Original Article

Thermal description of the business activity in Bogotá City

W. A. Rojas C.[1]

[1]*Departamento de Ciencias Básicas*
Fundación Universitaria Unipanamericana
Avenida (Calle) 32 No. 17 – 30 Bogota´, Colombia

Abstract - *According to The National Statistical System (Dane) in Colombia data taken from the 2005-2015 period, business activity is a very relevant economic sector that contributes to the gross domestic product (GDP) of Bogota City. With this in mind and by taking the econophysics laws, we could find the entropy of mixing for this sector, when we suppose that the size of the companies is equal to the size of molecules in the mix of ideal gases without any chemical reaction.*

Keywords - *Econophysics, Thermodynamics, Business activity, Bogotá.*

I. INTRODUCTION

Economic activity is a human manifestation that involves:

- production, consumption, and exchange of goods and services.
- Production and exchange of information: quantities and prices.

All of this has several thermodynamics features. Money, goods, and services exchange or transferences, are analog to the energy exchange and transferences between the system and reservoir. Then, we can obtain an adiabatic economy system as a closed system, where the money, goods, services, and companies are constant. These economic systems no exchange or transfer money, goods, services to other economic systems; a good example of an adiabatic economic system is the Democratic People's Republic of Korea. On the other hand, an open economic system is one that allows the transfer and exchange of energy E to other economic systems. So, we can identify energy as money, information, goods, and services that the economic system produces or consumes. An economic system has an internal energy U that can be altered when it exchanges energy with other economic systems. For an adiabatic economic system, the internal energy remains constant. The heat Q is interpreted as an exchange chaotic form energy between internal parts of the economic system or other systems. The route it takes will be interpreted as thermodynamic entropy S [2, 3]. An economic system does work W when it produces information, goods, and services. This work is interpreted as an orderly form of energy. The economic activity is very dynamic, even in a short time, we can assume that it has got a thermodynamic behavior. Under this latter condition, it is possible to state laws of physical economics like in a thermodynamic system [15]

- Zeroth law of physical economics: there is an equilibrium parameter that we define as an economic temperature. As a result, all parts of the system have the same economic temperature T

$$T = \frac{1}{\langle u \rangle} \qquad (1)$$

where hui is the average energy by an agent in the economic system [2, 3].

- The first law of physical economics: the internal energy of the system is altered in two ways, by transferring heat, *do*or work, between the economic system to other economic systems [1]

$$dU = dQ - dW. \qquad (2)$$

- The second law of physical economics: For an economic system the missing information is interpreted as entropy S [16]. So, when two economic systems are in contact, it takes to place an exchange heat with a temperature gradient dT between them.

Thischangeisasociatedtotheentropygrowth

$$\Delta S \geq 0. \quad (3)$$

For instance, an adiabatic economic system has the same behavior as the ideal gas with internal energy constant U = cte. Where the agents (molecules) compete by energy portions. Here the Second law gets the form [2, 3]

$$dS = \frac{\langle du \rangle}{\langle u \rangle} \quad (4)$$

For the adiabatic economic system, we have that the trade between two companies is analog according to the elastic collision between two particles in an ideal gas, due to the total energy being conserved

$$U_{total} = \sum_i u_i = cte \quad (5)$$

where UI is the energy for each one company, and the total number of companies remains constant

$$N = \sum_i n_i = cte. \quad (6)$$

Therefore, the internal energy is rewritten as

$$U = \sum_i n_i u_i = cte \quad (7)$$

and average energy per company

$$U = \sum_i u_i p_i. \quad (8)$$

Where we define a company choice probability for N companies of the adiabatic economic system,

$$p_i = \frac{n_i}{N}. \quad (9)$$

Then, the probability of a specific partition is equal to the several ways several companies n_i are distributed in one of the states with energy [2–4]

$$W = \frac{N!}{n_1 n_2 \cdots n_i!}. \quad (10)$$

If the number of the configuration is maximum, we find the partition function for the economic system

$$Z = \sum_i e^{-\beta u_i}. \quad (11)$$

Within the continuous limit and the normalization condition of probability, we obtain the average energy

$$\langle u \rangle = \int_0^\infty u p(u) du \quad (12)$$

and the Boltzmann-Gibbs distribution

$$p(u) = \beta e^{-\beta u} = \frac{1}{\langle u \rangle} e^{-\frac{u}{\langle u \rangle}}. \quad (13)$$

So here we have the economic temperature [2, 3]

$$\langle u \rangle = \frac{1}{\beta}. \quad (14)$$

In contrast, the loss of information of an adiabatic economy system is obtained from the Boltzmann principle

$$S = k_B ln|W|. \quad (15)$$

This loss of information is associated with the Second law of physical economics

$$dS = \frac{\langle du \rangle}{\langle u \rangle}. \quad (16)$$

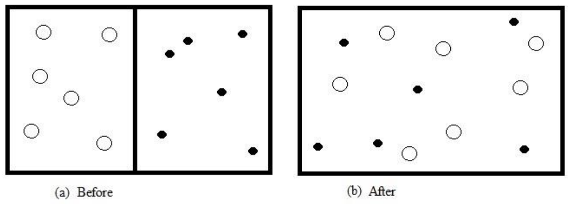

Fig. 1 These gases were isolated one each other by a wall. After we removed the wall, the gases are mixed without chemical reaction to reach the thermal equilibrium.

II. THE ENTROPY OF MIXING

Mind that two ideal gases with different molecular sizes are in a box. These gases were isolated one each other by a wall.

After we removed the wall, the gases are mixed without chemical reaction to reach the thermal equilibrium.

As a consequence, the change in the thermal entropy for both different types of ideal gases are

$$S = k_B ln|W|, \quad (17)$$

and the number of the configuration for two different species of ideal gases is

$$W = \frac{N!}{N_1 N_2!}. \quad (18)$$

Where N_1 and N_2 are type 1 and type 2. The total molecules of mix system are

$$N = N_1 + N_2. \quad (19)$$

Therefore, regarding the Stirling approximation, the change in entropy of mix is reduced to

$$\Delta S = -k_B N[X_1 ln[X_1] + X_2 ln|X_2|] + S_0 \quad (20)$$

Where

$$X_i = \frac{N_i}{N} \quad (21)$$

And

$$S_0 = k_B N ln|N|. \quad (22)$$

For three types of an ideal gas, we have the whole molecules are

$$N = N_1 + N_2 + N_3 \quad (23)$$

and the variation in the entropy density of mixing for three ideal gas is [13]

$$\Delta s = \frac{\Delta S}{N} = k_B ln|N| - \frac{k_B}{N}\left[N_1 ln\left|\frac{N_1}{N}\right| + N_2 ln\left|\frac{N_2}{N}\right| + N_3 ln\left|\frac{N_3}{N}\right|\right] \quad (24)$$

For (24), we can define the initial entropy density as

$$s_0 = \frac{S_0}{N} = k_B ln|N| \quad (25)$$

and the entropy density of mixing is

$$\Delta s_{mix} = \frac{\Delta S}{N} = -\frac{k_B}{N}\left[N_1 ln\left|\frac{N_1}{N}\right| + N_2 ln\left|\frac{N_2}{N}\right| + N_3 ln\left|\frac{N_3}{N}\right|\right]. \quad (26)$$

Likewise, we find

$$\Delta s = s_0 - \Delta s_{mix}. \quad (27)$$

III. STATE EQUATION FOR A MIXING OF IDEAL GASES

For an ideal gas, the state equation is

$$PV = nRT, \quad (28)$$

where P is the internal pressure, V is the volume of the system, R is the gas constant, n is the mole's number and T is the temperature of the system. Also, we have the following relations

$$R = k_B N_A \text{ and } N = nN_A \quad (29)$$

also, k_B is the Boltzmann's constant, and N_A is the Avogadrós constant. For an adiabatic economic system, we propose that the state equation is

$$F(E) = PV = k_B NT \quad (30)$$

Where $F(E)$ is an energy-dependent function. This is true if

- The types of ideal gas do not have a chemical reaction.
- The several types of an ideal gas have the same economic temperature.
- There is no transition of phases between the types of ideal gases.

Also, for the*i* type of ideal gas, we see that

$$F_i(E) = N_i k_B T \quad (31)$$

The eq (31) is rewritten as

$$\frac{F_i(E)}{N_i k_B T} = \frac{N_i}{N}. \quad (32)$$

The ratio between the*i* type and *j* type of ideal gases for a mix

$$\frac{F_i(E)}{F_j(E)} = \frac{N_i}{N_j}. \quad (33)$$

Replaces the (33) in the (24). So we find

$$s = -\frac{F_1(E)}{NT}\left[ln\left|\frac{F_1(E)}{T}\right|\left(1 + \frac{N_2}{N_1} + \frac{N_3}{N_1}\right) + ln\left|\frac{1}{k_B N}\right| + \frac{N_2}{N_1} ln\left|\frac{N_2}{k_B N N_1}\right| + \frac{N_3}{N_1} ln\left|\frac{N_3}{k_B N N_1}\right|\right] \quad (34)$$

In addition, the total number of companies (23) in the adiabatic economic system is rewritten as

$$\frac{N}{N_1} = 1 + \frac{N_2}{N_1} + \frac{N_3}{N_1} \quad (35)$$

and

$$\alpha = ln\left|\frac{1}{k_B N}\right| + \frac{N_2}{N_1} ln\left|\frac{N_2}{k_B N N_1}\right| + \frac{N_3}{N_1} ln\left|\frac{N_3}{k_B N N_1}\right|. \quad (36)$$

Then, by the (36) and (35), the equation (34) is reduced to

$$s = -\frac{F_1(E)}{NT}\left[ln\left|\frac{F_1(E)}{T}\right|\frac{N}{N_1} + \alpha\right] \quad (37)$$

This equation is quite important since it states a relation between the entropy density, the economic temperature, and the state equation of the economic system. The second law of physical economics (16) is rewritten as

$$\frac{ds}{dE} = \frac{ds}{dT}\frac{dT}{dE} = \frac{1}{T} \quad (38)$$

Regarding the equation (37)

$$\frac{ds}{dT} = \frac{F_1(E)}{2NN_1T^2}\left[N + N_1\alpha + Nln\left|\frac{F_1(E)}{T}\right|\right] \quad (39)$$

On the other hand, the variation of economic temperature (14) is

$$\frac{dT}{dE} = -E^{-2} \quad (40)$$

As a result, the (39) and (40), the equation (38) reduces to

$$F_1(E)\left[\beta + Nln\left|\frac{F_1(E)}{T}\right|\right] = \frac{2NN_1}{T} \quad (41)$$

for the first type of ideal gases. Where we define

$$\beta = N + N_1\alpha \quad (42)$$

Also, for three species of ideal gases, the total state equation is

$$F_T(E) = F_1(E)\left[\frac{N_1+N_2}{N} - \beta - N\ln\left|\frac{F_1(E)}{T}\right|\right] = \frac{2N_1}{T}. \qquad (43)$$

IV. THERMAL DESCRIPTION OF THE BUSINESS ACTIVITY IN BOGOTA CITY

The main economic activities that contributes to the growth domestic product (GDP) of Bogotá City in the period 2005-2015 were [5–8]

• Banking intermediation.

•Business activity.

•Industry sector.

Having said that, there is a Colombian law (Law 9905 of 2004) that defines the size of a company according to specific features [6]

• A large company is one that has got a staff of more than 201 workers and assets greater than 3000 monthly salaries legal minimums in force.

• Medium company that has a staff between 51 and 200 workers and assets between 5001 and 30000 monthly salaries legal minimums in force.

• Small company that has a staff between 11 and 50 workers and assets between 501 and 5000 monthly salaries legal minimums in force.

• Micro company that has a staff of 10 workers and assets of less than 500 monthly salaries legal minimums in force.

Thus, with this company's classification, we can model as a mix of ideal gases in an adiabatic box, where the different types of ideal gases do not have any chemical reaction. There are no transition phases and the total number of companies remains constant. That said, each company contributes to the gross domestic product (GDP) of Bogotá City. For our convenience, we consider small businesses and micro-businesses of the same one. Yet, we define the several companies the follows

- N_1 for large companies.
- N_2 for medium companies**.**
- 3. N_3 for small companies and micro-companies.

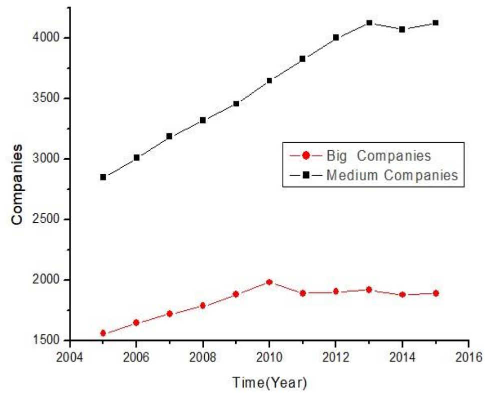

Fig. 2 The total whole of companies in Bogotá City during the 2005-2015 period. [6].

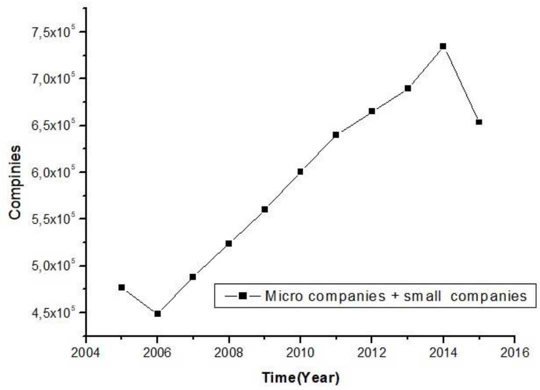

Fig. 3 The whole number of companies in Bogotá City in the 2005-2010 period. [6].

In Figures 2 y 3, we can observe the whole number of companies in Bogotá City during the 2005-2015 period. Along with this term, the city had three Mayors. They were Luis Eduardo Garzon, Samuel Moreno and Gustavo Petro respectively [6, 7]. In Figure 4, it is shown the relation between business activity and state equation for this system FT(E)Eq. (43) with $k_B \approx 10^{15}$ and $k_B \approx 10^{16}$.
From the latter, we can infer a Boltzmann economic constant as

$$10^{15} \leq k_B \leq 10^{16} \qquad (44)$$

In this context, an analogy between business activity and statistical physics is clearly shown, because the state equation is well approximated to real data, this is possible when we choose a correct Boltzmann economic constant. In statistical physics, this constant is associated with molecular kinetic activity and system temperature. Thus, we calculate the entropy density (37)for this system, what is can be seen in Figure 5. At this point, we assume that the economic system has a thermal behavior, the types of the ideal gas have the same economic temperature, the companies numbers are constant and they do not have any chemical reaction.

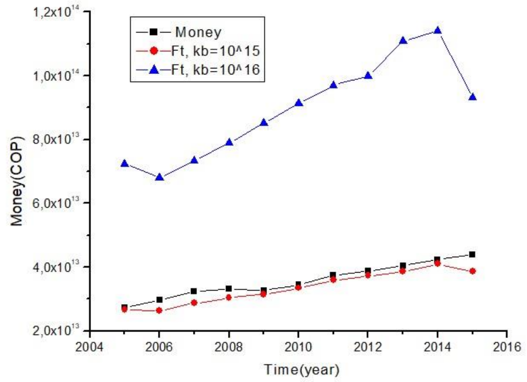

Fig. 4 Relation between business activity and state equation for this system $F_T(E)$ (43) with $k_B \approx 10^{15}$ and $k_B \approx 10^{16}$. [6].

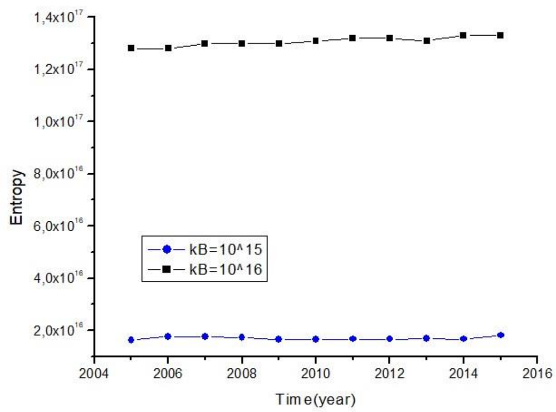

Fig. 5 Behavior of the entropy density [6].

Then again, means that Companies neither merge nor go bankrupt. Even they don't change their size. In Figure 6, the behavior of the economic temperature of Bogotá City for business activity is expressed.

V. SUMMARY AND DISCUSSION

The findings of this study can be understood as the description of the business activity in Bogotá City when there is an analogy between companies of different sizes and ideal gases in an adiabatic box.

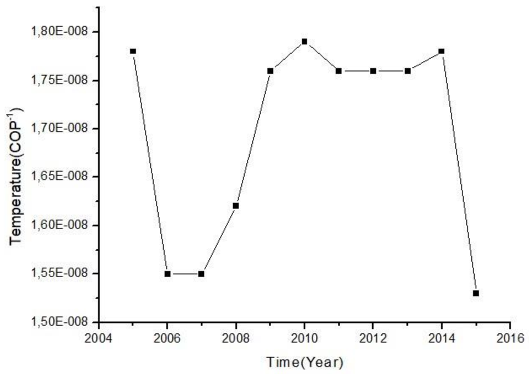

Fig. 6 The behavior of the economic temperature of Bogotá City for business activity.

The latter is true when the configuration space of the economic system has the same features as a physical system. This allows defines of thermal variables like energy (money), economic temperature, and entropy. The loss of information in the economic system is related to the Second law and thermal entropy. In this case for business activity in Bogotá city if the total number of companies remains constant only in a short period of time. Along with this term, we assume that this system gets a thermal behavior and does not have any chemical reaction over several companies. Accordingly, it is possible to define a Boltzmann economic constant in Business activity when the economic temperature is the same for all companies. Therefore, we could find that the state equation for this adiabatic economic system, works respectively with real data and its entropy density. Notably, the economy is a human activity that is possible to describe by using the Thermodynamics Laws.

In this study, the economic system has not had to exchange or transfer money, goods, and services with other systems. The whole number of staff of companies is remaining constant, we do not take into account the loss of jobs. Social movements and terrorism acts have an impact on the local economy and are not included here. Also, the change in taxes o laws affects to GDP of Bogota city.

The commercial activities are legal and do not take into account illegal activities such as drug trafficking, this latter is an important factor in the city.

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