real-world optimization problems that require fast computation. However, despite these advantages, GWO also has some limitations. One notable drawback is its tendency to become trapped in local optima, which may affect its performance in complex, high-dimensional search spaces.

3.2. Artificial Bee Colony Algorithm (ABC)

Proposed by Karaboga, the Artificial Bee Colony (ABC) algorithm draws inspiration from the natural foraging patterns observed in honeybee swarms. Its optimization performance is highly influenced by the choice of control parameters, where suitable parameter tuning plays a crucial role in enhancing search effectiveness. The algorithm categorizes the artificial bees into three primary roles: employed bees, onlooker bees, and scout bees.

3.2.1. Employed Bees Phase

Employed bees are linked to particular food sources, and they evaluate their quality and communicate this information to other members of the colony via the 'waggle dance' upon returning to the hive. They generate new candidate solutions in the neighborhood of their current positions by applying the following update equation:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{kj} - x_{ij}) \tag{12}$$

Where:

- $k \neq i$is the index of a randomly selected neighboring solution.
- $j \in \{1, \ldots, D\}$is the index of a dimension (variable) in the solution.
- ϕ_{ij} is a randomly generated number within the range [-1,1].

If the fitness of the new solution exceeds that of the current one, the employed bee adopts the new solution; otherwise, the previous solution is preserved.

3.2.2. The Second Phase: Onlooker Bees

Onlooker bees do not engage in direct exploration of food sources but stay within the hive to gather information shared by employed bees. Using this information, they choose a food source to exploit further and attempt to enhance its quality. Each onlooker bee selects a solution (food source) according to a probability determined by the following equation:

$$pi = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \tag{13}$$

Where:

- pirepresents the probability of selecting food source ith.
- fit_i denotes the fitness value of solution ith.
- $fit_n$is the total number of available solutions in the population.

Once a solution is selected, the onlooker bee generates a new solution using the same update Equation (12) as employed bees.

The fitness value *fit* used by Karaboga [11, 12] is defined using a well-known formula, as presented below. The purpose of this formulation is to ensure that the fitness value remains positive even when the objective function takes negative values.

$$fit_i = \begin{cases} \frac{1}{f_i+1} \; if \; f_i \geq 0 \\ 1 + |f_i| \; if \; f_i < 0 \end{cases} \tag{14}$$

3.2.3. Scout Bees Phase

The scout bee phase plays a vital role in the Artificial Bee Colony (ABC) algorithm by supporting exploration and preventing early convergence. In contrast to employed and onlooker bees, scout bees do not depend on information shared by other bees. Rather, they perform random searches within the solution space to identify new possible solutions. If a solution does not show improvement after a specified number of iterations (referred to as the limit), the associated employed bee is converted into a scout bee and generates a new solution through random exploration using (15):

$$x_{ij} = x\left(xj, min_{j,max}\right)_{j,min} \tag{15}$$

Where:

- $x_{j,min}$and $x_{j,max}$ represent the lower and upper bounds of the variable x, respectively.
- ϕ is a random number uniformly distributed within the range [0,1].

By replacing the entire solution vector with newly generated values, the scout bee contributes to increasing population diversity and helps reduce the likelihood of stagnation in local optima.

3.3. The Modified Artificial Bee Colony - mABC

Meta-heuristic algorithms are based on two core principles, exploration and exploitation, which work together to effectively search the solution space and generate solutions. Exploration aims to diversify the solutions by broadening the search space to include more varied candidate solutions. In contrast, exploitation focuses on narrowing the search scope by reducing differences between candidate solutions. Exploration relates to globalizing the search space, while exploitation localizes the search space [11]. Achieving a balance between these two attributes is crucial for enhancing the quality of solutions in optimization algorithms. If an algorithm places too much focus on exploration while underemphasizing exploitation, the high variability among solutions in successive iterations may prevent the algorithm from identifying the optimal solution. Ideally, the disparity between solutions should diminish progressively to facilitate convergence toward the best solution. Therefore, the performance of a meta-heuristic algorithm is primarily determined by achieving an appropriate trade-off between exploration and exploitation.

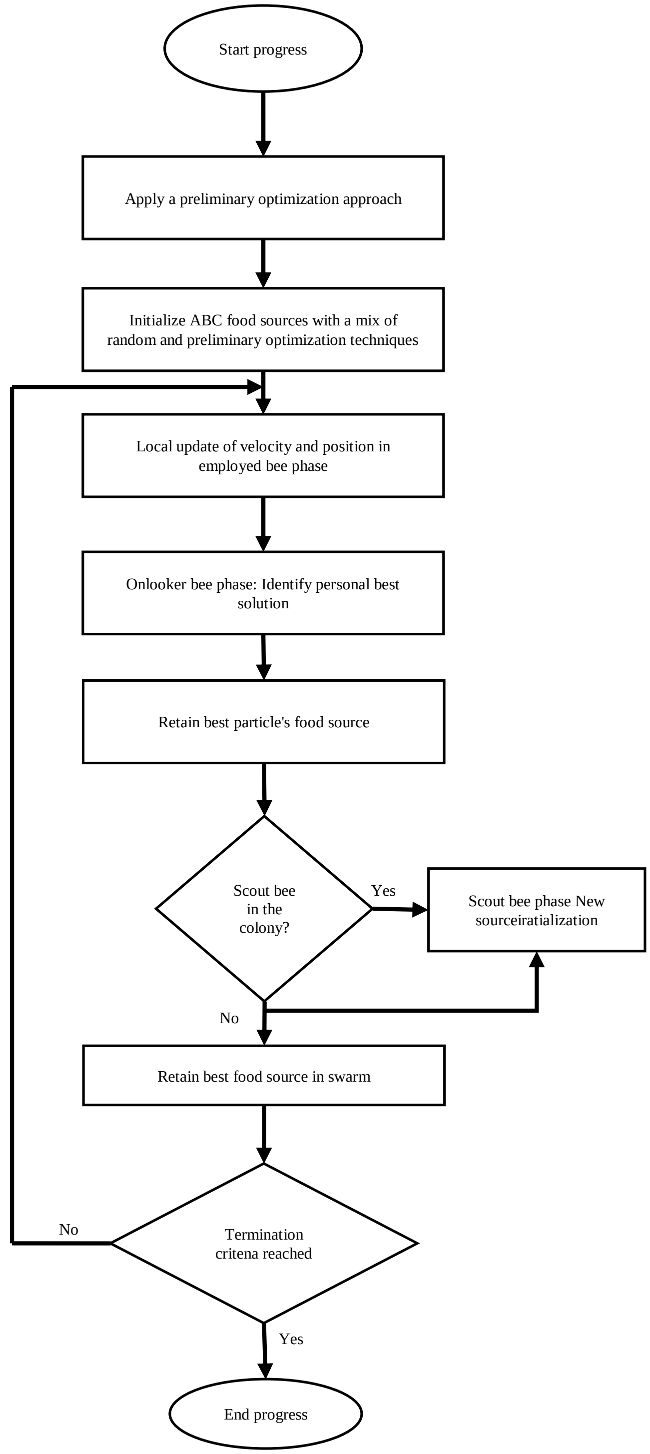

Fig. 4 The flowchart of the proposed modified ABC algorithm

Similarly, the Artificial Bee Colony (ABC) optimization algorithm functions as a meta-heuristic method. In cases where an optimization problem involves multiple local optima, an imbalance between exploration and exploitation may result in slow convergence and suboptimal solution quality. This paper presents an enhanced version of the ABC algorithm (mABC) aimed at reducing the computational time associated with the standard ABC algorithm. An additional phase is introduced as a parameter initialization step for the ABC process. As shown in Figure 4, mABC is initialized through a preliminary optimization step, which aims to generate the initial parameters for the ABC algorithm using an effective optimization technique. A viable approach for this purpose is to employ a faster optimization method, such as the GWO. Theoretically, GWO is a relatively simple optimization technique that can achieve faster execution times than ABC. The preliminary optimization process is designed to identify a viable solution for the problem, producing a set of parameters that converge towards the optimal solutions, either local or global.

4. Results and Discussion

This study proposes an optimization method for the Fuzzy Logic Controller (FLC) parameters using an enhanced ABC algorithm. The optimization process for determining the optimal parameters of the FLC using the mABC algorithm is compared with the GWO, ABC, and Ant Colony Optimization (ACO) algorithms [21], with the search space parameters described in Table 1. The effectiveness of the proposed algorithm is evaluated through two different experimental scenarios, each with specific control requirements for the inverted pendulum system. The simulation process is conducted using MATLAB/Simulink software to collect data and analyze the results. Table 2 outlines the experimental parameters of the inverted pendulum system utilized in this study. Figure 5 illustrates the optimization process, showing that the mABC algorithm achieves faster optimization speed compared to the other algorithms discussed. The mABC benefits from the fast initialization of parameters from the GWO algorithm, which shortens the initial optimization phase. It then capitalizes on the exploration capabilities to handle potential solution sources and discover new solutions when previous ones no longer provide optimal results. As a result, the mABC provides a better optimal solution than the GWO algorithm.

Scenario 1: Balancing control of the pendulum while maintaining the desired cart position at a fixed value. $x^* = 0.25(m)$

Scenario 2: Balancing control of the pendulum with the cart position varying across four discrete levels: $x_0^* = 0(m)$,$x_1^* = 0.2(m)$,$x_3^* = 0.45(m)$, $x_4^* = 0.3(m)$.

Table 1. Parameters to execute the mABC algorithm

Parameter	Symbol	Value
Maximum iteration	$Nmax$	50
Swarm size	$Npar$	20
The upper bound, UB	K_1	7
	K_2, K_3, K_4	2
	K_5	500
Lower bound, LB	K_1	2
	K_2, K_3, K_4	0.01
	K_5	40

Table 2. Physical parameters of the inverted pendulum system

Symbol	Parameters	Value
m_c	Mass of the mobile base (cart)	$2.4[Kg]$
m_p	Mass of the pendulum rod	$0.23[Kg]$
l	Length of the pendulum	$0.5[m]$
$x(t)$	Horizontal displacement of the cart	$[m]$
$\dot{x}(t)$	The velocity of the cart	$\left[\frac{m}{s}\right]$
$\ddot{x}(t)$	Acceleration of the cart	$\left[\frac{m}{s^2}\right]$
$\theta(t)$	Angular displacement of the pendulum	$[Rad]$
$\dot{\theta}(t)$	Angular velocity of the pendulum	$\left[\frac{rad}{s}\right]$
$\ddot{\theta}(t)$	Angular acceleration of the pendulum	$\left[\frac{rad}{s^2}\right]$
F	The external force applied to the cart.	$[N]$

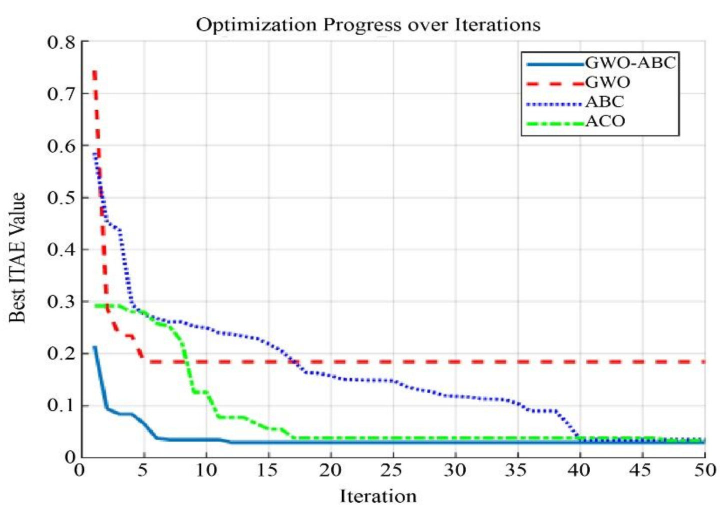

Fig. 5 Results for the comparative analysis of the proposed algorithm and standard GWO and ABC algorithm

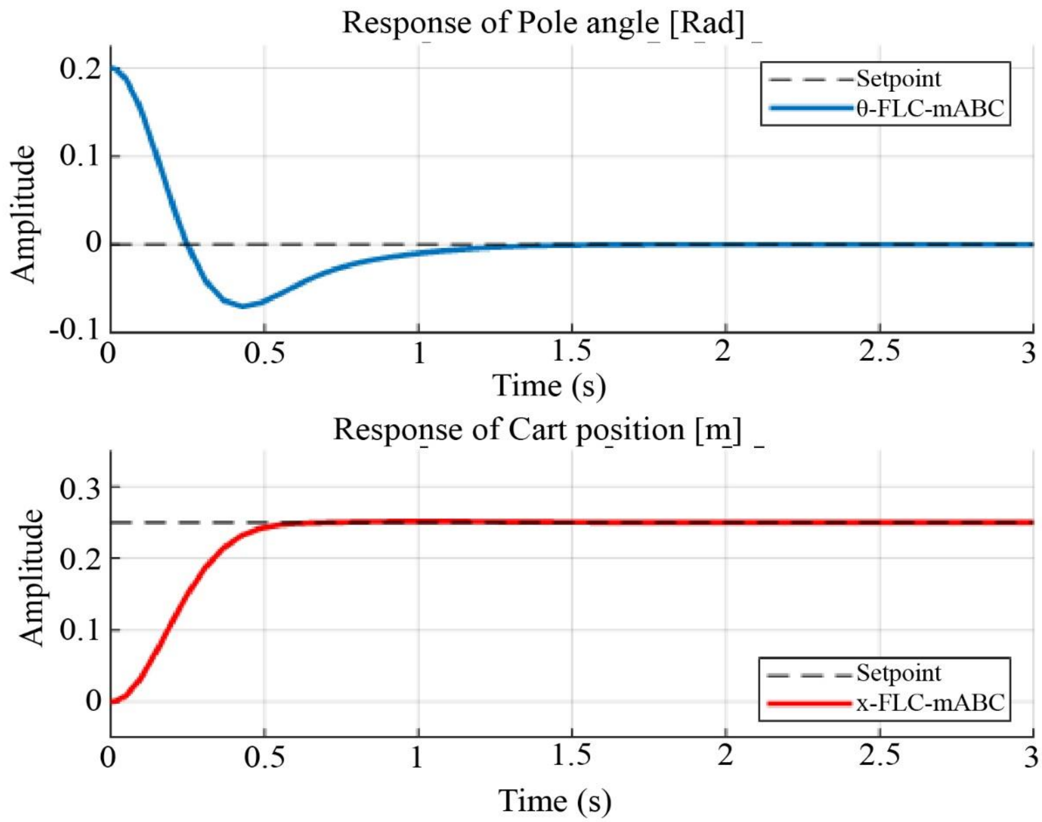

Fig. 6 Simulation results for the first scenario

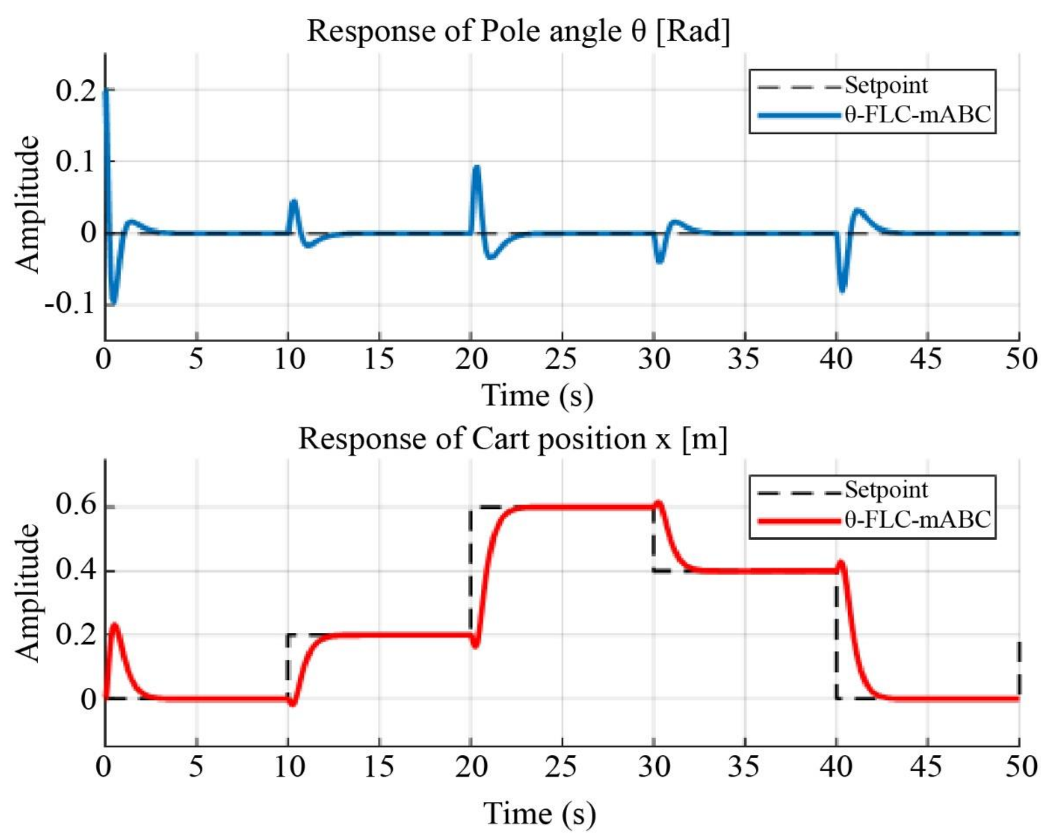

Fig. 7 Pole angle and cart position's dynamic responses for the second scenario

Figure 6 illustrates the cart position deviation and the angular deviation of the pendulum from the vertical axis. The results show that both the position error and the angular deviation are driven to zero within a stable settling time of less than 1.5 seconds, demonstrating the effectiveness of the proposed control method. The control response of the cart-pendulum system using the proposed optimization technique in the second scenario is illustrated in Figure 7. The results indicate that the proposed hybrid control strategy achieves high control performance, ensuring the pendulum quickly stabilizes even when the cart position changes at different levels over time. Not only does the actual position closely follow the control signal, but the pendulum's angular deviation is also completely eliminated within a reasonably short settling time.

5. Conclusion and Future Work

This study has presented an innovative hybrid metaheuristic optimization framework meticulously engineered through the synergistic integration of the GWO and the ABC algorithm. This fusion is strategically designed to address the inherent challenge of achieving an optimal equilibrium between exploratory diversification and exploitative intensification within the metaheuristic search process. The resultant hybrid algorithm is subsequently deployed to formulate an optimized control paradigm, wherein an FLC is employed as the primary mechanism for the stabilization of an inverted pendulum system. Rigorous simulations executed within the MATLAB/Simulink computational environment provide empirical validation of the performance efficacy of the developed hybrid control methodology. Prospective avenues for future scholarly inquiry encompass the refinement of the structural architecture of the ABC algorithm, with a specific focus on enhancing its adaptability and robustness.

Additionally, the exploration of seamless integration with other advanced optimization paradigms is warranted, aiming to harness the synergistic advantages afforded by their complementary search characteristics. Furthermore, substantive advancements in the design and parameterization of FLC components, particularly concerning the morphology of membership functions and the logical construction of rule bases, hold the potential to yield significant enhancements in both control performance metrics and computational efficiency, particularly within the context of real-time control implementations demanding stringent temporal constraints.

Acknowledgments

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