The Effect of Initial Inventory Level on the Rates of Deteriorating and Production

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Abstract

This study clarified the effect of initial inventory on the deterioration, thus on the rate of production. The study suggested a new technique to determine the deterioration items in the production-inventory system depending on the lifetime. In other words, divided stored items into groups depending on the production date to determine the deterioration. The optimal control model was derived under periodic review policy. Our numerical results show that new suggested techniques were efficient to determine the deterioration compared with other techniques.

Keywords: *Production-inventory system; Optimal control; Deterioration items.*

I. INTRODUCTION

The dynamic of the production-inventory planning problem, where units of the product are subject to deterioration, is concerned. The initial inventory level has an effect on the amount of deterioration, thus on the production rate. Therefore, techniques that used to determine the deterioration items must be different, according to the initial inventory level.

Several previous studies have investigated the deterioration rate in the inventory system as a time function. Reference [1] investigated continuous and periodic review policies with single item and time has an effect on the demand, holding costs and deterioration. Reference [8] developed a model, with and without shortage, and price has an effect on the demand. An inventory system with two items and inventory has an effect on the demand and the deterioration was studied by [5]. Also, [4] developed the model for the products, with the demand that rises quickly to peak in the middle and falls quickly at the end of planning time (quadratic demand rate), without shortage. A model has taken into account a shortage and costs of deterioration, holding and shortage was discussed by [10]. Reference [7] clarified the effect of inflation and production cycle on the total cost. A comparison between two models of inventory system, optimal control and linear programming, with linear function of demand was investigated by [2]. More recently, [3] discussed Hybrid production system with defective and deterioration items that follows probability distributions.

Our model suggests two techniques to determine the deterioration items, depending lifetime of the items. The stored items are divided into groups depending on the production date to determine the rates of deterioration and production. The first suggested technique is applied in the case of initial inventory level is less than the inventory goal, while the second one technique to the opposite case.

II. DETERIORATION RATE FUNCTION

The deterioration rate of the items in the production-inventory system must depend on the lifetime (production date), which means the inventory items different in the time period of storage. Most past studies have determined the deterioration rate without consider a difference in the items lifetime. In the case of initial inventory is less than inventory goal, every month the inventory will increase to reach the inventory goal, thus the items differ in the lifetime and deterioration. That means items must divided into groups depend on the production date. The suggested technique treatment this problem by determined the deterioration items or every month. The deterioration rate of the first month is as follows:

$$\vartheta_1 = q_1 I_1 \qquad (1)$$

The deterioration goal is as follows:

$$\hat{\vartheta}_1 = q_1 \hat{I}_1 \qquad (2)$$

The deterioration rate of the second month is divided into two parts for old and new items:

$$\vartheta_2 = q_1\{I_2 - (I_1 - \vartheta_1)\} + q_2(I_1 - \vartheta_1) \qquad (3)$$

Let:

$$A_t = I_t - \vartheta_t$$
$$B_t = I_t - A_{t-1} \qquad (4)$$

Suppose the inventory goal is fixed, yields:

$$\hat{A}_t = \hat{I} - \hat{\vartheta}_t \qquad \hat{B}_t = \hat{\vartheta}_{t-1} \tag{5}$$

Equation (3) become:

$$\vartheta_2 = q_1 B_2 + q_2 A_1 \tag{6}$$

$$\hat{\vartheta}_2 = q_1 \hat{\vartheta}_1 + q_2(\hat{I} - \hat{\vartheta}_1) \tag{7}$$

By the same manner, the deterioration items of the third month divided into three parts as follows:

$$\vartheta_3 = q_1 B_3 + q_2(B_2 - q_1 B_2) + q_3(A_1 - q_2 A_1) \tag{8}$$

$$\hat{\vartheta}_3 = q_1 \hat{\vartheta}_2 + q_2(\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1) + q_3\{\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1)\} \tag{9}$$

The deterioration rate of the fourth month is as follows:

$$\vartheta_4 = \begin{matrix} q_1 B_4 + q_2(B_3 - q_1 B_3) + q_3\{B_2 - q_1 B_2 - q_2(B_2 - q_1 B_2)\} \\ + q_4\{A_1 - q_2 A_1 - q_3(A_1 - q_2 A_1)\} \end{matrix} \tag{10}$$

$$\hat{\vartheta}_4 = \begin{matrix} q_1 \hat{\vartheta}_3 + q_2(\hat{\vartheta}_2 - q_1 \hat{\vartheta}_2) + q_3\{\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1 - q_2(\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1)\} \\ + q_4[\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1) - q_3\{\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1)\}] \end{matrix} \tag{11}$$

From the deterioration rates of the four months, we can deduce:

$$\vartheta_5 = \begin{cases} q_1 B_5 + q_2(B_4 - q_1 B_4) + q_3\{B_3 - q_1 B_3 - q_2(B_3 - q_1 B_3)\} \\ + q_4[B_2 - q_1 B_2 - q_2(B_2 - q_1 B_2) - q_3\{B_2 - q_1 B_2 - q_2(B_2 - q_1 B_2)\}] \\ + q_5[A_1 - q_2 A_1 - q_3(A_1 - q_2 A_1) - q_4\{A_1 - q_2 A_1 - q_3(A_1 - q_2 A_1)\}] \end{cases} \tag{12}$$

$$\hat{\vartheta}_5 = \begin{cases} q_1 \hat{\vartheta}_4 + q_2(\hat{\vartheta}_3 - q_1 \hat{\vartheta}_3) + q_3\{\hat{\vartheta}_2 - q_1 \hat{\vartheta}_2 - q_2(\hat{\vartheta}_2 - q_1 \hat{\vartheta}_2)\} \\ + q_4[\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1 - q_2(\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1) - q_3\{\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1 - q_2(\hat{\vartheta}_1 - q_1 \hat{\vartheta}_1)\}] \\ + q_5 \begin{bmatrix} \hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1) - q_3\{\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1)\} \\ - q_4\{\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1) - q_3\{\hat{I} - \hat{\vartheta}_1 - q_2(\hat{I} - \hat{\vartheta}_1)\}\} \end{bmatrix} \end{cases} \tag{13}$$

III. MATHEMATICAL MODEL

A. Notations

The following variables and parameters are used:

T : the length of the planning horizon (T>0).

I_t: The inventory level.

P_t: The production rate.

$\hat{P}_t$: The production goal rate.

D_t: The demand .

$\hat{I}$: The inventory goal level.

c : A Penalty of the inventory level.

k : A Penalty of the production rate.

B. Optimal Control Model

The objective function can be expressed as the quadratic form to be minimized [6, 9]:

$$2J = \sum_{t=1}^{T-1} c\{I_t - \hat{I}\}^2 + k\{P_t - \hat{P}_t\}^2 \tag{14}$$

subject to the state equations:

$$\Delta I(t) = \begin{cases} P_t - D_t - \vartheta_t & I_1 < \hat{I} \\ P_t - D_t - q_t I_t & I_1 \geq \hat{I} \end{cases} \tag{15}$$

with initial condition I_1,

From (15), there are two cases of the model: the first one represents the initial inventory level is less than the inventory goal level, and the opposite represent the second one. In the first case, we suggested a new technique to determine the deterioration rate by dividing the inventory items into many groups depending on the items lifetime (production date). In the second case, we used two techniques (general and suggested) to determine the production rate: the general technique that the production rate is equal to the demand and deterioration, while the suggested technique that the production rate is equal to the demand in some months, as well as demand and deterioration in the other months depending on the inventory level and demand rate.

To solve this problem, the Lagrangian function is:

$$L = \begin{matrix} \sum_{t=1}^{T-1} -\frac{1}{2}\left[c\{I_t - \hat{I}\}^2 + k\{P_t - \hat{P}_t\}^2\right] \\ + \sum_{t=1}^{T-1} \lambda_{t+1}[P_t - D_t - \vartheta_t - I_{t+1} + I_t] \end{matrix} \tag{16}$$

A Hamiltonian function is as follows:

$$H_t = \frac{-1}{2}\left[c\{I_t - \hat{I}\}^2 + k\{P_t - \hat{P}_t\}^2\right] + \lambda_{t+1}[P_t - D_t - \vartheta_t] \tag{17}$$

By using (17), we can write (16) as follows:

$$L = \sum_{t=1}^{T-1} H_t - \lambda_{t+1}\{I_{t+1} - I_t\} \tag{18}$$

The adjoint equations can be found by differentiating (18) with respect to I_t:

$$\Delta\lambda(t) = \begin{cases} c\{I_t - \hat{I}\} & I_1 < \hat{I} \\ c\{I_t - \hat{I}\} + q_t & I_1 \geq \hat{I} \end{cases} \qquad (19)$$

The production rate can be found by differentiating (18) with respect to P_t:

$$P_t = \hat{P}_t + \frac{1}{c}\lambda_{t+1};\ t = 1,2,\ldots,T-1 \qquad (20)$$

The production goal rate must satisfy (15):

$$\hat{P}_t = \begin{cases} D_t + \hat{\vartheta}_t & I_1 < \hat{I} \\ D_t & (1-q_t)I_t \geq \hat{I} \\ D_t - (1-q_t)I_t + \hat{I} & (1-q_t)I_t < \hat{I} \end{cases} \qquad (21)$$

We can write (15) as follows:

$$\Delta I_t = \begin{cases} -(\vartheta_t - \hat{\vartheta}_t) + \frac{1}{c}\lambda_{t+1} & I_1 < \hat{I} \\ -q_t I_t + \frac{1}{c}\lambda_{t+1} & (1-q_t)I_t \geq \hat{I} \\ \hat{I} - I_t + \frac{1}{c}\lambda_{t+1} & (1-q_t)I_t < \hat{I} \end{cases} \qquad (22)$$

The boundary value problem represented by (19 & 22), and can be solved numerically by Microsoft Excel, with the initial condition I_1 and terminal condition λ(T) =0.

IV. NUMERICAL SOLUTION

Consider an inventory system, with the following parameter values,

$T = 5\ month;\ c = 5\$;\ k = 10\$;\ Dt = 100 + 50 * Sin\ (t^2);\ q_t = 0.03t;$

Figures 1 and 2 show the solution in the case of initial inventory level (50 items) is less than inventory goal level (100 items).

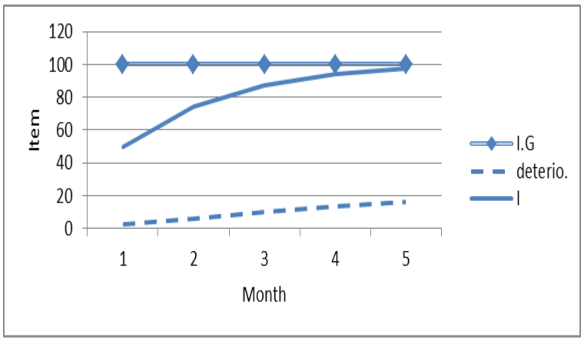

Fig. 1. The Inventory Level And Deterioration Items

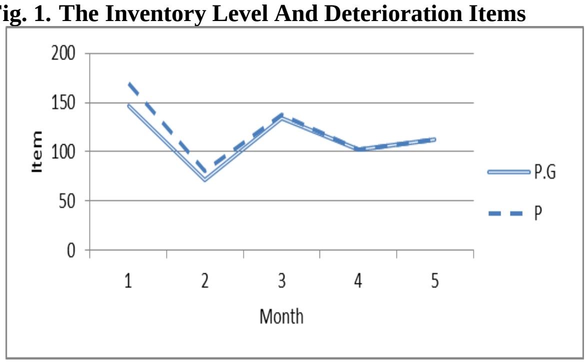

Fig. 2. The Production Rate

From Figures (1 & 2), the inventory levels and production rate (I & P) are converging to its goal level (I.G & P.G) over time. The deteriorating rate that increases with time, according to the suggested technique that divided the inventory items into many groups depending on the lifetime.

Figures 3 to 5 show the solution in the case of initial inventory level is greater than the inventory goal level with two techniques to find the inventory level and production rate.

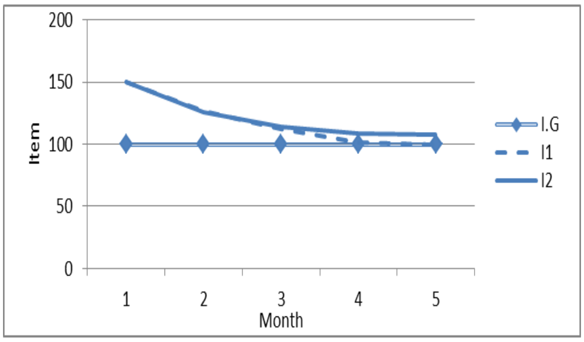

Fig. 3. The Inventory Levels And Its Goal

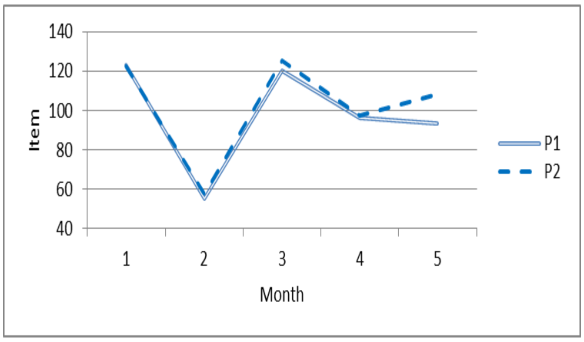

Fig. 4. Production Rates, According To The Two Techniques

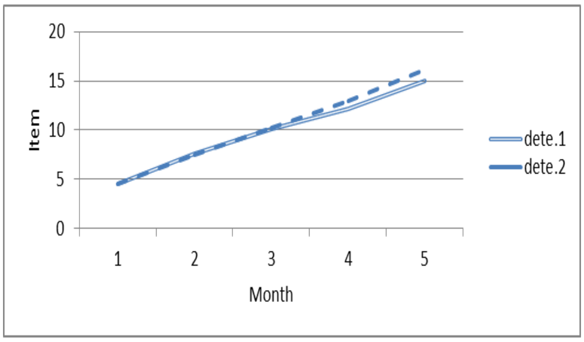

Fig. 5. Deterioration rates, according to the two techniques

The inventory level of the suggested technique (I1) is better than the other technique (I2) to reach its goal (Figure 3). Figures 4 and 5 show the rates of deterioration and production, according to the suggested technique (dete.1 & P1) are less than its rates compared with other technique (dete.2 & P2).

V. CONCLUSION

In this paper, an optimal control model that clarified the effect of initial inventory on the rates of deterioration and production was developed. We suggested two techniques to determine the deterioration rate: the first one in the case of initial inventory is less than the inventory goal, and the second one for the opposite case. The production date was taken into account to divide stored items into groups and then determined the deterioration rate. The deterioration rate has a positive relation to the control variable (production rate). Our results of new techniques were better than other techniques in the converging between the inventory level and its goal. Stochastic demand, shortage and multi items can be taken into account in future studies.

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