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Original Article

Model Order Reduction of Interval Systems in Z-Domain Using the Improvement of Simplified RAM

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Abstract *- This study emphasizes an improvement method for Simplified Routh Approximation with discrete-time SISO interval systems. The diminished order model's numerator and denominator are estimated using the θ interval table. The Kharitonov polynomial is also used to verify the proposed inferred interval model's stability. The enhanced approach sustains the reduced system's stability feature if the system of high-rise order intervals is sturdy. The limitation of calculating the reciprocal transformation and inverse reciprocal transformation, observed in the very recent Advanced Routh Approximation Method, has also been avoided in the suggested procedure. To substantiate the relevance of the initiated approach, plotting of impulse and step responses for the reduced models as well as the system. A mathematical example is comprehended to elucidate the proposed method and simplified to the Simplified Routh Method, Model order in discrete time order, uncertain method, and mixed method (α and β method) of integral square error values. To obtain the results, interpret the efficacy and accuracy of this intended approach.*

Keywords *- Interval Analysis, Discrete Systems, Improvement of Simplified Routh Approximation Method, Kharitonov Polynomials, Model Order Diminish, Integral Square Error.*

1. Introduction

In nearly every single empirical problem with control system concept, the system's order elaboration is a crucial topic; no semblance of higher order systems is much better. An approach is the model order reduction approximation for acquiring models at a lower level from high- order systems. In the literature, systems having prescribed parameters like Aggregation method [1], Continued fraction method [2], Moment matching Mechanism [3], Pade approximate model [4], Routh estimation [5], and L∞ enhanced optimization method [6] of continuous systems. However, very few methods are applicable to interval systems. Regarding interval systems, the range of specified intervals affects the numerator and denominator coefficients. In this case, interval coefficients are handled using interval arithmetic [7]. Compared to continuous time systems, digital systems have numerous advantages. This sparked a persistent interest in discrete interval systems research. A technique for reducing discrete interval systems has been suggested by B. Bandyopadhyay et al. [8]. Nevertheless, the simplified model's stability is not affected by this approach. Ismail et al. [9] suggested a Pade idea-based discrete system reduction to preserve dominating poles. In this method, the reduced model's denominator is created by keeping the provided discrete interval system's dominating poles. Although the numerator is dominated by retaining the model's initial moments of r with the system, this method increases computational complexity. Several techniques are furthermore suggested for order decrease in the discrete interval system, similar to straight series expansion established reduction [10], reduction based on pole clustering model [11], model minimization of systems with discrete intervals with multiple outputs and inputs [12], stability preservation method [13], and biased model reduction by differentiation technique [14], Sasty et al. and V.P. Singh Recommended methods [15, 16], Aditya Prasad Padhy et.al [17] proposed by decrease of the model order of Uncertain systems in discrete time domains. With this approach, the numerators and denominators of systems with discontinuous intervals decrease in the model order method, which is computed by approximating the Routh criteria and comparing with that of moments in time and the Markov framework. Conversely, the mixed method for lowering a discrete linear time interval system's order proposed by Kranthi Kumar etal [18] requires two tables. Diminishing a linear system with discontinuous time intervals, order obtained via the α table and the β table, yields the numerator polynomial. The recommended methodology in this research paper for decreasing the model sequence of systems with discontinuous intervals of high order is to improve the Simplified Routh

Approximation Method. The proposed approaches have required only one interval table. The suggested approach ascertains the denominator and numerator of lower models of order utilizing the θ–table. Stable models with a lower order of the initial high-order system are consistently produced by the proposed method. The suggested approach does not involve the reciprocal transformation. The proposed method results are contrasted with other current techniques through the integral square error. It can be observed that the values of integral square error using the suggested approach are lower than those of the other methods. This shows the accuracy of the proposed approach. In addition to the methods available earlier as analyzed above, the method [20] published very recently suffers from the serious drawback of computation of p-transformation, full numerator and denominator tables for the computation of k- number of interval parameters for obtaining the reduced order model and no guarantee on the retention of stability of the higher order system in its models has also been addressed and avoids in this proposed research procedure. The suggested method emphasizes reducing the computation time when compared to the available methods in the international literature. Also, it assures the retention of stability in the reduced-order models where most of the methods fail. Hence, the suggested method bridges the gap of the need raised by many of the controller designs. The latency of the method is much less compared to other significant methods in the recent literature.

2. Problem Statement

Let us focus on n^{th} order one input Higher order distinct period of time systems with a single output that are stable as follows.

$$T_n(Z) = \frac{[O_1^-,O_1^+]Z^{n-1}+[O_2^-,O_2^+]Z^{n-2}+......+[O_n^-,O_n^+]}{[L_1^-,L_1^+]Z^n+[L_2^-,L_2^+]Z^{n-1}+........+[L_n^-,L_n^+]} \quad (1)$$

Then the r^{th} order reduced model will be:

$$R_r(Z) = \frac{[a_1^-,a_1^+]Z^{r-1}+[a_2^-,a_2^+]Z^{r-2}+......+[a_r^-,a_r^+]}{[b_1^-,b_1^+]Z^r+[b_2^-,b_2^+]Z^{r-1}+.......+[b_r^-,b_r^+]} \quad (2)$$

3. Proposed Reduction Method

To acquire the lower-order model, the stages that include the stages are included in the suggested method.

Step 1:-

Higher-order system of bilinear transformation

$$Z=P+1 \quad (3)$$

In Equation (1), the higher-order system of n^{th} order is obtained as:

$$T_n(P) = \frac{[x_1^-,x_1^+]P^{n-1}+[x_2^-,x_2^+]P^{n-2}+......[x_n^-,x_n^+]}{[y_1^-,y_1^+]P^n+[y_2^-,y_2^+]P^{n-1}+......[y_n^-,y_n^+]} \quad (4)$$

Step 2:-

Formulation of the θ table.

The θ table's first two rows come from the denominator coefficient of $T_n(P)$, and the remaining coefficients are replaced by Equation (5). All the coefficients are calculated by using interval arithmetic rules [7].

Finally, construct the θ interval table as:

Table 1. Construct the θ interval table

$$[y_n^-,y_n^+][y_{n-1}^-,y_{n-1}^+][y_{n-2}^-,y_{n-2}^+]......$$
$$[y_{n-1}^-,y_{n-1}^+][y_{n-2}^-,y_{n-2}^+]$$
$$[c_1^-,c_1^+][c_2^-,c_2^+][c_3^-,c_3^+]$$
$$[c_2^-,c_2^+][c_3^-,c_3^+]$$
$$[d_1^-,d_1^+][d_2^-,d_2^+]$$

For i= odd

$$[c_i^-,c_i^+] = [y_i^-,y_i^+] \qquad i = 1, 3, 5........$$

$$[d_1^-,d_1^+] = [d_2^-,d_2^+]$$

$$[d_i^-,d_i^+] = [y_{i+1}^-,y_{i+1}^+] \qquad i = 1, 3, 5........$$

For i= even

$$[c_i^-,c_i^+] = \frac{[y_i^-,y_i^+]-\{[y_n^-,y_n^+][y_{i+1}^-,y_{i+1}^+]\}}{[y_1^-,y_1^+]}$$

$$[d_i^-,d_i^+] = \frac{[d_{i+1}^-,d_{i+1}^+]-\{[d_1^-,d_1^+][d_{i+2}^-,d_{i+2}^+]\}}{[d_2^-,d_2^+]} \quad (5)$$

Step 3:-

The parameters with θ values are delivered from the first column's ratios of the interval θ table.

Step 4:-

A suggested approach for obtaining the polynomials within the denominator and numerator.

(i). The diminished order numerator polynomial $N_r(P)$, in general is characterized as:

$$N_r(P) = \frac{[\theta_k^-,\theta_k^+]!}{[b_r^-,b_r^+]}\{[a_r^-,a_r^+]+[a_{r-1}^-,a_{r-1}^+]P+......\quad (6)$$

(ii). The polynomial denominator of the lowered order model, $D_r(S)$, is generally characterized as:

$$D_r(S) = [1,1]P^K + \frac{[\theta_K^-,\theta_K^+]!}{[b_r^-,b_r^+]}\{[b_r^-,b_r^+] + [b_{r-1}^-,b_{r-1}^+]P...... \quad (7)$$

Step 5:-

Check the stability of Equation (7), using Kharitonov polynomials [19]

Step 6:-

Lastly, use the inverse bilinear transformation by replacing

$$P = Z-1 \quad (8)$$

In Equations (6) and (7), we get the $R_r(Z)$

Step 7:-

From the initial system's step response with its model of diminished order, the integral square error calculations were done as:

$$ISE = \int_0^{\infty} [T_n(t) - T_r(t)]^2 dt \quad (9)$$

Here, $T_n(t)$ – The initial discrete-time system's step reactions

$T_r(t)$ – Reduced order discrete time systems' step responses

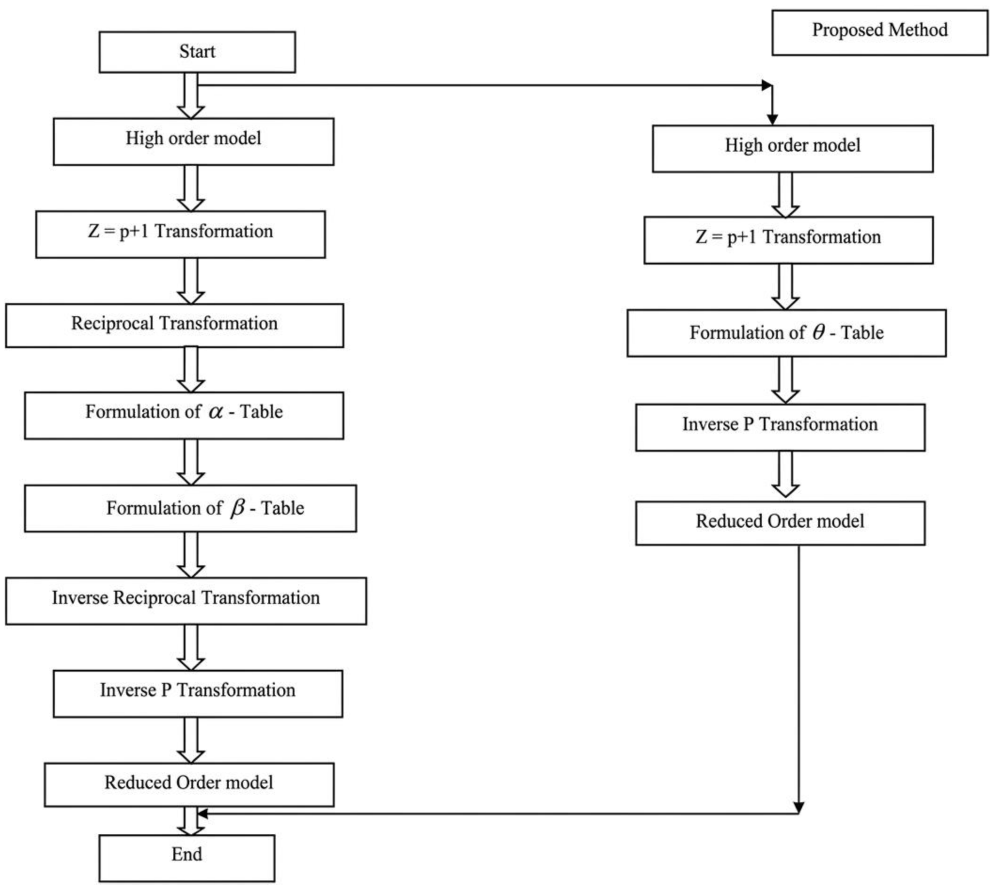

Fig. 1 Comparisons of the Advanced Routh approximation and the proposed method

4. Examples

Example 1

Examine the initial steady discrete system's interval, which is provided below

$$G_6(Z) = \frac{[0.85,0.95]Z^5+[1.10,1.250]Z^4+[2.050,2.150]Z^3+[3.15,3.45]Z^2+[3.4,3.5]Z+[2.6,3.0]}{[[10.15,10.35]Z^6+[8.1,8.45]Z^5+[6.2,6.65]Z^4+[5.3,5.6]Z^3+[1.1,1.2]Z^2+[1.3,1.6]Z+[2.20,2.251]} \quad (10)$$

For this interval systems model, a simplified 2nd-order model is advised. Applying the bilinear transformation Z=P+1, then it is conceivable to transfer the TF, G (Z), in the P-domain as:

$$G_6(P) = \frac{[0.85,0.95]P^5+[5.35,6.0]P^4+[14.95,16.65]P^3+[24.4,26.9]P^2+[24.5,26.6]P+[13.15,14.3]}{[10.15,10.35]P^6+[69.0,75.55]P^5+[198.95,204.15]P^4+[314.1,323.7]P^3+[287.45,297.67]P^2+[145.6,151.75]P+[34.35,36.1]} \quad (11)$$

Using the proposed θ interval table, the θ interval parameters are:

$$\theta_1 = [0.2263, 0.2479] \quad \theta_2 = [0.6426, 0.7324]$$

$$\theta_3 = [1.0005, 1.2870] \quad \theta_4 = [1.3851, 2.0906]$$

$$\theta_5 = [1.9822, 3.1944] \quad \theta_6 = [3.8435, 4.9229] \tag{12}$$

For K = 2 retaining θ_1 and θ_3 values, the proposed reduced order model

$$R_2(P) = \frac{[0.1536,0.2470]P+[0.08245,0.1328]}{[1,1]P^2+[0.912912,1.409454]P+[0.2153,0.3352]} \tag{13}$$

Using the inverse Transformation in two dimensions, when P=Z-1, Next in the Z-domain, the transfer function of $R_2(P)$ can be conveyed as:

$$R_2(Z) = \frac{[0.15361,0.24701]Z+[-0.164550,-0.02080]}{[1.1,1.1]Z^2+[-1.0870880,-0.5905460]Z+[-0.1941541,0.4222880]} \tag{14}$$

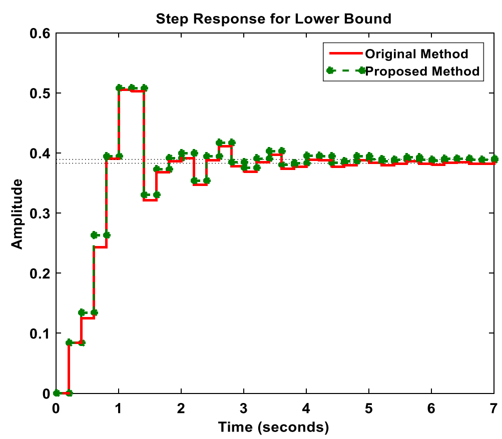

Fig. 2 The Lower Bound's Step response nature

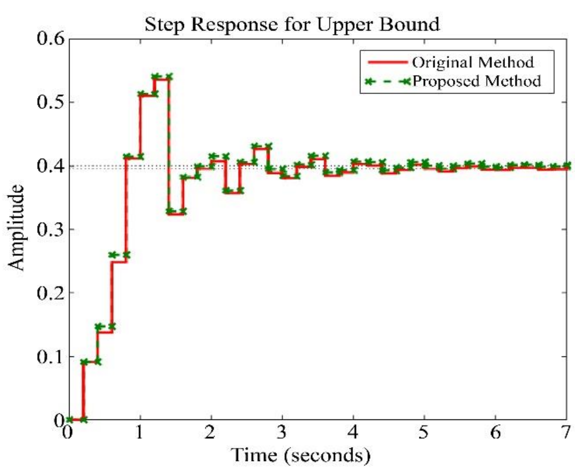

Fig. 3 The Upper Bound's Step response nature

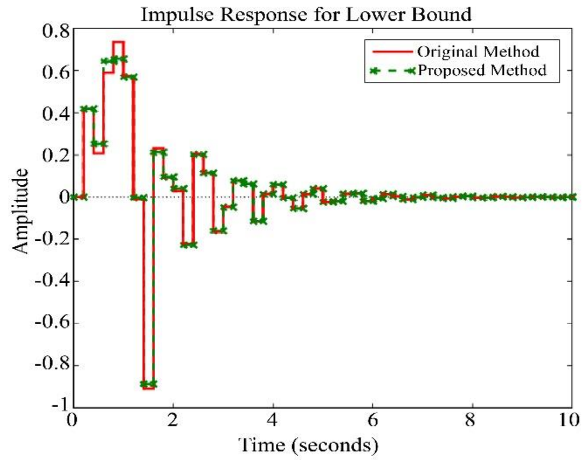

Fig. 4 Lower Bound's Impulse Response

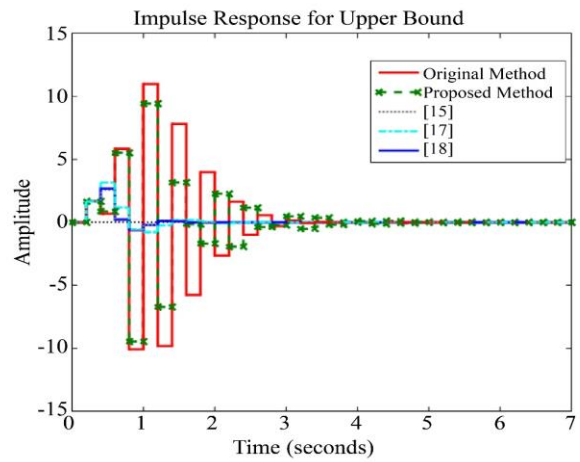

Fig. 5 Upper Bound's Impulse Response

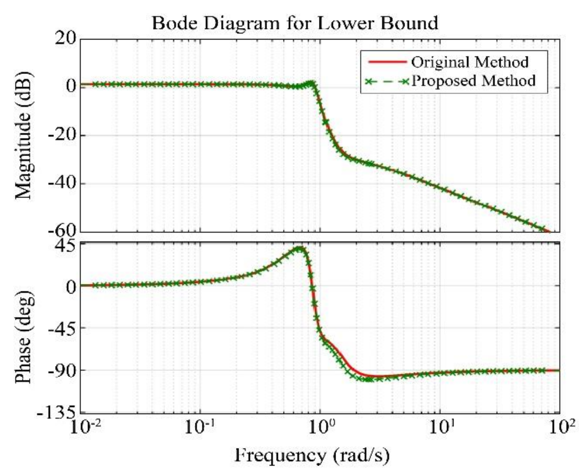

Fig. 6 Lower Bound's Bode response

Figures 2 and 3 display the step output of both decreased order and original models' upper and lower boundaries of interval systems, respectively. The impulse response of decreased order and original models of the system of discrete intervals are plotted in Figures 4 and 5, respectively.

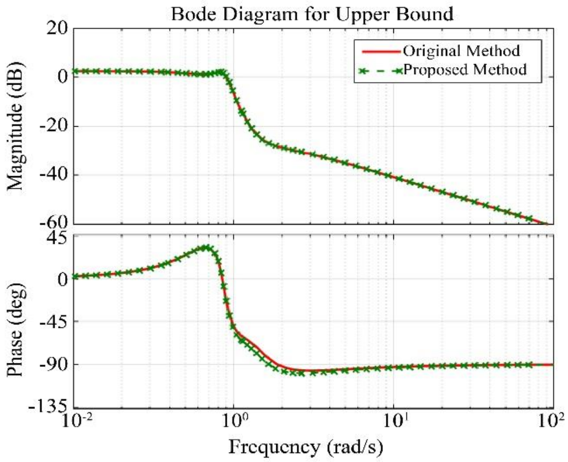

Fig. 7 Upper Bound's Bode response

Figures 6 and 7 display the Bode plot reactions of the initial and shortened order discrete models of interval systems.

Plots of reduced-order models and original systems are all quite close, as can be seen in Figures 2 through 7. This demonstrates how accurate the suggested approach is.

The advantageous features have been illustrated with this physical example, such as the retention of the critical response characteristics of time and frequency domains.

Example 2:-

Examine a 3rd-order TF that describes a discrete system's interval. [8]

$$G_3(Z) = \frac{[1.0,2.0]Z^2+[3.0,4.0]Z+[8.0,10.0]}{[6.0,6.0]Z^3+[9.0,9.50]Z^2+[4.90,5.01]Z+[0.80,0.850]} \quad (15)$$

The system Equation (15) can be transformed into a recommender system by:

Z = P + 1

$$G_3(P) = \frac{[1.0,2.0]P^2+[5.0,8.0]P+[12.0,16.0]}{[6.0,6.0]P^3+[27.0,27.50]P^2+[40.90,42.0]P+[20.70,21.350]} \quad (16)$$

Making use of the proposed θ table, the θ interval parameters are:

$\theta_1 = [0.4928, 0.5220]$

$\theta_2 = [1.6665, 1.7596]$

$\theta_3 = [3.978, 4.090]$ (17)

For K=2, retaining θ_1 and θ_3 values, the proposed reduced order model

$$R_2(P) = \frac{[0.459,0.8248]P+[1.1016,1.6496]}{[1,1]P^2+[3.7546,4.3302]P+[1.9002,2.2011]} \quad (18)$$

Using the Bilinear inverse transformation (IBTM), when P = Z-1, consequently, the purpose of transfer of $R_2(P)$ can be composed as in the Z- domain as:

$$R_2(Z) = \frac{[0.4590,0.82480]Z+[0.27680,1.19060]}{[1.0,1.0]Z^2+[1.75160,2.33020]Z-[1.430,0.55350]} \quad (19)$$

Decreased function of order transfer by the G. V K. Sastry et al. method [15] is:

$$R_2(Z) = \frac{[0.4260,0.35440]Z+[0.6680,0.28280]}{[1.0,1.0]Z^2+[1.48460,-0.13940]Z+[0.9030,-1.53880]} \quad (20)$$

Aditya Prasad Padhy et al.'s method [17] proposed a reduced-order transfer function.

$$R_2(Z) = \frac{[4.49,9.15]Z+[2.48,12.01]}{[27,27.5]Z^2+[-19.36,-15.97]Z+[9.67,10.82]} \quad (21)$$

And the reduced order model by Kranthi Kumar et al. method (α and β method) [18] may be evaluated like

$$R_2(Z) = \frac{[0.2307,0.3352]Z+[0.1409,0.4577]}{[1,1]Z^2+[-0.3336,-0.2403]Z+[0.0615,0.2522]} \quad (22)$$

Praveen Kumar et al.'s method [20] proposed a reduced model.

$$R_2(Z) = \frac{[0.20,0.34]Z+[0.14,0.48]}{[1,1]Z^2+[-0.34,-0.25]Z+[0.07,0.25]} \quad (23)$$

Table 2. ISE contrast for models with a lower and upper value

Methods	ISE
Proposed Method	[1.1204, 1.0099]
G.V.K.R.Sastry et.al method [15]	[1.1313, 1.2523]
Aditya Prasad Padhy et.al method [17]	[1.2737, 1.3617]
Kranthi Kumar et.al method [18]	[1.3555, 1.2071]
Praveen Kumar [20]	[1.4499, 1.1607]

Table 2 shows the ISE contrast for models with lower and upper values. The proposed method has a minimum error compared to the other existing methods.

The original Step answers and diminished order systems with discontinuous intervals are charted in Figures 8 and 9, respectively.

Figures 10 and 11 show the impulsive reaction between the original and diminished order systems of discrete intervals, respectively. Step and impulsive reactions of the lower discrete order system of intervals utilizing the suggested technique are shown in Figure 8-11, and the original systems are extremely similar to each other when compared to the current techniques. (i.e Sastry et.al, Aditya Prasad Padhy et.al, Kranthi kumar et.al and Praveen kumar et.al). This demonstrates the suggested method's accuracy.

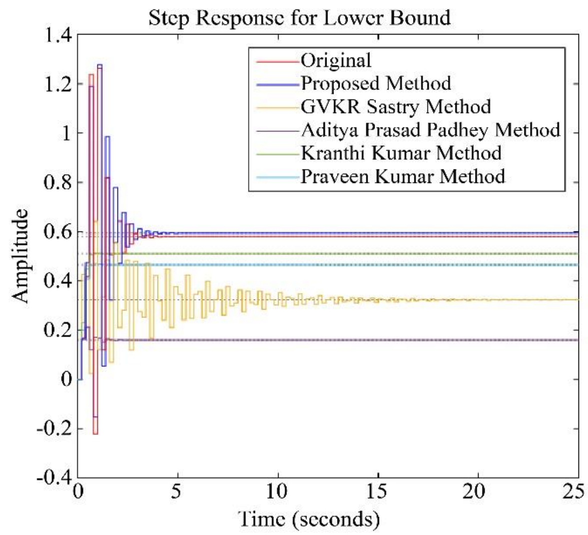

Fig. 8 Lower Bound's Step response

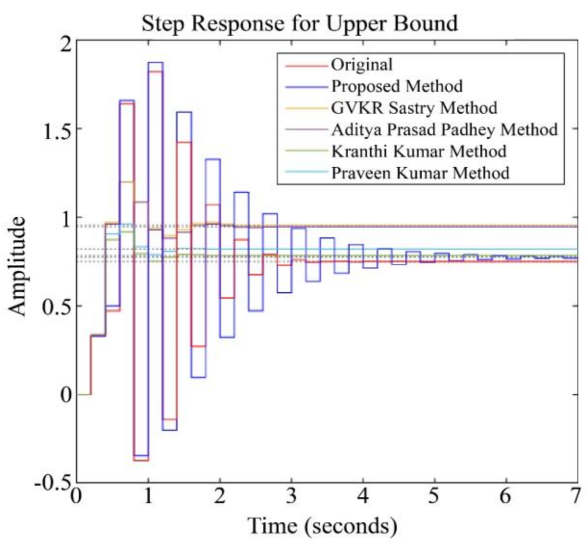

Fig. 9 Upper Bound's Step response

The findings of the suggested method are collated with SRAM to guarantee the work's originality, Aditya Prasad Padhy et al., α and β methods, Advanced Routh Approximation Methods [15, 17, 18, 20] through integral square error as depicted in the second table.

The second table exhibits that the suggested approach has a lower integral square error than the other approaches. [15, 17, 18, 20]. This numerical example shows the effectiveness of the proposed method over the recently appearing article [20] by retaining the stability, computing the reduced order model very fast, as it is computationally simple.

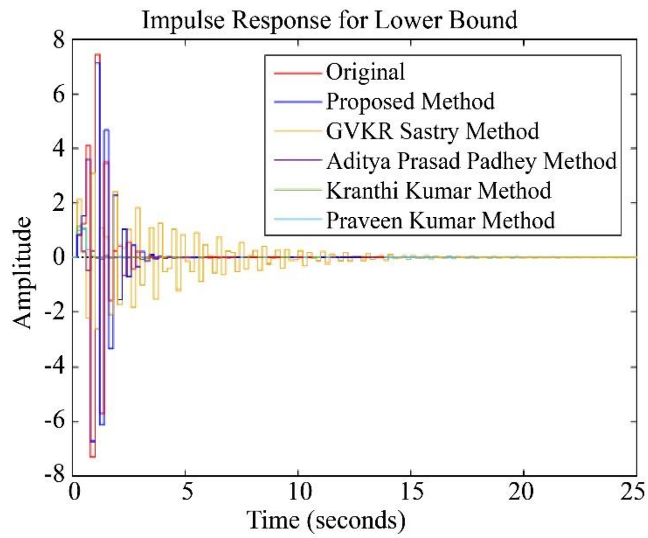

Fig. 10 Lower Bound's Impulse Response

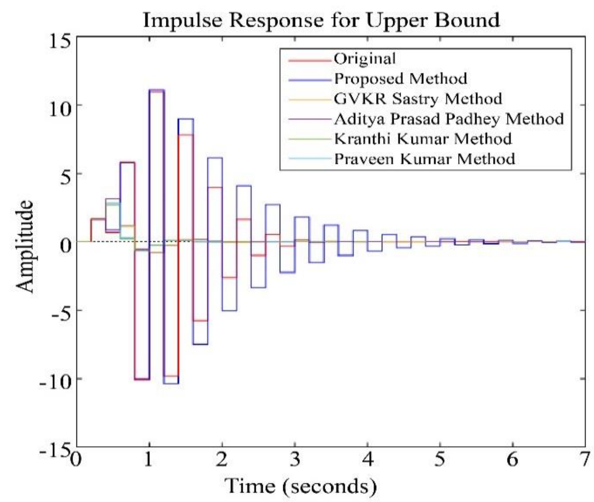

Fig. 11 Upper Bound's Impulse Response

It can be stated that by using the suggested method, in addition to retaining the stability in the models, the number of computations is much less than that of the other methods. The original method's impulse energy is represented in the model to exhibit that the impulse energy criteria are met through retaining the ISE criteria.

5. Conclusion

This work presents a decrease in the model order technique given a discrete interval system with a single input and single output-improvement of Simplified Routh Approximation. The strategy is used to ascertain the denominator as well as the numerator of the lower interval Discrete interval systems method. Sixth-order and third-order discrete-time frame test systems are lowered to a second-level model in order to illustrate the suggested strategy. Also, the procedure proposed in this article is superior to the methodology reported recently. The consistency of the resulting discrete model of lower-order intervals is ensured by the suggested method. The test system was used to validate the method that was described. Step and impulse responses are plotted to illustrate the advantages of the suggested approach. The effectiveness and applicability of the suggested strategy were demonstrated by the data obtained and the comparative analysis carried out utilizing integral square error.

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Conflict-of-Interest Statement

The authors have no Conflicts of interest while preparing the article.

References

[1] M. Aoki, "Control of Large Scale Dynamic Systems by Aggregation," *IEEE Transactions on Automatic Control*, vol. 13, no. 3, pp. 246-253, 1968. [CrossRef] [Google Scholar] [Publisher Link]

[2] Y. Shamesh, "Continued Fraction Methods for the Reduction of Linear Time-Invariant Systems," *IEE Conference Publication*, no. 96, pp. 220-227, 1973. [Publisher Link]

[3] N.K. Sinha, and B. Kuszta, *Modelling and Identification of Dyanamic Systems*, Springer Dordrecht, pp. 1-334, 1983. [Google Scholar] [Publisher Link]

[4] Y. Shamesh, "*Order Reduction of Linear Systems by Pade Approximation Method*," Doctoral Thesis, University of London, 1973. [Google Scholar]

[5] M. Hutton, and B. Friedland, "Routh Approximation for Reducing Order of Linear Time Invariant System," *IEEE Transaction on Automatic Control*, vol. 20, no. 3, pp. 329-337, 1975. [CrossRef] [Google Scholar] [Publisher Link]

[6] Keith Glover, "All Optimal Hankel-Norm Approximations of Linear Multivariable Systems and their L, ∞ -Error Bounds†," *International Journal of Control*, vol. 39, no. 6, pp. 1115-1119, 1984. [CrossRef] [Google Scholar] [Publisher Link]

[7] Eldon Hausen, "Interval Arithmetic in Matrix Computations- Part I," *Journal of the Society for Industrial and Applied Mathematics Series B Numerical Analysis*, vol. 2, no. 2, pp. 308-320, 1965. [CrossRef] [Google Scholar] [Publisher Link]

[8] B. Bandyopadhyay, O. Ismail, and R. Gorez, "Routh- Pade Approximation for Discrete Interval Systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 12, pp. 2454-2456, 1994. [CrossRef] [Google Scholar] [Publisher Link]

[9] O. Ismail, B. Bandyopadhyay, and R. Gorez, "Discrete Interval System Reducing using Pade Approximation to Allow Retention of Dominant Poles," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, no. 11, pp. 1075-1078, 1997. [CrossRef] [Google Scholar] [Publisher Link]

[10] Vinay Pratap Singh, and Dinesh Chandra, "Model Reduction of Discrete Interval System using Dominant Poles Retention and Direct Series Expansion Method," *2011 5th International Power Engineering and Optimization Conference*, Shah Alam, Malaysia, pp. 27-30, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[11] V.P. Singh, and D. Chandra, "Model Reduction of Discrete Interval System using Clustering of Poles," *International Journal of Modelling Identification and Control*, vol. 17, no. 2, pp. 116-123, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[12] A.P. Padhy, V.P. Singh, and S. Pattnaik, "On Model Reduction of Multi Input Multi Output Discrete Interval Systems," *2018 3rd IEEE International Conference on Recent Trends in Electronics, Information & Communication Technology (RTEICT)*, Bangalore, India, pp. 1842-1845, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[13] Younseok Choo et al., "A Note on Discrete Interval System Reduction via Retention of Dominant Poles," *International Journal of Control, Automation and Systems*, vol. 5, no. 2, pp. 208-211, 2007. [Google Scholar] [Publisher Link]

[14] N. Pappa, and T. Babu, "Biased Model Reduction of Discrete Interval System by Differentiation Technique," *2008 Annual IEEE India Conference*, Kanpur, India, pp. 258-261, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[15] G.V.K.R. Sastry, and P. Mallikarjuna Rao, "A New Method for Modelling of Large Scale Interval Systems," *IETE Journal of Research*, vol. 49, no. 6, pp. 423-430, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[16] V.P. Singh, P.D. Dewangan, and S.L. Sinha, "Improved Approximation of SISO and MIMO Continuous Interval Systems," *International Journal of System Control and Information Processing*, vol. 3, no. 3, pp. 246-261, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[17] Aditya Prasad Padhy, Varsha Singh, and Vinay Pratap Singh, "Model Order Reduction of Discrete Time Uncertain System," *Journal of Information and Optimization Sciences*, vol. 41, no. 2, pp. 661-668, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[18] Kranthi Kumar Deveraetty, and S.K. Nagar, "Mixed Methods for Reducing the Order of a Linear Discrete Time Interval System," *Proceedings of International Conference on Advances in Computing, Communication and Information Technology*, pp. 49-53, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[19] V.L. Kharitonov, "Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations," *Differential Uravneniya*, vol. 14, pp. 2086-2088, 1978. [Google Scholar]

[20] Praveen Kumar, Pankaj Rai, and Amit Kumar Choudhary, "Order Reduction of z-Domain Interval Systems by Advanced Routh Approximation Method," *Circuits, Systems, and Signal Processing*, vol. 43, pp. 6911-6930, 2024. [CrossRef] [Google Scholar] [Publisher Link]