Algorithm of Mathematical Modeling of Layered Block Medium with Combined Hierarchical Inclusions for Acoustic Monitoring, Taking into Account Convective Mixing in Fluid-Saturated Inclusions

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Abstract

Background: A new method of modeling acoustic monitoring of a layered-block elastic medium with several inclusions of various physical-mechanical and phase hierarchical structures has been developed. Methods: An iterative process of solving a direct problem for the case of three hierarchical inclusions of l, m, s ranks based on the use of 2D-integro differential equations has been developed. Results: The degree of hierarchy of inclusions is determined by the values of their ranks, which may be different. Hierarchical inclusions are located in different layers one above the other: the upper anomalously stressed, the second-fluid-saturated and the third anomalously dense. The degree of filling with inclusions of each rank is different for all three hierarchical inclusions. At the same time, the question of dynamic processes in fluid-saturated hierarchical inclusions related to convective mixing of a single-component fluid is investigated. Conclusions: The simulation results can be used when conducting monitoring studies of fluid return control of oil fields. The results can help explain the excessive water flooding of oil reservoirs.

Keywords - *hierarchical environment, acoustic field, iterative algorithm integral-differential equations, direct problem, free convection effects, oscillatory perturbations in a fluid-saturated inclusion.*

I. INTRODUCTION

This document is a template. A significant result of geomechanical - geodynamic studies of the past time was the discovery of a close relationship between global geodynamic and local geomechanical processes caused by mining, especially in tectonically active zones. No less important result of research was also the conclusion about the fundamental role of the block-hierarchical structure of rocks and massifs for explaining the existence of a wide range of nonlinear geomechanical effects and the emergence of complex self-organizing geosystems.

The hierarchical structure is characteristic for many systems, especially for the Earth's lithosphere, where more than 30 hierarchical levels from tectonic plates thousands of kilometres to mineral grains are distinguished [1]. Thus, the crust is a discrete system of blocks and, like any synergetic discrete ensemble, has the properties of hierarchy and self-similarity [2].

The development of oil and gas fields associated with the movement of multiphase multicomponent media, which are characterized by non-equilibrium and nonlinear rheological properties. The actual behaviour of reservoir systems is determined by the complexity of the rheology of moving fluids and the morphological structure of the porous medium, as well as the diversity of interaction processes between the fluid and the porous medium [3]. Consideration of these factors is necessary for a meaningful description of fluid filtration processes in the presence of nonlinearity, noneqiulibrium and heterogeneity inherent for real systems. At the same time, new synergistic effects (loss of stability with the appearance of oscillations, formation of ordered structures) are revealed. This requires the development of new methods for monitoring and managing complex natural systems that are tuned to account for the observed phenomena. Thus, the reservoir system from which oil is to be extracted is a complex dynamic hierarchical system.

When constructing a mathematical model of a real object, it is necessary to use active and passive monitoring data obtained during the current operation of the object as a priori information. In [4,5], simulation algorithms were constructed in the electromagnetic case for 3-D heterogeneities, in the seismic case for 2-D heterogeneities for an arbitrary type of N-layer medium excitation source with a hierarchical elastic inclusion located in the J-th layer. In [6], a new 2D modeling algorithm was developed for diffraction of sound on an elastic and porous fluid-saturated inclusion of a hierarchical structure located in the J-th layer of an N-layer elastic medium. In work [7], modeling algorithms were constructed in the acoustic case for 2-D heterogeneities for an arbitrary type of excitation source of an N-layer medium with a separate hierarchical anomalously density, stress, and plastic inclusion located in the J-th layer.

In this paper, we developed an algorithm for modeling the acoustic field (longitudinal acoustic wave) in the form of an iterative process for solving a direct problem for the case of three hierarchical inclusions l, m, s –th ranks based on the use of 2D integral and integral-differential equations. The degree of hierarchy of inclusions is determined by the values of their ranks, which may be different. Hierarchical inclusions are located in different layers one above the other: the upper anomalously stressed, the second fluid-saturated and the third anomalously dense (Figure. 1). At the same time, the question of dynamic processes in fluid-saturated hierarchical inclusions connected with convective mixing of a one-component fluid is investigated.

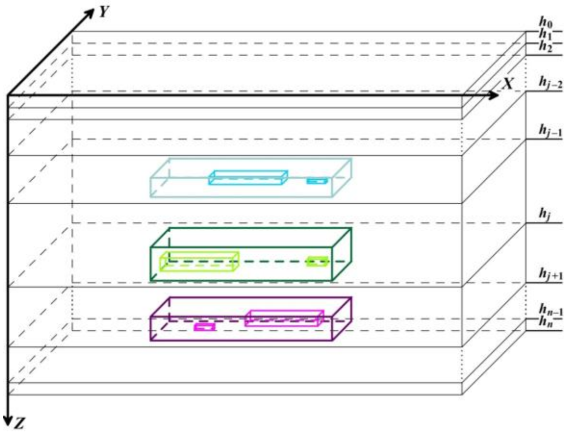

Figure 1. Scheme of the combined anomalously stressed (upper), fluid-saturated (medium) and anomalously dense (lower) heterogeneities of the hierarchical type, located in an N-layered elastic medium.

II. ALGORITHM FOR MODELING THE DIFFRACTION OF SOUND WAVE ON A TWO-DIMENSIONAL COMBINED: ANOMALOUSLY STRESSED, FLUID-SATURATED AND ANOMALOUSLY DENSE HETEROGENEITY OF A HIERARCHICAL TYPE, LOCATED IN AN N-LAYERED ELASTIC MEDIUM.

The authors of [5, 7] described an algorithm for modeling the diffraction of sound on a two-dimensional elastic hierarchical inclusion, located in the J-th layer of the N-layered medium. - $G_{Sp,j}(M, M^0)$ the source function of the seismic field, the boundary problem for which was formulated in [8-9], $k_{1ji}^2 = \omega^2(\sigma_{ji}/\lambda_{ji})$ is the wave number for a longitudinal wave, in the above expression the index *ji* denotes that the properties of the medium are inside the heterogeneity, *ja* is outside the heterogeneity, λ is the Lame constant, σ medium density, ω is the circular frequency, $\vec{u} = grad\,\varphi$ is the displacement vector, φ^0 is the potential of the normal seismic field in a layered medium in the absence of the heterogeneity: $\varphi_{ji}^0 = \varphi_{ja}^0 \cdot \varphi_l^0$ - potential of a normal seismic field in a layered medium in the absence of heterogeneity of the previous rank l, if l = 2 ... L $\varphi_l^0 = \varphi_{l-1}$, if l = 1 $\varphi_l^0 = \varphi^0$, which coincides with the corresponding expression [7].

We assume that the density of the first hierarchical inclusion for all ranks l and the enclosing the layers are the same, and the elastic parameters of the hierarchical inclusion for all ranks differ from the elastic parameters of the containing medium. Let the values of the ranks for all hierarchical inclusions: l = m = s = 1, then the system of equations describing the propagation of a longitudinal acoustic wave in the first inclusion is rewritten as:

$$\frac{(k_{1(j-1)il}^2 - k_{1(j-1)}^2)}{2\pi} \iint_{S1cl} \varphi_l(M) G_{Sp,(j-1)}(M, M^0) d\tau_M + $$

$$+ \varphi_{l-1}^0(M^0) = \varphi_l(M^0), M^0 \in S_{1Cl}$$

(

$$\frac{\sigma_{(j-1)il}(k_{1(j-1)il}^2 - k_{1(j-1)}^2)}{\sigma(M^0)2\pi} \iint_{S1cl} \varphi_l(M) G_{Sp,(j-1)}(M, M^0) d\tau_M + $$

$$+ \varphi_{l-1}^0(M^0) = \varphi_l(M^0), M^0 \notin S_{1Cl}, M^0 \in \Pi_{j-1}$$

1)

$$\frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0)2\pi} \iint_{S1cl} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + $$

$$\varphi_{l-1}^0(M^0) = \varphi_l(M^0), M^0 \in \Pi_j$$

(2)

Let us calculate $\varphi_l(M^0), M^0 \in \Pi_j$ in the layer where the second hierarchical elastic inclusion is located using expression (2), then the normal potential of the acoustic field for the second inclusion is written in the form: $\varphi_{m-1}^0(M^0) = \varphi_l(M^0), M^0 \in \Pi_j$ .The system of equations for the second elastic hierarchical inclusion of rank *m* = 1 has the form according to [4,5]:

$$\frac{(k_{1jim}^2 - k_{1j}^2)}{2\pi} \iint_{S2Cm} \varphi_m(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{jim}} \varphi_{m-1}^0(M^0) -$$

$$- \frac{(\sigma_{ja} - \sigma_{jim})}{\sigma_{jim} 2\pi} \int_{C2m} G_{Sp,j} \frac{\partial \varphi_m}{\partial n} dc = \varphi_m(M^0), M^0 \in S_{2Cm};$$

$$\frac{\sigma_{jim}(k_{1jim}^2 - k_{1j}^2)}{\sigma(M^0)2\pi} \iint_{S2Cm} \varphi_m(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{m-1}^0(M^0) -$$

$$- \frac{(\sigma_{ja} - \sigma_{jim})}{\sigma(M^0)2\pi} \int_{C2m} G_{Sp,j} \frac{\partial \varphi_m}{\partial n} dc = \varphi_m(M^0), M^0 \notin S_{2Cm}; \quad (3)$$

$$\frac{\sigma_{(j+1)im}(k^2_{1(j+1)im}-k^2_{1(j+1)})}{\sigma(M^0)2\pi}\iint_{S_{2Cm}}\varphi_m(M)G_{Sp,(j+1)}(M,M^0)d\tau_M$$

$$+\varphi^0_{m-1}(M^0)-\frac{(\sigma_{(j+1)a}-\sigma_{(j+1)im})}{\sigma(M^0)2\pi}\int_{C_{2m}}G_{Sp,(j+1)}\frac{\partial\varphi_m}{\partial n}dc=$$

$$=\varphi_m(M^0),M^0\notin S_{2Cm},\in\Pi_{j+1}; \qquad (4)$$

Let us calculate $\varphi_m(M^0), M^0\notin S_{2Cm},\in \Pi_{j+1}$ in the layer, where the third hierarchical anomalously density inclusion is located using expression (4), then the normal potential of the acoustic field is:

$\varphi^0_{s-1}(M^0)=\varphi_m(M^0), M^0\in\Pi_{j+1}$.

We assume that the elastic parameters of the third hierarchical inclusion for all ranks s and the enclosing layer are the same and the density of inclusions differs from the density of the surrounding medium, then the system of equations for the third hierarchical inclusion of rank s = 1 has the form according to [7]:

$$\frac{(k^2_{1(j+1)is}-k^2_{1(j+1)})}{2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,(j+1)}(M,M^0)d\tau_M+$$

$$+\frac{\sigma_{(j+1)a}}{\sigma_{(j+1)is}}\varphi^0_{s-1}(M^0)-\frac{(\sigma_{(j+1)a}-\sigma_{(j+1)is})}{\sigma_{(j+1)is}2\pi}\int_{C_{3s}}G_{Sp,(j+1)}\frac{\partial\varphi_s}{\partial n}dc=$$

$$=\varphi_s(M^0),M^0\in S_{3Cs};$$

$$\frac{\sigma_{(j+1)is}(k^2_{1(j+1)is}-k^2_{1(j+1)})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,(j+1)}(M,M^0)d\tau_M+$$

$$+\varphi^0_{s-1}(M^0)-\frac{(\sigma_{(j+1)a}-\sigma_{(j+1)is})}{\sigma(M^0)2\pi}\int_{C_m}G_{Sp,(j+1)}\frac{\partial\varphi_s}{\partial n}dc=$$

$$=\varphi_s(M^0),M^0\notin S_{3Cs}\in\Pi_{j+1}; \qquad (5)$$

$G_{Sp,(j+1)}(M,M^0)$ - the source function of the seismic field, it coincides with the function [8-9], $k^2_{1(j+1)is}=\omega^2(\sigma_{(j+1)is}/(\lambda_{(j+1)is});\ \lambda_{(j+1)is}=\lambda_{(j+1)a};$ - is the wave number for the longitudinal wave and elastic parameters, in the above expression the index ji denotes that the properties of the medium are inside the heterogeneity, ja is outside the heterogeneity, s = 1 ... S is the number of hierarchical level, $\varphi^0_{s-1}(M^0)=\varphi_m(M^0), M^0\in\Pi_{j+1}$ is the potential of the normal acoustic field in the layer j + 1 in the absence of the third heterogeneity of the previous rank. Let us calculate in the layer j-1 using expression (6)

$\varphi_s(M^0), M^0\notin S_{3Cs}\in\Pi_{j-1}$

$$\frac{\sigma_{(j-1)is}(k^2_{1(j-1)is}-k^2_{1(j-1)})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,(j-1)}(M,M^0)d\tau_M+$$

$$+\varphi^0_{s-1}(M^0)-\frac{(\sigma_{(j-1)a}-\sigma_{(j-1)is})}{\sigma(M^0)2\pi}\int_{C_m}G_{Sp,(j-1)}\frac{\partial\varphi_s}{\partial n}dc=$$

$$=\varphi_s(M^0),M^0\notin S_{3Cs}\in\Pi_{j-1}; \qquad (6)$$

The values of L, M, S-are the maximum values of the ranks of the hierarchy for the three inclusions. In this paper, L = 3, M = 3, S = 4. (6 ’)

$l = l + 1; m = m + 1; s = s + 1$. If l <3 or l = 3

$\varphi^0_{l-1}(M^0)=\varphi_{s-1}(M^0), M^0\in\Pi_{j-1}$

Then we go to algorithm (1) - (6). If l> 3, and m = 2, then we calculate in layer j using expression (7):

$\varphi_s(M^0), M^0\notin S_{3Cs}\in\Pi_j$

$$\frac{\sigma_{jis}(k^2_{1jis}-k^2_{1j})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,j}(M,M^0)d\tau_M+\varphi^0_{s-1}(M^0)- \qquad (7)$$

$$-\frac{(\sigma_{ja}-\sigma_{jis})}{\sigma(M^0)2\pi}\int_{C_m}G_{Sp,(j-1)}\frac{\partial\varphi_s}{\partial n}dc=\varphi_s(M^0),M^0\notin S_{3Cs}\in\Pi_j;$$

$\varphi^0_{m-1}(M^0)=\varphi_{s-1}(M^0), M^0\in\Pi_j$

and we go to algorithm (3 ') - (6), if m = 3, then we go to algorithm (3) - (6) and

$\varphi^0_{m-1}(M^0)=\varphi_{s-1}(M^0), M^0\in\Pi_j$

If m> 3, and s <4 or s = 4, then we go to (5) - (6) and

$\varphi^0_{s-1}(M^0)=\varphi_{s-1}(M^0), M^0\in\Pi_{j+1}$

If s> 4, then go to (7 ’).

$$\frac{(k^2_{1jim}-k^2_{1j})}{2\pi}\iint_{S_{2Cm}}\varphi_m(M)G_{Sp,j}(M,M^0)d\tau_M+\frac{\sigma_{ja}}{\sigma_{jim}}\varphi^0_{m-1}(M^0)-$$

$$-\frac{(\sigma_{ja}-\sigma_{jim})}{\sigma_{jim}2\pi}\int_{C_{2m}}G_{Sp,j}\frac{\partial\varphi_m}{\partial n}dc=(\varphi_m(M^0)+\alpha p_2),M^0\in S_{2Cm}; \qquad (3’)$$

$$\frac{\sigma_{jim}(k^2_{1jim}-k^2_{1j})}{\sigma(M^0)2\pi}\iint_{S_{2Cm}}\varphi_m(M)G_{Sp,j}(M,M^0)d\tau_M+\varphi^0_{m-1}(M^0)-$$

$$-\frac{(\sigma_{ja}-\sigma_{jim})}{\sigma(M^0)2\pi}\int_{C_{2m}}G_{Sp,j}\frac{\partial\varphi_m}{\partial n}dc=\varphi_m(M^0),M^0\notin S_{2Cm};$$

Where

$$\alpha=1-\chi-\frac{K}{K_0}$$

K-module of all-round compression, porosity- χ , the true module of the phase compressibility K_0 , pore hydrostatic pressure $p2$. The first equation of system (3) is rewritten as (3 ’) in accordance with the fact that in the second inclusion, second-rank heterogeneities become fluid or oil-saturated and the inclusions are completely filled with liquid and it is not mobile. Then go to (5) - (6).

$$\frac{\sigma_{jis}(k^2_{1jis}-k^2_{1j})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,j}(M,M^0)d\tau_M+$$

$$+\varphi^0_{s-1}(M^0)--\frac{(\sigma_{ja}-\sigma_{jis})}{\sigma(M^0)2\pi}\int_{C_m}G_{Sp,j}\frac{\partial\varphi_s}{\partial n}dc= \qquad (7’)$$

$$=\varphi_s(M^0),M^0\notin S_{3Cs}\in\Pi_j;$$

Let us calculate in all layers j = 1, N using the expression (7 ’):

$\varphi_s(M^0), M^0\notin S_{3Cs}\in\Pi_j$

The algorithm stops if the ranks of the hierarchy become larger than the given numbers (6 ’). If at a certain hierarchical level, the structure of local heterogeneity splits into several heterogeneities, then the double and contour integrals in expressions (1-7 ’) are taken over all heterogeneities of a given rank. Consider the case of thermal convection in fluid-saturated inclusions located in the layer h_j .

A. *The problem of convection occurring in the fluid saturated inclusion*

In an unevenly heated one-component fluid located in a gravity field, mechanical equilibrium is possible. If the temperature heterogeneity is such that the conditions of hydrostatic equilibrium are violated, then the equilibrium becomes unstable and, as a result of the development of perturbations, is replaced by convective motion. Under the same conditions, when the conditions of hydrostatic equilibrium are violated, convection occurs at arbitrarily small temperature heterogeneity. This convection is called free heat. In this model, the macroscopic motion of a fluid is described by the following system of equations: the Navier-Stokes equation of motion, the heat transfer equation, and the continuity equation. For a real compressible fluid in a field of gravity, this system has the form [11]:

$$\sigma\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\nabla)\vec{v}\right]=-\nabla p+\eta\Delta\vec{v}+\left(\frac{\eta}{3}+\varsigma\right)\nabla\, div\,\vec{v}+\sigma\vec{g},$$
$$\sigma T\left(\frac{\partial s}{\partial t}+\vec{v}\nabla s\right)=\kappa\Delta T+D,\qquad (8)$$
$$\frac{\partial\sigma}{\partial t}+div(\sigma\vec{v})=0.$$

Here $\vec{v}$ is the velocity vector, p-pressure, σ-density, *T*-absolute temperature, *s*-entropy per unit volume of fluid; $\vec{g}$ - gravity acceleration; and η - shear and ς volume viscosity coefficients; κ - thermal conductivity coefficient (coefficients η , ς , κ are constants; D-dissipative function:

$$D=\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\,div\,\vec{v}\right)^2+\varsigma\left(div\,\vec{v}\right)^2$$

δ_{ik} -Kronecker symbol; over indices *i*, *k* summation is assumed. To the written system of equations it is necessary to add the equation of state of the medium and the boundary conditions: in our approximation of one-component fluid:

$$\sigma=\sigma(T,p)\qquad (9)$$

In our case

$\sigma=\sigma_{jim}$

m = 2 (3 ’). In the following formulas, we will use the notation for density σ.

Entropy can be expressed through two variables *T* and *p*. If the compressibility of the medium is not significant, the original system can be simplified. Starting to derive the equations of free convection, we begin by simplifying the equation of state (9). Imagine *T* and *p* in the form:

$T=\overline{T}+T',\ p=\overline{P}+P'$

where $\overline{T}$ and $\overline{P}$ are constant averages, taken as a reference point, T' and P' we will assume small, so σ' that deviations from the average value $\sigma_0=\sigma(\overline{T},\overline{P})$ are small compared to it. We confine ourselves to a small T' and P' approximation. Then the equation of state (9) takes the form:

$$\sigma=\sigma_0+\left(\frac{\partial\sigma}{\partial T}\right)_p T'+\left(\frac{\partial\sigma}{\partial p}\right)_T P'=\sigma_0(1-\beta T'+\alpha P')\qquad (10)$$

Where β and α are the coefficients of thermal expansion and isothermal compressibility:

$$\beta=-\frac{1}{\sigma_0}\left(\frac{\partial\sigma}{\partial T}\right)_p,\alpha=\frac{1}{\sigma_0}\left(\frac{\partial\sigma}{\partial p}\right)_T\qquad (11)$$

The requirement of smallness σ΄ in comparison with σ_0 means from (10) that:

$$|\beta T'|<<1,|\alpha P'|<<1\qquad (12)$$

We will further assume that changes in density due to pressure heterogeneity are small compared with changes due to temperature heterogeneity:

$$|\alpha P'|<<|\beta T'|$$

(13)

Then the equation of state (10) will be rewritten in the form:

$$\sigma=\sigma_0(1-\beta T')$$

(14)

Thus, we neglect the dependence of density on pressure; temperature dependence cannot be neglected; since this dependence leads to convection. Condition (13) means that the pressure along the fluid should not change significantly. Hence, the vertical scale of the area *l* in which convection takes place should not be large, and the hydrostatic pressure drop will have the order of $\sigma_0 gl$. Conditions (12), (13) will be rewritten as:

$$\sigma_0 gl\,\alpha<<\beta\Theta<<1$$

(15)

Θ-is a difference of the characteristic temperature. The condition of density changes smallness allows writing approximately the equation of continuity (8) in the form for an incompressible fluid:

$$div\,\vec{v}=0$$

(16)

We will similarly neglect the change in entropy due to pressure and write it down as:

$$s=s_0+\frac{c_p}{\overline{T}}T',\quad c_p=\overline{T}\left(\frac{\partial s}{\partial T}\right)_p$$

(17)

Substituting (17) into (8), (the second equation of the system), neglecting D we receive:

$$\rho_0\overline{T}\left(\frac{c_p}{\overline{T}}\frac{\partial T'}{\partial t}+\vec{v}\frac{c_p}{\overline{T}}\nabla T'\right)=\kappa\Delta T'\qquad \frac{\partial T'}{\partial t}+\vec{v}\nabla T'=\frac{\kappa}{\sigma_0 c_p}\Delta T'$$
$$\frac{\partial T'}{\partial t}+\vec{v}\nabla T'=\chi\Delta T'\qquad (18)$$
$$\chi=\frac{\kappa}{\sigma_0 c_p}$$

- thermal diffusivity. If there is an internal heat source, then to (1.13) in the right part a member is added: $\frac{Q}{\sigma_0 c_p}$

Q is the power of the sources. Let us proceed to the transformation of equation (8) (the first equation). Substitute (14) in (8) with (16):

$$\sigma = \sigma_0(1-\beta T')$$

$$\sigma\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\nabla)\vec{v}\right] = -\nabla p + \eta\Delta\vec{v} + \left(\frac{\eta}{3}+\varsigma\right)\nabla\, div\,\vec{v} + \sigma\vec{g}$$

$$div\,\vec{v} = 0$$

then we receive:

$$\sigma_0(1-\beta T')\frac{d\vec{v}}{dt} = -\nabla p + \eta\Delta\vec{v} + \sigma_0(1-\beta T')\vec{g} \quad (19)$$

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}$$

Let

$p = \tilde{p} + p'$,

$\nabla\tilde{p} = \rho_0\vec{g}$,

$\tilde{p}$ - hydrostatic pressure;

$$\sigma_0(1-\beta T')\frac{d\vec{v}}{dt} = -\nabla p' + \eta\Delta\vec{v} - \sigma_0\beta T'\vec{g} \quad (20)$$

$\frac{dv_z}{dt}$ If it is little compared with the acceleration of gravity, we can write:

$$\sigma_0\frac{d\vec{v}}{dt} = -\nabla p' + \eta\Delta\vec{v} - \sigma_0\beta T'\vec{g} \quad (21)$$

For free convection, this condition is satisfied. We divide (21) by the average density:

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = -\frac{1}{\sigma_0}\nabla p' + \nu\Delta\vec{v} + g\beta T'\vec{\gamma}$$

$$\nu = \frac{\eta}{\sigma_0}$$

- coefficient of kinematic viscosity, $\vec{\gamma}$ - a unit vector directed upwards. System (22) - (24) is called the heat convection system in the Businesque approximation:

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = -\frac{1}{\sigma_0}\nabla p + \nu\Delta\vec{v} + g\beta T\vec{\gamma} \quad (22)$$

$$\frac{\partial T}{\partial t} + \vec{v}\nabla T = \chi\Delta T \quad (23)$$

$$div\,\vec{v} = 0 \quad (24)$$

We omitted the strokes in *T* and *p*, but remember that *T* is measured from the average constant value, and *p* is the deviation from the hydrostatic $\tilde{p}$, corresponding to temperature $\overline{T}$ and density σ_0. This system corresponds to weak convection. Density deviations from the average value are assumed so small that we neglected them in all equations except the equation of motion, where it is taken into account only in the term with the lifting force (22). If the fluid fills a cavity surrounded by a solid array, then the system should be added the conditions of an ideal thermal contact:

$$\frac{\partial T_m}{\partial t} = \chi_m\Delta T_m \quad (25)$$

T_m - the temperature of the massif, χ_m - its thermal diffusivity. We formulate the boundary conditions: at the boundary *S* of a liquid with a solid massif: $\vec{v}$ =0, and the temperature and the normal component of the heat flux are:

$$T = T_m; \quad (26)$$

$$\kappa\frac{\partial T}{\partial n} = \kappa_m\frac{\partial T_m}{\partial n}$$

(27)

If the temperature and heat flux are set directly at the boundary of the cavity, then the equations will contain the following values: *L*-length of the cavity, Θ-characteristic temperature difference, τ-time characterizing unsteady external conditions and fluid parameters:

$\nu, \chi, g\beta$. From these quantities, three dimensionless combinations can be constructed:

$G = \frac{g\beta\Theta L^3}{\nu^2}$, $P = \frac{\nu}{\chi}$, $F = \frac{\chi\tau}{L^2}$ - Grashof, Prandtl and Fourier numbers.

B. Mechanical equilibrium

In an unevenly heated fluid, as a rule, convective motion occurs. There are special conditions for heating a fluid under which it can be in a state of mechanical equilibrium, that is, it can remain stationary. In this case, there will be no thermodynamic equilibrium: the spatial heterogeneity of temperature will lead to the appearance of a heat flow. Let us turn to the system (22) - (24) Let:

$\vec{v}$ = 0. We will look for stationary distributions of temperature and pressure in equilibrium, let T_0, p_0 be equilibrium distributions of temperature, neglecting of internal sources of heat, and pressure:

$$-\frac{1}{\sigma_0}\nabla p_0 + g\beta T_0\vec{\gamma} = 0 \quad (28)$$

$$\Delta T_0 = 0 \quad (29)$$

Applying an operation *rot* to equation (28)

$$rot\left(-\frac{1}{\sigma_0}\nabla p_0 + g\beta T_0\vec{\gamma}\right) = 0$$

We obtain:

$$\nabla T_0 \times \vec{\gamma} = 0 \quad (30)$$

We omit the trivial case:

$\nabla T_0 = 0$

then

∇T_0

is parallel to

$\vec{\gamma}$, i.e. it is vertically directed and

$$\frac{\partial T_0}{\partial x} = \frac{\partial T_0}{\partial y} = 0 \quad (31)$$

$$T_0 = T_0(z)$$

(32)

Turning to (29):

$$\frac{\partial^2 T_0}{\partial z^2} = 0.$$

Then:

$$T_0(z) = -Az + B \quad (33)$$

A and *B* are constant.

$$\nabla T_0 = -A\vec{\gamma} \quad (34)$$

The equilibrium is stable if all perturbations extinguish with time. If one or several perturbations increase with time, then the equilibrium is unstable with respect to these perturbations, and their development over time will lead to no balance and convection. Under real conditions, various disturbances arise; therefore the equilibrium of a fluid can be observed only when it is stable.

C. Unstable equilibrium

Unstable equilibrium is quickly replaced by convection. Consider the temperature and pressure fields that are different from the equilibrium: $T_0 + T_1, p_0 + p_1$,

T_1,p_1- perturbations. Deviations of temperature and pressure from equilibrium distributions result in convective motion with velocity v_1. We will consider small nonstationary deviations in the linear approximation. In system (22) - (24) we neglect terms that are quadratic in perturbations and, taking into account (28) and (29), we obtain:

$$-\frac{1}{\sigma_0}\nabla p_0 + g\beta T_0\vec{\gamma} = 0$$

$$\Delta T_0 = 0$$

Let us write out a system of small deviations from the state of mechanical, hydrostatic equilibrium in the form:

$$\frac{\partial \vec{v}_1}{\partial t} = -\frac{1}{\sigma_0}\nabla p_1 + \nu\Delta\vec{v}_1 + g\beta T_1\vec{\gamma} \quad (35)$$

$$\frac{\partial T_1}{\partial t} + \vec{v}_1\nabla T_0 = \chi\Delta T_1$$

$$div\,\vec{v}_1 = 0$$

If the inclusion is surrounded by a heat-conducting massif, then this system must also be supplemented with an equation for the perturbation of the temperature T_{m1} of the massif, which follows from (25):

$$\frac{\partial T_{m1}}{\partial t} = \chi_{m1}\Delta T_{m1} \quad (36)$$

At the boundary between the liquid and the massif at the boundary *S* of the liquid with a solid massif v_1=0, and the temperature and the normal component of the heat flow are:

$$T_1 = T_{m1};$$

$$\kappa_1\frac{\partial T_1}{\partial n} = \kappa_{m1}\frac{\partial T_{m1}}{\partial n}$$

Let us write the system of equations for perturbations (35) - (36) in a dimensionless form, for this we choose the following units of measure: distances - characteristic linear size of the inclusion *L*, time - L^2/ν, velocity- χ/L, pressure- $\sigma_0\nu\chi/L^2$, temperature-*AL* (*A*-equilibrium temperature gradient, defined by the relation (2.7):

$$\frac{\partial v}{\partial t} = -\nabla p + \Delta v + RT\vec{\gamma}$$

$$P\frac{\partial T}{\partial t} - (v\vec{\gamma}) = \Delta T$$

$$divv = 0$$

$$P\tilde{\chi}\frac{\partial T_m}{\partial t} = \Delta T_m \quad (37)$$

System (37) includes dimensionless perturbations: v, p, T, T_m,

and all derivatives are taken along dimensionless coordinates and time. The system includes three dimensionless parameters: the Prandtl number *P*, the ratio of the thermal diffusivity of the fluid and the array:

$$\tilde{\chi} = \frac{\chi}{\chi_m}$$

and the Rayleigh number *R* = *G* * *P*. This system has particular solutions that depend on time exponentially: $v, p, T, T_m \sim \exp(-ft)$.

$$-fv = -\nabla p + \Delta v + RT\vec{\gamma}$$

$$-fPT - (v\vec{\gamma}) = \Delta T \quad (38\text{-}41)$$

$$div\,\vec{v} = 0$$

$$-fP\tilde{\chi}T_m = \Delta T_m$$

The boundary conditions for the amplitudes: at the boundary S of the liquid with a solid massif, v = 0, the temperature and the normal component of the heat flux:

$$T = T_m;\ \tilde{\kappa}\frac{\partial T}{\partial n} = \frac{\partial T_m}{\partial n};\ \tilde{\kappa} = \frac{\kappa}{\kappa_m};\ T_m \to \infty = 0;$$

$$\vec{v}\big|_S = 0 \quad \left.\frac{\partial\vec{v}}{\partial n}\right|_S = 0; \quad (42)$$

III. RESULTS

Linear homogeneous boundary value problem (38) - (42) –is a problem of eigenvalues. The eigenvalues of the decrements of normal disturbances are f. Eigen functions are the corresponding amplitudes. The formulated boundary value problem determines the spectrum of normal perturbations of the equilibrium of a fluid in the inclusion of a given geometry, which depends on four parameters entering the equations: Rayleigh numbers, Prandtl numbers, thermal conductivities and thermal diffusivity ratios. The dependence of normal disturbances on time lies in the multiplier *exp(-ft)*. If the decrement *f* is real, then the perturbation changes monotonically with time: at *f*> 0, the perturbation decreases, and at f <0 it increases. If the decrement is complex, it can be represented as $f = f_r + f_i$. In this case, the perturbations will oscillate with a frequency equal to the imaginary part of the decrement. In the case of a closed inclusion, the spectrum of normal disturbances is discrete and consists of several frequencies. As is shown in [10], fluctuations in temperature perturbations occur when the fluid is heated from above, but a similar effect occurs when the fluid is intermixed by an acoustic vertical effect. These temperature fluctuations will also affect the fluctuations in the density of the liquid:

$$\sigma_{jim} = \sigma^0_{ijm} + \sum_k \sigma^{k'}_{ijm}(2\pi f_k)$$

rank *m* = 2, which will affect the values of the acoustic potential of the longitudinal acoustic wave. That effect was probably recorded in [11].

IV. CONCLUSIONS

Iterative simulation algorithms are constructed for the seismic case in the acoustic approximation for the composite hierarchical heterogeneity. When building a complex seismic gravitational model without taking into account the anomalous influence of the stress-deformation state inside the hierarchical inclusion, called the elastic cushion, an analysis of the anomalous acoustic effect using longitudinal wave propagation data shows that the effect of anomalous elastic parameters in the seismic model cannot be neglected, since they affect on the values of the desired anomalous densities. If these values are used in constructing the density gravitational model without taking into account the influence of elastic parameters, these density values will not reflect the material composition of the analysed medium. When constructing an anomalously stressed geomechanical model without taking into account the anomalous influence of density heterogeneities within the hierarchical inclusion, which is the substrate for a two-phase field, the values of the desired anomalous elastic parameters causing an anomalous stress in the pillow using data on the propagation of the transverse wave will not be distorted. These values of the elastic parameters will not reflect the real stress state of the analysed medium over the fluid-containing field, which in turn seems to be a hierarchical multi-granular medium. Analysis of dynamic phenomena in hierarchical inclusions containing fluids (oil or water) in the form of convection, leading to density and potential oscillations of a longitudinal wave with large values of hierarchy ranks in a composite hierarchical structure. This phenomenon can be caused not only by thermal effects, but also by using a source of vertical exposure to an acoustic longitudinal wave. The set of additional frequencies will depend on the geometry of the inclusions, as well as on the composition of the fluid.

The first proposed iterative algorithm for modeling a hierarchically complex two-phase medium with account of convective intermixing can be used to control the production of viscous oil in mine conditions and light oil in sub horizontal wells [12].

REFERENCES

[1] I. Prangishvilli, F.Paschenko, B. Busigin, System laws and regularities in electrodynamics, nature and society, Nauka, Publ.: Moscow, Russia, 2001.

[2] G.Kocharjan, A.Spivak, Dynamics of deformation of block massive, IKZ “Akademkniga”: Moscow, Russia, 2003.

[3] M.Khasanov, G.Bulgakova, Nonlinear and noneqiulibrium effects in rheological complex environments, Institute of computer research: Moscow, Izhevsk, Russia, 2003.

[4] O.A.Khachay, A.Y. Khachay, On the integration of seismic and electromagnetic active methods for mapping and monitoring the state of two-dimensional heterogeneities in an N-layer medium, Vestnik SSSU. Series «Computer technologies, management, radio electronics », 2011, No.2 (219).

[5] O.A.Khachay, A.Y.Khachay, Modeling of electromagnetic and seismic fields in hierarchically heterogeneous media. Vestnik of SSSU, series "Computational mathematics and informatics", 2013, V.2, No.2.

[6] O.A.Khachay, A.Y. Khachay, Reflection of noneqiulibrium two-phase filtration processes in oil saturated hierarchical media in active wave geophysical monitoring data, Mining information and analytical bulletin, 2014, No.4.

[7] O.A. Khachay, O.Y. Khachay, A.Y. Khachay, Integration of acoustic, gravitational, and geomechanical fields in hierarchical environments, Mining information and analytical bulletin, 2017, No.4.

[8] A.Y. Khachay, Algorithm for solving the direct dynamic seismic problem when excited by a horizontal point force located in an arbitrary layer of an n-layer elastic isotropic medium. In Informatics and mathematical modeling. USSU: Ekaterinburg, Russia, 2006.

[9] A.Y. Khachay, Algorithm for solving the direct dynamic seismic problem when excited by a vertical point force located in an arbitrary layer of an n-layer elastic isotropic medium. In Informatics and mathematical modeling. USSU: Ekaterinburg, Russia, 2006.

[10] G. Gershuni, E. Zhukhovitskiy, Convective stability of incompressible fluid, Nauka, Main editors of Phys. and Math. Literature: Moscow, Russia, 1972.

[11] V.V. Dryagin, The use of caused acoustic emission collectors for the detection and extraction of hydrocarbons. Georesources, 2018, V.20, No.3, Part2. DOI: https://doi.org./10/18599/grs.2018.3.134-140.

[12] Y. Khachay, M. Mindubaev, Effect of convective transport in porous media on the conductions of organic matter maturation and generation of hydrocarbons in trap rocks complexes. Energy Procedia, 2016, No. 74.