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Original Article

A Simple Approach for Loss Allocation in Radial Distribution Systems

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Abstract - *Transmission conductors offer resistance and reactance. Loss in the power system is obvious in the process of meeting the load demand due to the resistance offered by the conductors. For a restructured system, which is operated with market-based electricity prices, a reasonable apportionment of losses between the consumers is desirable. This should be done in line with the usage of electrical power by them in an accurate manner. An effortless and lucid method to apportion the losses to various consumers or load points in the system is presented in this work. The proposed method is a straightforward and computationally efficient technique to allocate active power losses in radial distribution systems. Unlike complex iterative methods, the approach distributes losses proportionally among consumers based on their real power contributions and network topology. Primitive bus-branch matrices of load flow studies are used in the proposed approach for loss allocation. This method eases and reduces the complexity of solving and analyzing the problem, unlike the available traditional methods in the literature. The method ensures fairness by considering branch currents and nodal voltages while avoiding the need for exhaustive power flow analysis. Case studies on standard test systems (e.g., 9-bus, 15-bus and 33-bus radial distribution systems) validate the method's accuracy and practicality for utilities. A comparative evaluation between the proposed and Pro-rata method has been performed in this work to project the effectiveness of the approach.*

Keywords - *Load flow analysis, Loss allocation, Primitive bus-branch matrix, Radial distribution system, Restructured electricity market.*

1. Introduction

Restructuring of the electricity market has changed the traditional operation of the market. Prior to deregulation, the electricity market was operated in a vertically integrated manner. In a vertically integrated system, all the subsections of the power system are operated by a monopoly. On the contrary, the deregulated system divides the generation, transmission and Distribution System (DS) into separate entities. This facilitates competition among different generation entities and distribution companies. The competition among the entities will improve the quality, quantity and economy of power produced, which ultimately leads to social welfare maximization [1].

Donald et al. have used the Turvey-Shapley method to allocate the network costs among the DS consumers [2]. The approach used by the authors is a probabilistic method and not an exact method. This can land up giving deviated results from their actual values. Sobhy et al. have developed a consumer incentive-based Loss Allocation (LA) approach in a DG-integrated DS [3]. The incentive-based loss calculation may not consider the actual loss contribution by the loads. Himesh has proposed a τ–value–based LA approach for a 3-φ distribution network [4]. This method focuses only on unbalanced systems. Usman et al. used decomposition-based LA in a distribution network [5]. This method uses weighting factors, rather than actual contributions. Zeinab et al. have done LA in a DG distribution network using the power flows in the lines [6]. This method initially allocates zero losses to a group of buses in the system, then calculates losses in other nodes in a relative manner.

In this way, the actual allocation will not be done. Satish et al. proposed an analytical formulation of the shapley value for loss administration in a meshed system [7]. This method uses an analytical approach and focuses only on weakly meshed DS. Various loss management strategies have been reviewed by Coppo et al. in [8]. The authors have provided a glance at various approaches adopted and proposed by the work done in this area. Nivedita et al. have addressed the challenges involved in the LA approach by performing a literature review of various articles [9]. The authors have provided an insight into the work done in the literature. Yu et al. have performed multi-phase LA in an unbalanced DS [10]. The literature review on LA in unbalanced systems is provided by the authors of this work.

Mishra et al. reviewed LA approaches used in Radial Distribution Systems (RDS) [11]. The authors of this paper do a detailed review of the works done on the subject title. Nikolaidis et al. have used a graph-based approach for an unbalanced distribution network operated with market policy [12]. The authors of this work have used a graph-based approach to find the LA in unbalanced systems. Ishan et al. have proposed a transaction-tracing-based method, which introduces a flexible coordination mechanism for prosumers [13]. This approach is based on the transaction trading done in the electricity markets rather than the actual power consumed. Abbagouni et al. performed LA for an unbalanced network [14]. This method accounts only for the current flowing in the feeders, not the power consumed. Dadashzade et al. have performed LA using a decomposition approach [15].

This method focuses on the unbalanced systems alone and finds the alternative load buses to mitigate the losses due to the unbalancedness of the system. Chintada et al. used pro-rata and decomposition methods for loss dispersion in DS [16]. The power flow direction is likely to change in a part or whole feeder after installing the Distributed Generators (DGs) in the system. The authors of this paper have worked on LA in a system integrated with DGs. Kushal has used the basic concepts of electrical circuits to arrive at average marginal LA in DS [17]. The method proposed in their work provides an average value of LA and not its exact value. Though most of the work has been done on loss allocation, the complexity involved in framing the problem is observed to be more. This paper proposes a simple and fair approach to allocating the losses among various load point consumers.

The main highlights of this paper are:

- Simple mathematical approach.
- An easy method to find the nodes beyond a branch.
- Comparative analysis of the proposed method and the Pro-rata method.

The authors of this paper have done LA in a simple DS without comparative analysis in [18]. In this manuscript, a detailed approach with comparative analysis is provided. The subsequent part of this manuscript outlines the following sections: Section 2 discusses the formulation of the LA problem, Section 3 and 4 describes the pro-rata method and solution methodology, Section 5 analyses the results obtained by using the proposed approach to various test systems considered in this work and Section 6 concludes the overall work presented in this paper.

2. Mathematical Formulation for the Proposed Method

The proposed method uses injection matrices as primitive bus-branch matrices. Along with these primitive matrices, node voltages obtained from a converged load flow solution and the information about the nodes connected beyond each branch is used for allocating the losses to each consumer connected to the load point.

The mathematical formulation for BIBC and BCBV matrices, load flows using primitive bus-branch matrices and LA to the load points is as follows:

2.1. Formation of BIBC and BCBV Matrices

Each bus in a DS is connected to complex loads. The complex power at bus ‘n’ is expressed as

$$S_n = P_n + jQ_n = V_n I_{Ln}^* \text{ for n=1, 2, …, N} \tag{1}$$

Where, P_n represents the true power connected to the load at bus ‘n’, Q_n represents the reactive power drawn by the load at bus ‘n’, V_n represents the voltage at node ‘n’, I_n represents the current used by the load at node ‘n’ and ‘N’ represents the system size.

From Equation (1), the current injected at node ‘n’ can be expressed as

$$I_{Ln} = \left(\frac{S_n}{V_n}\right)^* = \left(\frac{P_n+Q_n}{V_n}\right)^* = \left(\frac{P_n-Q_n}{V_n^*}\right) \tag{2}$$

Equation (2) can be used to find the load current from the load connected at the bus and the voltage at it. The obtained load currents can be used to find the branch currents using the BIBC matrix.

Consider a simple 6-bus DS shown in the Figure 1 to get the relationship between load and branch currents using the BIBC matrix and that between branch currents and bus voltages using the BCBV matrix. Bus 1 in this system represents the slack bus.

The branch currents I_{12}, I_{23}, …, I_{36} for the system shown in the Figure 1 can be written in terms of load currents I_{L2}, I_{L3}, …, I_{L6} using KCL as

$$\begin{bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ I_{45} \\ I_{36} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{L2} \\ I_{L3} \\ I_{L4} \\ I_{L5} \\ I_{L6} \end{bmatrix} \tag{3}$$

Equation (3) in a simple form can be written as

$$[I_B] = [BIBC][I_L] \tag{4}$$

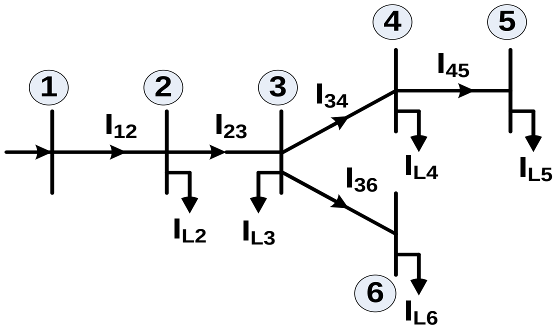

Fig. 1 One-line diagram of a 6-bus test system

For the system shown in Figure 1, the node voltages can be obtained from branch currents as

$$V_{b2} = V_{b1} - I_{12}Z_{12} \quad (5)$$

$$V_{b3} = V_{b2} - I_{23}Z_{23} \quad (6)$$

$$V_{b4} = V_{b3} - I_{34}Z_{34} \quad (7)$$

$$V_{b5} = V_{b4} - I_{45}Z_{45} \quad (8)$$

$$V_{b6} = V_{b3} - I_{36}Z_{36} \quad (9)$$

Substituting Equations (10), (11) and (12) in Equation (13), Equation (13) can be rewritten in terms of V_1 as

$$V_{b5} = V_{b1} - I_{12}Z_{12} - I_{23}Z_{23} - I_{34}Z_{34} - I_{45}Z_{45} \quad (10)$$

Similarly, the voltages at other non-slack buses can be written as

$$V_{b4} = V_{b1} - I_{12}Z_{12} - I_{23}Z_{23} - I_{34}Z_{34} \quad (11)$$

$$V_{b3} = V_{b1} - I_{12}Z_{12} - I_{23}Z_{23} \quad (12)$$

$$V_{b6} = V_{b1} - I_{12}Z_{12} - I_{23}Z_{23} - I_{36}Z_{36} \quad (13)$$

Equations (5), (10), (11), (12) and (13) can be expressed in vector-matrix form as

$$\begin{bmatrix} V_{b1} \\ V_{b1} \\ V_{b1} \\ V_{b1} \\ V_{b1} \end{bmatrix} - \begin{bmatrix} V_{b2} \\ V_{b3} \\ V_{b4} \\ V_{b5} \\ V_{b6} \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ I_{45} \\ I_{36} \end{bmatrix} \quad (14)$$

Equation (14) in a simple form can be written as

$$[\Delta V] = [BCBV][I_B] \quad (15)$$

Where V_{b1} to V_{b6} represent bus voltages and BCBV represents the branch-current to bus-voltage matrix.

2.2. Load Flow Analysis (LFA) Using Primitive Bus-Branch Matrices

Substituting Equation (4) in Equation (15) leads to the following expression:

$$[\Delta V] = [BCBV][BIBC][I_L] \quad (16)$$

By replacing the product of BCBV and BIBC matrices with a single matrix (DLF), Equation (16) becomes

$$[\Delta V] = [DLF][I_L] \quad (17)$$

Equation (22) can be used to update the node voltages in each iteration (k) as

$$[V]^{k+1} = [V]^k + [\Delta V]^k \quad (18)$$

Equations (2), (17) and (18) can be iteratively solved for load flow convergence.

2.3. Loss Allocation to the Load Points

The load current at load point ‘n’ can be written in rectangular representation of phasor form as

$$I_{Ln} = |I_{Ln}|\angle(-\delta_{Ln}) \quad (19)$$

$$\Rightarrow I_{Ln} = |I_{Ln}|\,cos(\,\delta_{Ln}) - j\,|\,I_{Ln}|\,sin(\,\delta_{Ln}) \quad (20)$$

$$\Rightarrow I_{Ln} = I_{real,Ln} - j\,I_{imag,Ln} \quad (21)$$

Where δ_{LN} represents the angle of current I_{Ln}, $I_{imag,}$ and $I_{real,\ Ln}$ respectively represent the imaginary and real components of load current I_{Ln}.

The current flowing in a branch ‘mn’, if N_{mn} represents the total number of buses connected to this branch, then it may be written as

$$I_{mn} = \sum_{n=1}^{N_{mn}} I_{Ln} \quad (22)$$

True power loss in branch m-n can be expressed as

$$P\,Re\{(V_m - V_n)I^*{}_{mn}\}_{loss,mn} \quad (23)$$

Let V_m-V_n=a-jb (Complex notation for potential difference between nodes ‘m’ and ‘n’). Then, using Equation (27), Equation (28) can be rewritten as

$$P\,Re\left\{(a - jb) \times \sum_{n=1}^{N_{mn}} I^*{}_{Ln}\right\}_{loss,mn} \quad (24)$$

Substituting Equation (21) in Equation (24), it gets modified as,

$$P\,Re\left\{(a - jb) \times \sum_{n=1}^{N_{mn}}\left(I_{real,Ln} + j\,I_{imag,Ln}\right)\right\}_{loss,mn} \quad (25)$$

$$\Rightarrow P\sum_{n=1}^{N_{mn}}\{aI_{real,Ln} + bI_{imag,Ln}\}_{loss,mn} \quad (26)$$

Equation (26) can be used to dispense real power loss in branch m-n to the consumers or load points beyond branch m-n. The amount of loss in branch m-n assigned to the consumer connected at load point ‘n’ can be found as

$$preal, Ln_{imag,Ln}{}_{loss,n} \quad (27)$$

3. Pro-Rata Method

The pro-rata method is the basic method used for loss distribution. The LA to the consumers in this method is done by allocating the losses in context to the real power load connected at the load point [18]. The impact of reactive power flow on the loss dissemination can be accounted for by taking kVA demand instead of kW demand.

The total kVA consumed by the load in the given system can be expressed as

$$kVA_{Total} = \sum_{n=2}^{N_{bus}} kVA_n \quad (28)$$

In the above expression, kVA_{Total} represents the total kVA demand in the given system, kVA_n represents the kVA demand at node ‘n’. N_{bus} represents the total number of buses in the system. In this equation, the lower limit for ‘n’ is taken as 2 because bus number 1 is usually allocated to the slack bus, which has no load connected to it. Using this approach, the loss allocated to the consumer connected at load point ‘n’ is expressed as

$$p_{loss,n} = TPL \times \frac{kVA_n}{kVA_{Total}} = \frac{TPL}{kVA_{Total}} \times kVA_n \quad (29)$$

$$\Rightarrow p_{loss,n} = K_D \times kVA_n \quad (30)$$

In Equation (29), TPL represents the total real power loss in the system. Since the ratio of total true power loss and total kVA load in the system is constant, this ratio in Equation (30) has been replaced by a constant K_D, known as the demand loss factor.

4. Solution Methodology

The solution for load flows can be obtained using BIBC and BCBV matrices using the procedure shown in the flowchart in Figure 2. As seen in the flowchart given in Figure 2, BIBC and BCBV matrices can be used to find the DLF matrix and voltage correction vector (ΔV). ΔV can ultimately be used to update the voltages during the iteration. This way, voltages are updated iteratively, and the load flow solution is obtained when the convergence criteria are met. BIBC and BVBV matrices can be easily found if the information about the nodes beyond each branch is known a priori. In previous study, a manual approach is applied to calculate the LA in small-sized systems. It would be cumbersome to use that technique for large systems. On the contrary, in this work, a logical method to find the nodes beyond a particular branch is proposed as given in the flowchart in Figure 3. Instead of manual calculations, the proposed approach is simulated in a MATLAB environment using the logical steps used in the flowchart. The code written using the method shown in the flowchart can be applied to a system of any size for LA calculation. In this flowchart, NBDB is the matrix containing the information about the nodes beyond the different branches.

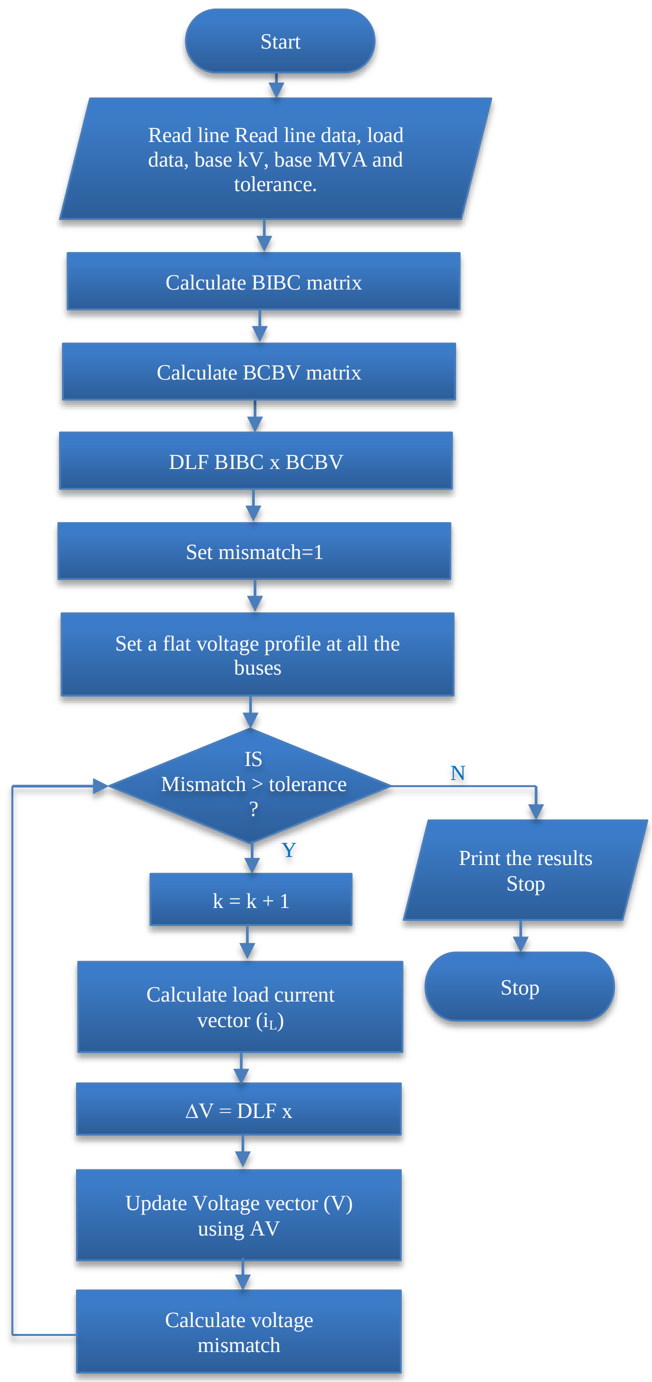

Fig. 2 Flowchart for DSLFA

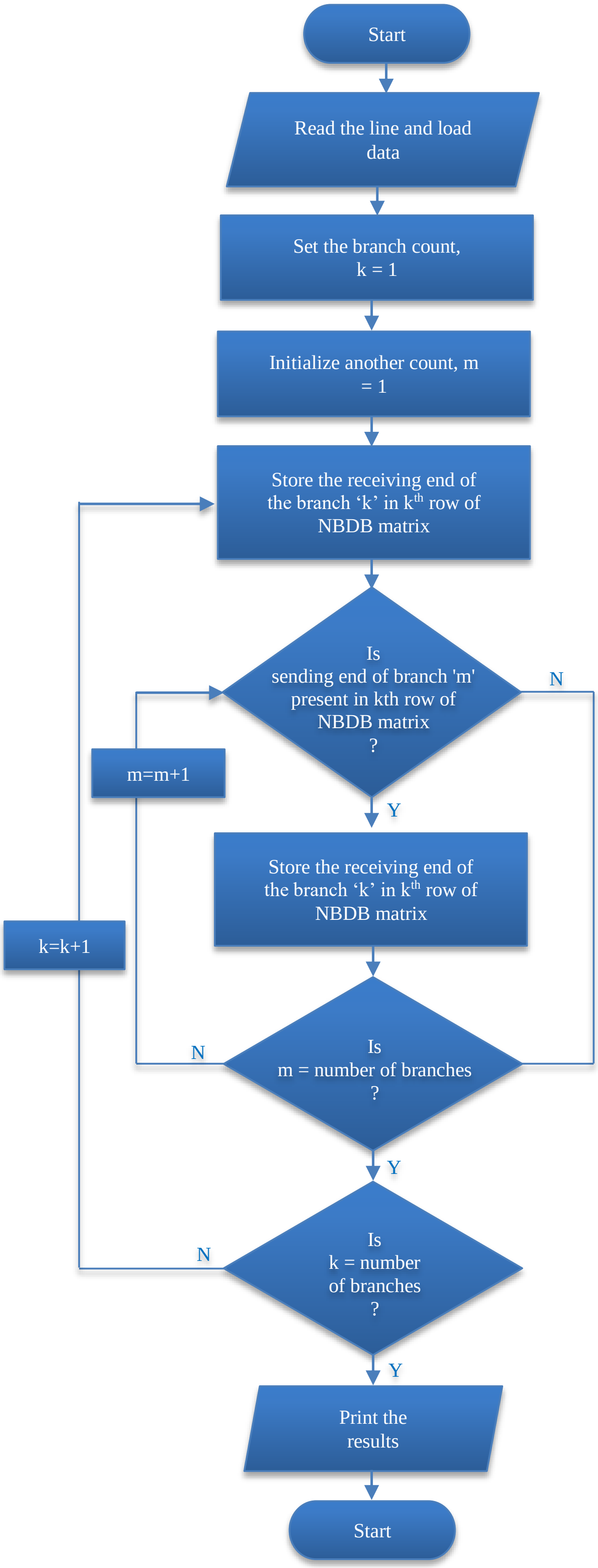

Fig. 3 Flowchart for detecting the nodes beyond a branch

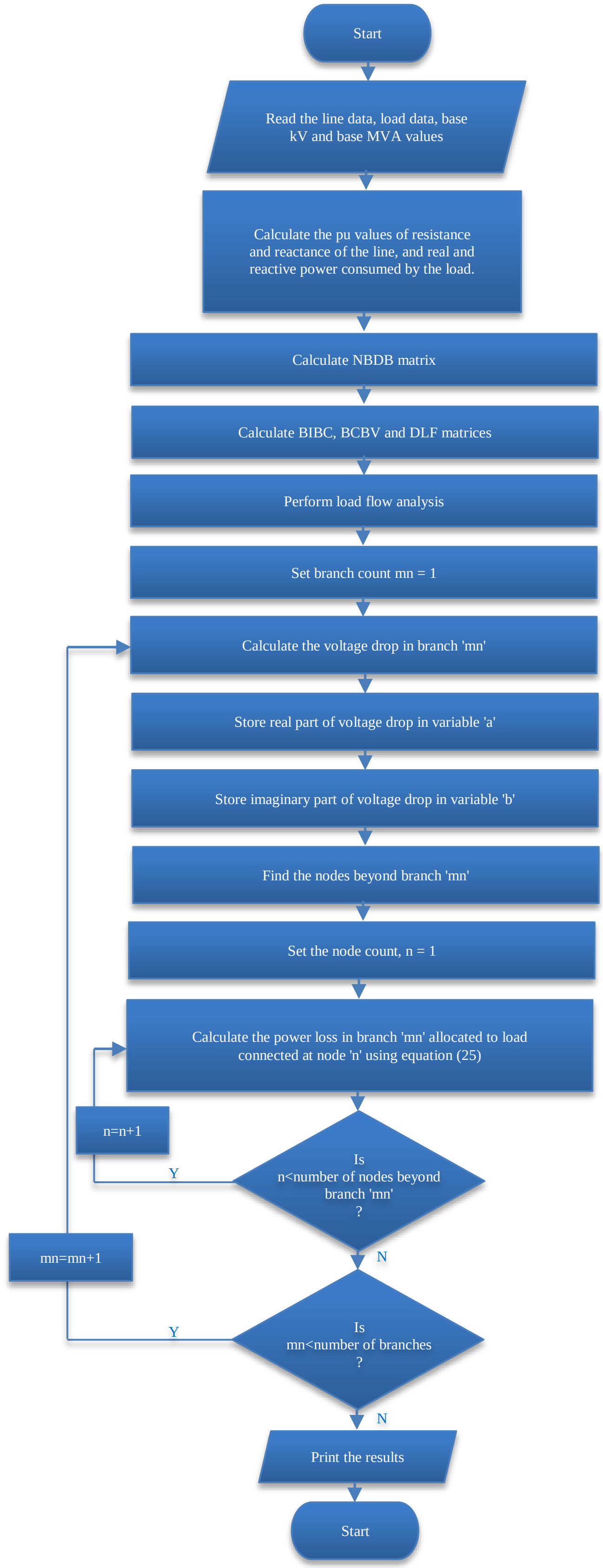

Fig. 4 Flowchart for LA using the proposed approach

LA can be done to various customer load points using the NBDB matrix using the mathematical formulae derived in Section 3. The overall work exhibited in this paper, including the proposed LA method, is shown in the flowchart in Figure 4.

5. Results and Discussion

The proposed approach is applied to 9-bus [19], 15-bus [20] and 33-bus [21] DS. The single line diagrams of these three systems are shown in Figure 5, Figure 7 and Figure 9, respectively. The input data of the above systems are provided in Tables 1, 2, and 3 in the Appendix section of this paper.

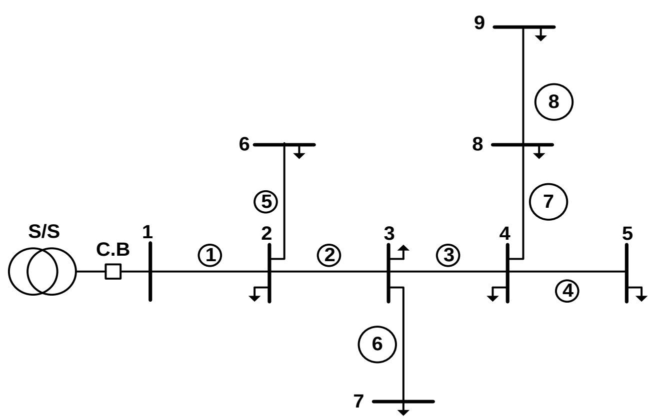

Fig. 5 Single line diagram of a 9-bus radial distribution system [19]

5.1. Loss Allocation for 9-Bus Distribution System

The DS shown in Figure 5 consists of 9 buses and 8 lines. Buses in this Figure are marked normally, and branches are encircled. This system's line and load data are provided in Table 1 in the Appendix section. For the LFA of this system, the base kV is taken as 11 and the base MVA is taken as 100. After performing the LFA using the approach explained in Section 2.2 of this paper, the voltages at various nodes in the system are as shown in Figure 6, and the total real power loss in the system is found to be 24.04 kW.

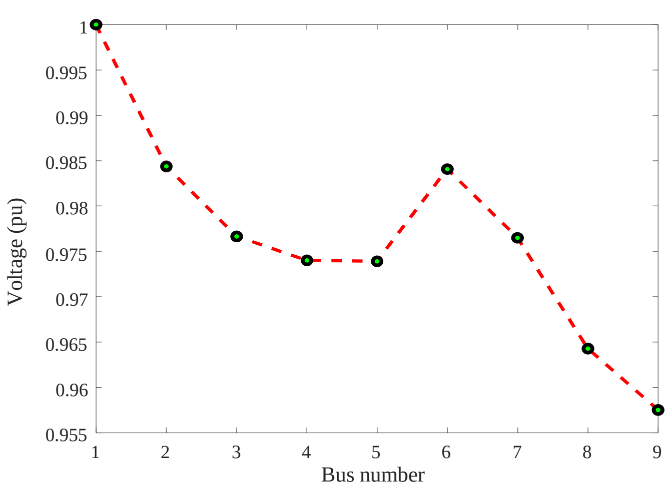

Fig. 6 Node voltage magnitudes in 9-bus RDS

The total loss obtained using load flow analysis is allocated to the consumers of load points in the 9-bus DS using the proposed and Pro-rata approaches. The comparative results are shown in Table 4.

Table 4. Real power loss allocated to consumers in the 9-bus RDS

Bus/Node Number	KVA Demand	Allocated Real Power Loss (KW)	
		Proposed Method	Pro-Rata Method
1	0	0	0
2	188.3083	2.6412	4.8349
3	203.8235	3.7693	5.2332
4	13.99428	0.3311	0.3593
5	34.98571	0.8307	0.8983
6	56.64203	0.7633	1.4543
7	13.41641	0.2970	0.3445
8	228.2809	7.4565	5.8612
9	196.8146	7.9497	5.0533

If two loads of similar kVA demand are considered, one load being nearer to the source (sub-station) and the other being relatively far, the Pro-rata method allocates the same loss to both the consumer loads. The results in Table 4 show that the loss allocated to the consumer connected at load point 3 is more than that allocated to the consumer connected at load point 9. This is because the Pro-rata method allocates the loss based on the amount of load connected at the node.

However, the contribution of load to the total loss in the system depends on its connectivity; far-end loads contribute more to the system loss than the nearer ones. In this sense, the kVA demand connected at bus 3 and bus 9 being almost the same, and bus 9 being relatively farther away, should incur more loss. The proposed method takes care of this feature, but not the Pro-rata method. Thus, it is seen that the formulation of the pro rata method, though simple, results in outcomes that are not fair enough. In this regard, the proposed method aims to give a fair result for LA among the consumers connected to the system.

5.2. Loss Allocation for 15-Bus RDS

The DS shown in Figure 7 consists of 15 buses and 14 lines. Buses in this Figure are marked normally, and branches are encircled. This system's line and load data are provided in Table 2 in the Appendix section. For the LFA of this system, the base kV is taken as 13.2, and the base MVA is taken as 100. After performing the load flow analysis using the approach described in Section 2.2 of this paper, the voltages at various nodes in the system are shown in Figure 8, and the total real power loss in the system is 41.61 kW.

The total loss obtained using LFA is allocated to the consumers of load points in the 15-bus DS using the proposed approach and the Pro-rata approach. The comparative results are shown in Table 5.

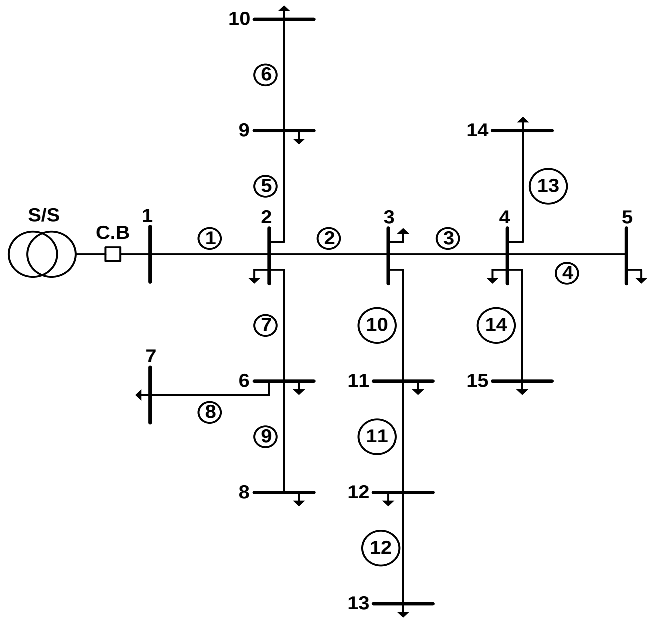

Fig. 7 Single line diagram of a 15-bus RDS [20]

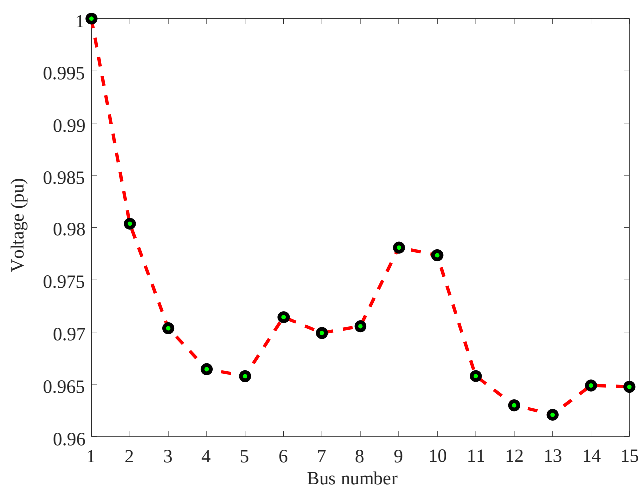

Fig. 8 Node voltage magnitudes in 15-bus RDS

From the results shown in Table 5, it can be seen that using the Pro-rata method, the losses allocated to the consumer at bus number 4 are the same as those to the consumer at bus number 15, though bus number 15 is relatively farther from the source than bus number 4 from the source end. A similar observation can be made for bus numbers 3, 12 and 14, and 2, 5, 8 and 13. But a fair justification is made in LA for the cases discussed above using the proposed method.

Table 5. Real power LA in 15-bus RDS

Bus/Node Number	kVA Demand	Allocated Real Power Loss (kW)	
		Proposed Method	Pro-Rata Method
1	0	0	0
2	62.9993	1.0056	1.4961
3	99.9969	2.2851	2.3747
4	200.0010	5.0665	4.7495
5	62.9993	1.7058	1.4961
6	99.9969	2.3314	2.3747
7	99.9969	2.4428	2.3747
8	62.9993	1.5435	1.4961
9	200.0010	3.3947	4.7495
10	200.0010	3.5296	4.7495
11	200.0010	5.2939	4.7495
12	99.9969	2.9550	2.3747
13	62.9993	1.9517	1.4961
14	99.9969	2.7276	2.3747
15	200.0010	5.3721	4.7495

5.3. Loss Allocation for 33-Bus RDS

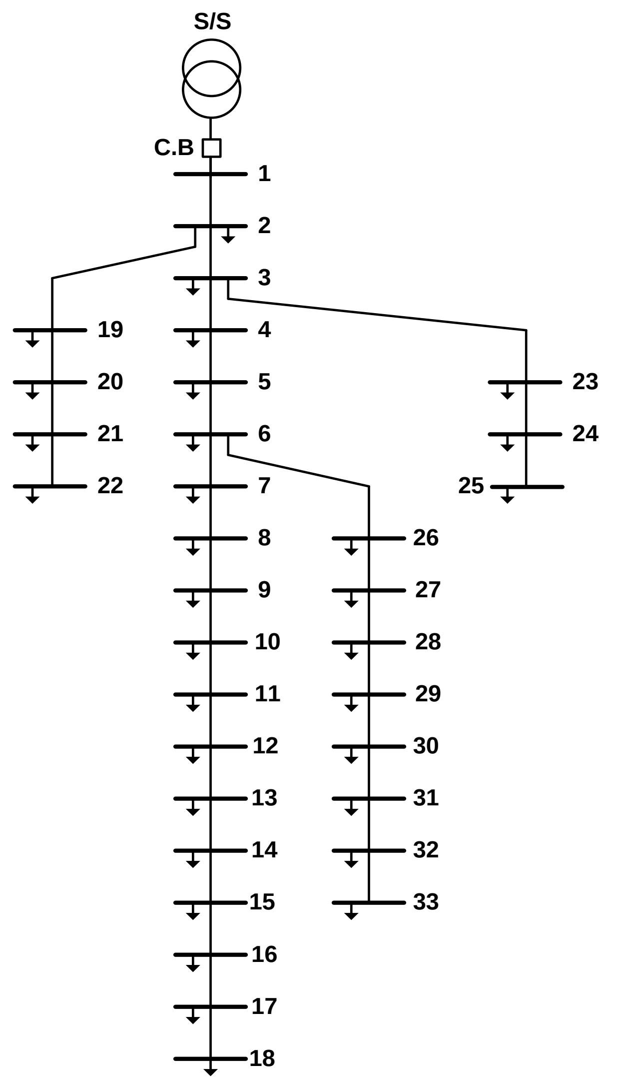

Fig. 9 Single line diagram of a 33-bus RDS [21]

The DS shown in Figure 9 consists of 33 buses and 32 lines. This system's line and load data are provided in Table 2 in the Appendix section.

For load flow analysis of this system, the base kV is 12.66, and the base MVA is 100. After performing the LFA using the approach described in Section 2.2 of this paper, the voltages at various nodes in the system are shown in Figure 10, and the total real power loss in the system is 202.67 kW.

The total loss obtained using load flow analysis is allocated to the consumers of load points in the 33-bus DS using the proposed and Pro-rata approaches. The comparative results are shown in Table 6.

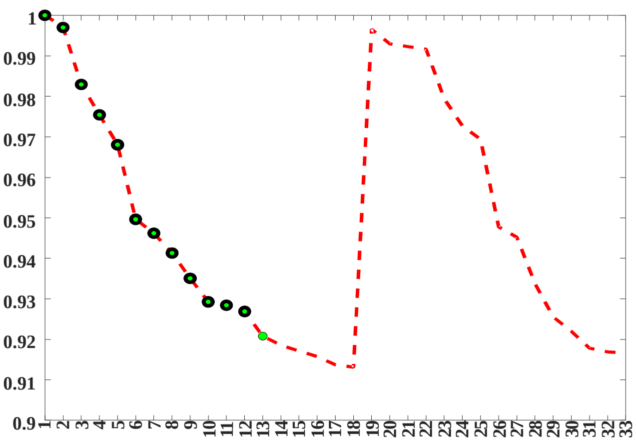

Fig. 10 Node voltage magnitudes in 33-bus RDS

Table 6. Real power LA in 33-bus RDS

Bus/Node Number	KVA Demand	Allocated Real Power Loss (KW)	
		Proposed Method	Pro-Rata Method
1	116.61904	0.31286	5.19608
2	98.48858	1.63027	4.38826
3	144.22205	3.25011	6.42596
4	67.08204	2.10257	2.98891
5	63.24555	3.22949	2.81797
6	223.60680	11.19907	9.96302
7	223.60680	12.35289	9.96302
8	63.24555	4.11680	2.81797
9	63.24555	4.49429	2.81797
10	54.08327	3.36436	2.40973
11	69.46222	4.61552	3.09496
12	69.46222	4.98348	3.09496
13	144.22205	10.11609	6.42596
14	60.82763	5.34902	2.71024
15	63.24555	5.36422	2.81797
16	63.24555	5.47902	2.81797
17	98.48858	8.18356	4.38826
18	98.48858	0.31830	4.38826
19	98.48858	0.59658	4.38826
20	98.48858	0.64725	4.38826
21	98.48858	0.69114	4.38826
22	102.95630	1.95535	4.58732
23	465.18813	11.71098	20.72692
24	465.18813	13.03414	20.72692
25	65.00000	3.38857	2.89614
26	65.00000	3.58619	2.89614
27	63.24555	4.37427	2.81797
28	138.92444	10.17014	6.18992
29	632.45553	22.55075	28.17969
30	165.52945	13.97840	7.37533
31	232.59407	19.77138	10.36346
32	72.11103	5.74791	3.21298
33	116.61904	0.31286	5.19608

From the results shown in Table 6, it can be seen that using Pro-rata method the losses allocated to consumer at bus number 6 is same as that to the consumer at bus numbers 9, 10, 16, 17 and 28 though these bus numbers (9, 10, 16, 17 and 28) are relatively far away than bus number 6 from source end. A similar observation can be made for some other bus numbers with the same kVA demand. Even in this case, it can be seen that a fair justification is made in LA using the proposed method.

6. Conclusion

LA has been done so until now using various methods for transmission and DS. In this paper, a simple and effective method for LA in DS has been proposed. This paper contributes to two unique features: a simple mathematical analysis and an easier logical approach to finding the nodes beyond any branch in the DS. A lucid way of solving the problem has been explained using a flowchart for every part of it. The proposed method has been applied to 9-bus, 15-bus and 33-bus RDS. The results obtained using the proposed approach are compared with those of the Pro-rata method. The proposed method shows a fair means of allocating the losses. A reasonable difference in the allocation of losses for the consumers at the near and far ends is found using the proposed method.

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Appendix

Table 1. Input data for 9-bus system

Line Number	S.E	R.E	R (Ohms)	X (Ohms)	P_{load} (kilowatts)	Q_{load} (kilovars)
1	1	2	1.632	1.1019	162	96
2	2	3	1.088	0.7346	150	138
3	3	4	0.544	0.3673	12	7.2
4	4	5	0.272	0.1836	30	18
5	2	6	0.544	0.3673	45.6	33.6
6	3	7	1.376	0.3896	12	6
7	4	8	2.752	0.7792	180	140.4
8	8	9	4.128	1.1688	156	120

Table 2. Input data for 15-bus system

Line Number	S.E	R.E	R (Ohms)	X (Ohms)	P_{load} (kilowatts)	Q_{load} (kilovars)
1	1	2	1.3531	1.3235	44.1	44.99
2	2	3	1.1702	1.1446	70	71.41
3	3	4	0.8411	0.8227	140	142.83
4	4	5	1.5235	1.0276	44.1	44.99
5	2	9	2.0132	1.3579	140	142.83
6	9	10	1.6867	1.1377	140	142.83
7	2	6	2.5573	1.7249	70	71.41
8	6	7	1.0882	0.734	70	71.41
9	6	8	1.2514	0.8441	44.1	44.99
10	3	11	1.7955	1.2111	140	142.83
11	11	12	2.4485	1.6515	70	71.41
12	12	13	2.0132	1.3579	44.1	44.99
13	4	14	2.2308	1.5047	70	71.41
14	4	15	1.197	0.8074	140	142.83

Table 3. Input data for the 33-bus system

Line Number	S.E	R.E	R (Ohms)	X (Ohms)	P_{load} (kilowatts)	Q_{load} (kilovars)
1	1	2	0.0922	0.047	100	60
2	2	3	0.493	0.2511	90	40
3	3	4	0.366	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.819	0.707	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	0.7114	0.2351	200	100
8	8	9	1.03	0.74	60	20
9	9	10	1.044	0.74	60	20
10	10	11	0.1966	0.065	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.468	1.155	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.591	0.526	60	10
15	15	16	0.7463	0.545	60	20
16	16	17	1.289	1.721	60	20
17	17	18	0.732	0.574	90	40
18	2	19	0.164	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40

Line Number	S.E	R.E	R (Ohms)	X (Ohms)	P_{load} (kilowatts)	Q_{load} (kilovars)
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.898	0.7091	420	200
24	24	25	0.896	0.7011	420	200
25	6	26	0.203	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.059	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.963	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0.341	0.5302	60	40