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Original Article

Architecture Design Rules of 'Moons' of Planets in our Solar System

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Abstract *- In our Solar System,* the *Sun, the Planet and its natural satellite constitute a Circular Restricted 3-Body Problem (CRTBP), which has fixed point solutions consisting of L1, L2, L3, L4 and L5. Planet, its respective natural satellite and a test particle* also *constitute a CRTBP and it has its corresponding 5 Lagrange Points. L4 and L5 of the Sun and Planet invariably have asteroids trapped, which constitute the Trojans of the respective Planet. There are 'regular moons', there are 'irregular moons', and there are transitional moons' Regular moons have arisen as a by-product of the planet's formation within the circum-planetary disk of gas and particles. The outer moons in the Hill Sphere of the respective Planet are irregular moons which are more distant and have inclined and elliptical orbits and which are captured celestial bodies from the asteroid belt from the Kuiper Belt or from the Oort's Cloud. In the year 2000 invention of wide field Charge Coupled Device (CCD) led to a spike in the discovery of irregular moons. There are transitional 'moons' which lie between regular moons and irregular moons, for example Iapetus, a moon of Saturn. Finally, there are ring moons. Jupiter, Saturn, Uranus and Neptune have rings surrounding them. At the outer edge of these rings are fully formed moons. These are called ring moons. Sun and Planets (except Mercury and Venus) have captured bodies(asteroids) at L4 and L5. These captured bodies at L4 and L5 are called Trojans. Till date February 2024, Jupiter, Saturn, Uranus and Neptune have 228 irregular moons orbiting the outer four planets. The largest of these are Himalia of Jupiter, Pheobe of Saturn, Syacorax of Uranus and Triton of Neptune. Each planet has a gravitational sphere of influence known as the Hill Sphere. If the Hill Sphere is spacious enough, it captures a natural satellite.*

Keywords - *Circular restricted three body problem, Trojans, Regular moons, Oort's Cloud, Hill Sphere, Asteroid belt, KuiperBelt.*

1. Introduction

There was a supernova explosion in the neighbourhood of our Solar System. The supernova caused a substantial injection of dust and radioactive nucleoides into a passing pre-solar giant molecular cloud. The shock wave from the supernova explosion set the molecular cloud into a rapid rotation, and rapid rotation flattened the solar nebula like a pancake into a disc of accretion [1]. 4567.30My ±0.16My ago, the solar system was formed [2]. The precipitation of the solar system from the Solar Nebula is defined as the crystallization age of Calcium and Aluminum – rich Inclusions(CAI). This nebular system dissipated in 1 to 3My after the Solar System formation. This was followed by the formation of Gas Giants Jupiter and Saturn, followed by the formation of Ice Giants Neptune and Uranus, followed by the formation of terrestrial planets, namely Earth, Venus, Mars and Mercury[3]. Since the birth of our Solar system 4.567Gy ago and the rapid rotation of the Solar Nebula into circumstellar gas and particle disc, also referred to as proto-planetary discs of gas and dust, the birth and evolution of the planetary system is completed in the first 100 million years (as per latest finding about metal isotopes in Earth and Moon Bulk Silicate). The planets are born and formed by wrapping the planetary embryos, and planetary embryos are formed from planetesimals and planetesimals are formed from dust constituting the circumstellar proto-planetary disc. Within a narrow time slot of 30 million years, the Gas Giants, namely Jupiter, Saturn, and Ice Giants, namely Neptune and Uranus, were born. The remaining gas and dust disc was dissipated by photo-evaporation (this was responsible for pushing out sub-micron size particles) and Robertson-Poynting drag (this drag was responsible for the in-spiral of particles larger than micrometer size)[3]. In the first 30 million years after the birth of the Solar Nebula, all the Jovian Planets are formed sequentially by runaway gravitational accretion of gas by the planetary embryo. This was followed by the era of giant impacts in the first 100My after the Solar System precipitated[4]. Terrestrial Planets are formed by the infrequent and powerful impact of planetary embryos subsequently.

This was the era of giant impacts, which culminated into the formation of Terrestrial Planets, namely Mercury, Venus, Earth and Mars. Every planet in our Solar system, except Mercury and Venus, has one or more moons (the natural satellites of Planets). These moons are classified according to their proximity to the host planet and formation process.

1.1. Every planet has a Hill Sphere

The Hill Sphere is the gravitational sphere of influence of the given planet in the presence of the Sun.

$$(\textbf{Hill Sphere Radius})\boldsymbol{R_H} = (\frac{\boldsymbol{m}}{\boldsymbol{3M}})^{\boldsymbol{1/3}} \quad (\mathbf{1})$$

Where m= mass of the planet,
M= mass of the Sun;

If the Hill Sphere is spacious enough, the given planet will capture a natural satellite and that satellite will become the moon of the given planet.

There is a certain criterion [5] by which it is determined that the Hill Sphere is spacious enough to host a natural satellite.

R1=(Planet Radius/Hill Radius) is highly correlated with the probability of natural satellite capture.

So for a given Planet, the semi-major axis of captured moon 'a_{moon}' << Hill Radius of the given Planet. From the Author's paper [5] we get:

Table 1. (Planet Radius/Hill Radius) ratio=R1 and (a_R(Sun-Planet)/a_P) ratio=R2, R1/R2 and comment on planet's acceptability of natural satellite or on satellite's acceptability of a sub-satellite

	R1	R2	R1/R2	Comment
Sun-Mercury	0.01	0.0193	0.5181	Mercury can accept satellites with low probability
Sun-Venus	5.9862×10^{-3}	0.01045	0.5728	Venus can accept satellites with low probability
Sun-Earth	4.26×10^{-3}	7.434×10^{-3}	0.57	Earth has a satellite.
Sun-Mars	3.4566×10^{-3}	6.025×10^{-3}	0.5737	Mars has two satellites
Sun-Jupiter	1.345×10^{-3}	2.297×10^{-3}	0.5855	Much higher probability of satellites It has 67 natural satellites
Earth-Moon	0.0283	0.04936	0.5733	Moon cannot accept a sub-satellite
Mars-Phobos	0.66	1.16	0.5689	Phobos cannot accept sub-satellite
Mars-Deimos	0.2735	0.477815	0.5723	Deimos cannot accept a sub-satellite

Ratio R1 must be less than 0.006 in order to qualify as a natural satellite host.

1.2. Laplace Plane

For any perturbed orbit of a satellite there exists a Laplace Plane around which the orbital plane of the perturbed orbit precesses.

In the inner part of the Planet-satellite system, a tidally oblate planet dominates the orbital dynamics, and the equatorial plane of the planet is the Laplace Plane.

In the outer part of the system, the Sun's perturbation dominates the orbital dynamics and the natural satellite orbits in the Ecliptic plane (The ecliptic plane is the imaginary plane in the solar system, which contains the orbital planes of the major planets and their moons.)

1.3. Laplace Plane Transition

In between the inner part and outer part of the perturbed orbit, there is Laplace Plane Transition where the oblateness of the planet and Sun's perturbation balance each other.

This is called the Laplace Plane Transition (rL) orbit. For low-obliquity planets, the transition is smooth, but for high-obliquity planets, the transition can be complex.

Satellites on a circular orbit around high obliquity planets migrating through Laplace Plane Transition orbit can acquire substantial eccentricities and inclinations. This has happened in the case of our Earth-Moon system.

1.4. Formalism of Laplace Plane Transition

Laplace Plane Transition is defined as follows:

$$r_L(laplace\ plane\ transition\ orbit\ radius) = (2J_2\frac{M_E}{M_S}R_E^2a_E^3 \times (1-e^2)^{3/2})^{1/5} \quad (2)$$

J_2 = oblateness moment of Earth. As Earth's spin slows down, oblateness decreases, leading to r_L moving inward.
R_E = volumetric mean radius of Earch or the planet in

question,
M_E= mass of Earth,
M_S= mass of Sun,

a_E= semi major axis of Earth or the planet in question.

'e' is the eccentricity of the orbital path of Earth or the planet in question.

$$\boldsymbol{r_L = 16 \sim 22R_E}\ .$$

Natural satellites within
$\boldsymbol{r_L(laplace\ plane\ transition\ orbit\ radius)}$

lie in the equatorial plane of the planet. They are formed in-situ from the dust accretion disc around the planet, and they are called regular 'moons'. Natural satellites beyond 'r_L' lie in the ecliptic plane.

These satellites are irregular satellites. These are captured bodies. During planetary formation, a lot of small bodies were left over from giant impacts.These were captured from heliocentric orbits into host planets orbits.

These are primitive bodies. The captured body from the heliocentric orbit during the early period of Solar System history by Restricted Three Body Problem(RTBP) dynamics, but it has a temporary feature.

Therefore, there must be an auxiliary capture mechanism which sets these irregulars in permanent stable orbits. The auxiliary capture mechanisms can be any of the following:

Through gas-drag capture mechanisms [6,7,8].Collisional capture[9]; Chaos-assisted capture from low energy orbit[10]; Or various binary capture scenarios [11,12,13,14,15]

The Table containing Jovian Planets and their respective Hill Radii and Laplace Plane Transition orbits is given in Appendix I

All the moons within 'r_L' are under the influence of the HOST PLANET, hence orbiting in the equatorial plane and synchronous orbits; moons beyond 'r_L' are influenced by solar perturbations and hence are orbiting in the Ecliptic plane and are not in synchronous orbits.

1.5. Regular Satellites are Synchronous

Moon-Earth: synchronous;
Phobos-Deimos-Mars: synchronous;
Galilean Satellites-Jupiter: synchronous;
Regular Satellites-Saturn: synchronous;
Regular Satellites-Uranus: synchronous;
Regular Satellites-Neptune: synchronous;
Charon-Pluto: is in triple synchrony [3], and Charon has evolved to outer Clark's orbit since q(mass ratio) is greater than 0.2

BUT 4 small moons of PLUTO are not synchronous-most surprising. Why is this so?

1.6. Regular Moons and Irregular Moons

The inner moons in the Hill Sphere of a given Planet are regular moons and are formed by accretion from circum-planetary impact generated debris disk[16,17]. Regular moons have arisen as a by-product of the planet's formation within the circum-planetary disk of gas and particles. These regular moons orbit in the equatorial plane of the host planet in a prograde fashion with orbital radii of tens of the planet's radius. The outer moons in the Hill Sphere of the respective Planets are irregular moons which are more distant and have inclined and elliptical orbits and which are captured celestial bodies from the asteroid belt or from the Kuiper Belt or from the Oort's Cloud [18]. There are transitional 'moons' which lie between regular moons and irregular moons, for example Iapetus, a moon of Saturn[19]. Finally, there are ring moons. Jupiter, Saturn, Uranus and Neptune have rings surrounding them. At the outer edge of these rings are fully formed moons. These are called ring moons[20]. Satellite origin is important both for their history(including volcanic Io[21] and Europa[22], which is likely to hold a sub-surface ocean and organic-rich Titan[23], and they hold clues to the origin of gas giants and ice giants.

Massive circumstellar disk reduced to protoplanetary disk.

The protoplanetary disk had the jovian planets, terrestrial planets, and debris disk [24].

1.7. Protoplanetary Disk Reduced to Planetary System and Debris Disk

The debris disk gave birth to the asteroid belt between Mars and Jupiter and the Kuiper Belt beyond Uranus. The asteroid belt and Kuiper Belt comprised the majority of the debris disk. The remaining part of the debris disk constituted the Zodiacal Cloud and Oort's Cloud.[25]

1.8. Giant Impact on Mercury

The fact that Mercury has a molten iron core which constitutes 60 % of its total mass in comparison to Venus, Earth and Mars, which have a metallic core which is 30% of the total mass, testifies to the fact that there must have been a Giant Impact on the emerging differentiated Mercury where an Earth-sized impactor must have stripped away the Silicate mantle leaving a iron core dominant Planet but then the volatiles should have been lost but MESSENGER Mission testifies to the fact that pockmarked Mercury is rich in moderately volatile elements such as Potassium and Sulphur. Mercury has a high concentration of Sodium and Chlorine.

This is contrary to a post-impacted Mercury. Mercury, because of its proximity to the Sun, has a very limited Hill sphere and hence incapable of retaining a natural satellite; hence, it has no moon of its own. More research will unveil the process which has created these contradictory phenomena- the presence of volatile materials and massive molten iron core [26]. Mercury is locked in 3:2 spin: orbit resonance. It has a very thin atmosphere. It has 0.38% of Earth's gravity. Surface Temperature 450^0C and liquid iron core extend upto 3/4 of Mercury radius =2,449.5 Km.

1.9. Giant Impact on Venus

Venus's slow and retrograde rotation shows that a large head-on collision took place early in the late veneer period(LVP- 4.467bya to 4bya) or in the early Late Heavy Bombardment (LHB-4bya to3.8bya) period. Numerical simulation and comparative study of Venus with Earth, Mars and Mercury show that Venus experienced intense bombardment by all sorts of bodies ranging from small sizes to 100 km sized impactors or even larger. However, no natural satellite was formed, or if formed, it could not be retained by Venus because of its proximity to the Sun[27].

1.10. Giant Impact on Earth

Right at the end of the formation of Earth 4.467Gya, Mars-size impactor glancing angle collision with a high obliquity, high angular momentum Earth created a circum-terrestrial debris disk. This debris disk is predominantly made of impactor material. In a post-impact state, Earth's mantle, atmosphere and disk are not dynamically isolated from one another. As a consequence, they are well-mixed and equilibrated.

This ensures the identical nature in the isotopic signature of the newly accreted moon and impacted re-solidified Earth. In the debris disk moon is accreted beyond Roche's limit, which in the E-M system is at 15,000Km. This full-sized moon is catapulted, through the gravitational slingshot effect, on an expanding spiral path from its orbit of accretion, which in this case is 18,000Km. Moon is spiralling out to outer geo-synchronous at ~ 550,000Km. This model gives an explanation for the near identity between the isotopic signature of Earth and Moon and also gives a pathway to reach Earth's climatically favourable low obliquity of 23.44^0[28].

1.11. Giant Impact on Mars

Mars is struck by a protoplanet one-third its size—a debris disk forms within a few hours. The elementary building blocks of Phobos and Deimos (grains smaller than a micrometer) condense directly from gas in the outer part of the disk. The debris disk soon produces a moon near Mars that moves further away and propagates its two areas of dynamical influence like ripples, which over the course of a few thousand years causes the accretion of more dispersed debris into two small moons, Phobos and Deimos. Under the effect of the tidal pull of Mars, the large moon falls back to the planet within approximately five million years, while smaller Phobos and Deimos take up their current positions in the ensuing billions of years [29]. Through Smooth-Particle-Hydrodynamic Simulation, it is proposed that an asteroid impacted Mars produced a circum-martian debris disk. This was massive enough to produce Phobos and Deimos at $6R_{Mars,}$ where R_{Mars} is the volumetric mean radius of Mars, and $6R_{Mars}$ is inner Clarke's orbit (a_{G1}) in Kinematic Model parlance. Phobos is on the inner side of (a_{G1}); hence, it is launched on a collapsing spiral orbit doomed to its sure destruction through its glancing angle collision with Mars in 10My from now, leading to a hail storm on Earth because of powerful Martian ejecta directed towards Earth as a result of glancing angle collision of Phobos with Mars.

Alternatively, any time from now to a time earlier than 10My, Phobos will tidally pulverize and spread around Mars like Saturn's ring. From this Martian ring, there will be a moon shower as the Martian Atmosphere erodes the Martian ring. Deimos accreted on the far side of (a_{G1}), and hence, it was launched on an outward expanding spiral path, and presently, it is orbiting $6.9R_{Mars}$. The simulation also proposes that the impactor must be from a Vesta-to-Ceres-sized asteroid to produce a massive enough impact-generated circum-martian debris disk which can support Phobos and Deimos[30].

1.12. Giant Impact on Jupiter

From the Juno mission, accurate data has been obtained about the composition and internal structure of Jupiter [31]. This data suggests that Jupiter has a diluted core with a total heavy element mass ranging from 10 to a few 10s of Earth's mass (about 5 to 15% of the Jovian mass), and heavy elements are distributed within a region extending to nearly half of Jupiter's radius. In the planetary formation process, first, the compact core is accreted and subsequently, on reaching a critical mass the core wraps itself with gas by runaway gas accretion process. The finding of Jupiter's diluted core combined with high heavy element enrichment extending upto half the radius forces us to assume that a giant impact occurred early in the formation process.

The energetic head-on collision between a large planetary embryo and an impactor and the proto Jupiter shattered the original primordial compact core and thoroughly mixed the heavy elements with the inner envelope of the planet. Jupiter's gravitational focussing effect led more often to head-on collision as compared to glancing angle collision.

The inner part of the envelope becomes convectively driven by the steep temperature gradient near the core. This leads to vigorous turbulent mixing between the heavy elements and the H-He envelope. This giant impact led to impact impact-generated circum-jovian debris disk, and from this debris disk, the Jupiter ring and regular moons were born. Through the capture process, irregular moons were formed[32].

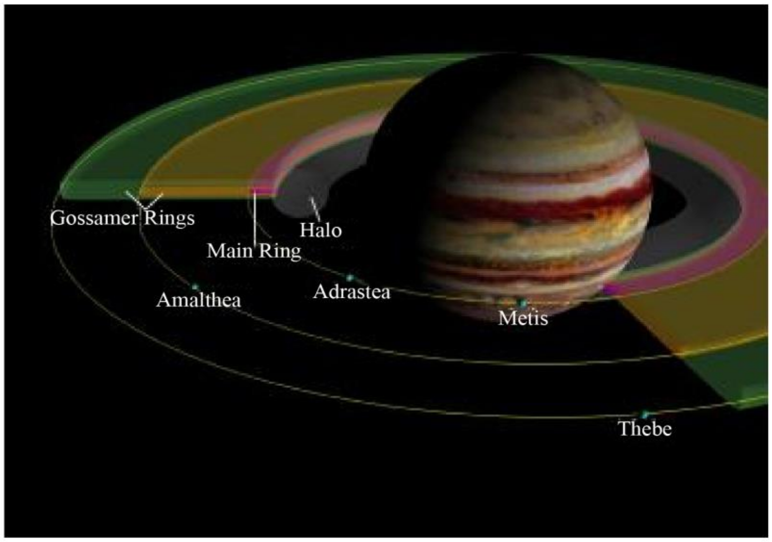

Fig. 1 The main ring and the Gossamer ring encircle Jupiter

There were similar impacts generated by circumplanetary debris discs corresponding to each planet, and each circumplanetary debris disc was the fertile ground for regular moon birth and formation.

2. Materials and Methods

2.1. The Birth of Regular Moons

The eight planets have their corresponding circumplanetary debris disk. Each disk gave birth to regular moons, which are large, spherical (except Phobos and Deimos – the moons of Mars but which are irregular in shape), nearly circular orbits, co-planar with the equatorial plane of the host planet and the regular moons are born from the impact-generated circumplanetary disk[33] except for Mercury and Venus.

2.2. Regular Satellites of Jupiter

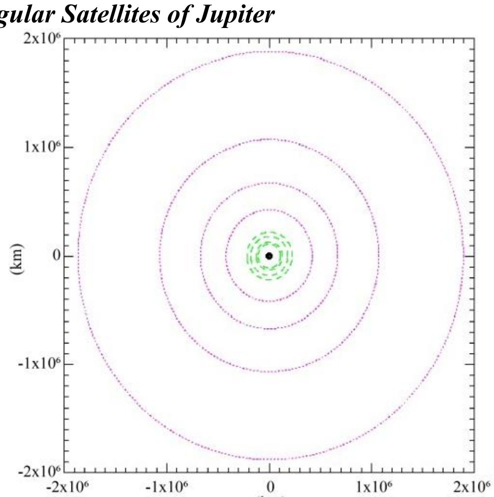

Fig. 2 Regular satellites of jupiter

Figure 2 shows a plan view for the orbits of the regular satellites of Jupiter. They have small circular orbits and low inclinations. These objects probably formed in an early circumjovian disk of gas and dust around Jupiter during Jupiter's formation.

The Table containing the four Galilean satellites is given in Appendix II.

The Table containing the regular moons of Saturn is given in Appendix III.

The Table containing the regular moons of Uranus is given in Appendix IV

The Table containing the regular moons of Neptune is given in Appendix V

The Table containing the regular moons of Pluto is given in Appendix VI

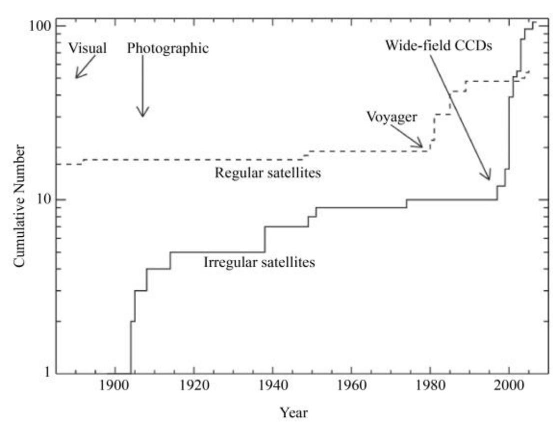

Fig. 3 A sudden spike since the year 2000 in the discovery of irregular moons after the invention of wide-field Charge Coupked Devices Telescopes

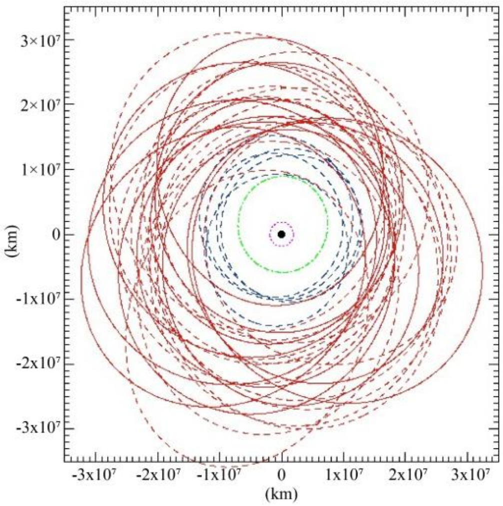

Fig. 4 Irregular satellites of Jupiter

Figure 4 shows the plan view of the orbits of all 31 known outer irregular satellites of Jupiter known before 2002. Irregular satellites have large orbits, inclinations and eccentricities.

Black Dot is Jupiter's location.

Purple dotted line is the orbit of the outermost Galilean satellite, Callisto.

Green dotted and dashed line is the innermost irregular prograde satellite Themisto.

Blue dashed lines are the 5 irregular satellites in the prograde group known before 2002.

Red solid lines are the 11 discovered irregular satellites of 2001 in the retrograde group.

Red dashed lines are the 14 previously known irregular satellites in the retrograde group.

2.3. Jupiter Magnetosphere

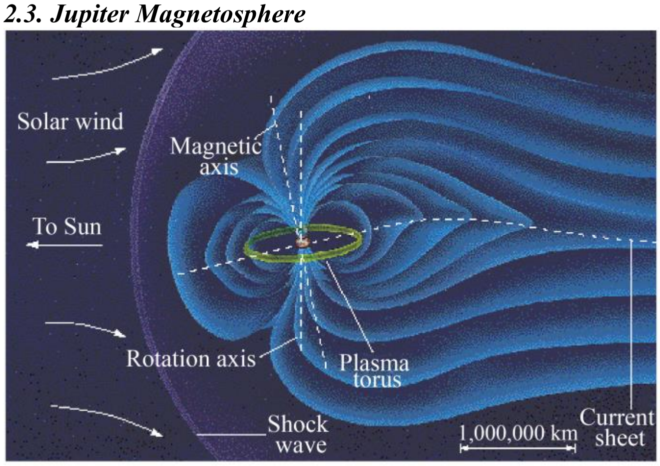

Fig. 5 Jupiter Magnetosphere

Io (the most volcanic in our Solar System)
Io - It is the innermost Galilean moon, and it is the most volcanically active in our whole solar system.

It has over 100 active volcanoes that erupt and alter the surface of the moon.

The surface looks bright yellow because of sulphur and sulphur compounds. Because of the close proximity of Jupiter, Io is subjected to very large tidal forces leading to squeezing and stretching of the interior which in turn causes volcanic activity.

Materials thrown up can escape from Io and form a plasma torus in Io's orbit around Jupiter, as seen in Figure 4.

2.4. Ring System of Jovian Planets

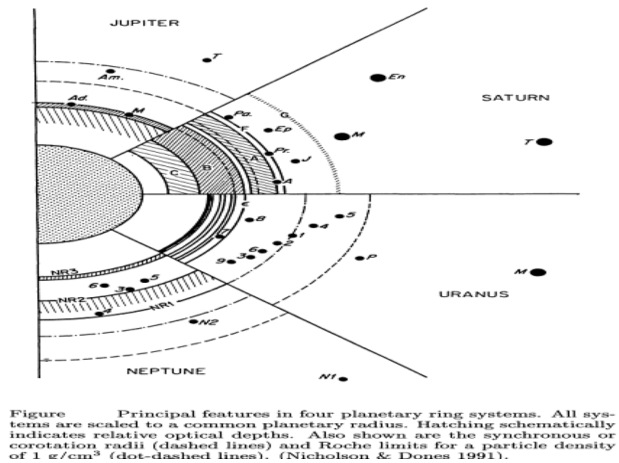

Fig. 6 Principal features of four planetary ring systems. The ring systems have been scaled to a common planetary radius. The hatching scheme indicates relative optical depths. Also shown are the synchronous co-rotating radii (dashed lines) and Roche's limits for a particle of 1 $/cm^3$(dot-dashed lines) [courtesy: Nicholson & Damos 1991]

The Roche's Limit roughly divides the domain of rings and satellites though there are numerous exceptions to the rule as shown in Figure 6.

Dot-dashed lines are Roche's limit in Figure 6.

Results of ring particle and nearby satellite interaction - Resonantly controlled outer edges of Saturn's A and B rings; The narrow Encke and Keeler gaps in the outer A ring; Numerous eccentric ringlets at Saturn and Uranus; And the curious arcs embedded in Neptune's Adam ring

Saturn + Phoebe binary pair-

Phoebe is beyond Laplace Plane Transition orbit. Hence, it orbits in the ecliptic plane.

Phoebe is orbiting in a retrograde fashion opposite to the orbital direction of all the other moons.

Phoebe is a heavily cratered and scarred outpost of the Saturn system, about four times farther than Iapetus (the nearest major neighbor of Phoebe).

Phoebe could be a captured asteroid in a wide eccentric orbit in the ecliptic plane.

It seems to be one of the original chunks of rocks which precipitated from the solar nebula 4.567Gy ago when the solar system was born.

It may be very primitive and one of the KBO(Kuiper Belt Object).

A lot of projectiles smaller than 100m have hit Phoebe and these projectiles may be from outside or from inside the Saturnian system.

The projectiles have chipped off ejecta from Phoebe which have become the retrograde, smaller outermost moons of Saturn. In that sense, they are the progeny of Phoebe.

2.5. Phoebe Outlying Status

Saturn's $\boldsymbol{r_L}$=2.5× $\mathbf{10^9}$ m.
Phoebe semi-major axis a= 12.952× $\mathbf{10^9}$ m.

Iapetus semi-major axis a= 3.5613× $\mathbf{10^9}$ m

Saturn's Hill Radius $\boldsymbol{R_H}$ = 65.4727× $\mathbf{10^9}$ m.

Both Phoebe and Iapetus are well within Saturn's Hill Radius; hence, they remain captured but they are far enough from oblate Saturn to experience solar perturbation since they are well beyond the Laplace Plane Transition orbit of 2.5×10^9m.

Hence, they orbit in the ecliptic plane and not in the equatorial plane of Saturn.

Fig. 7 Photo of Phoebe from Voyager 2

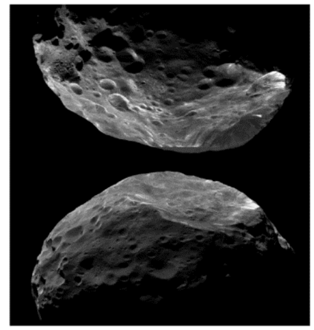

Fig. 8 Photo of Iapetus from Cassini in 2000Km

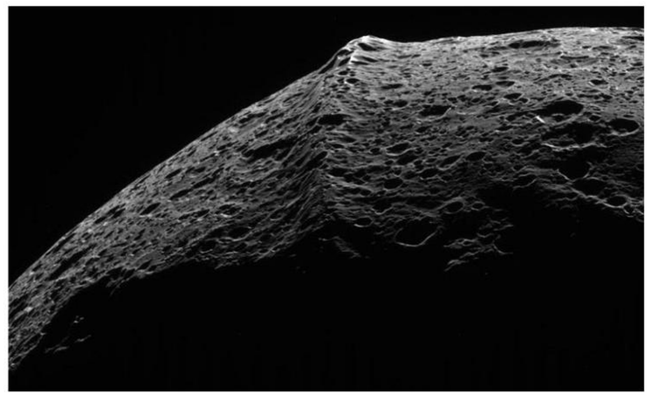

Fig. 9 Equatorial ridge of Iapetus image from Cassini

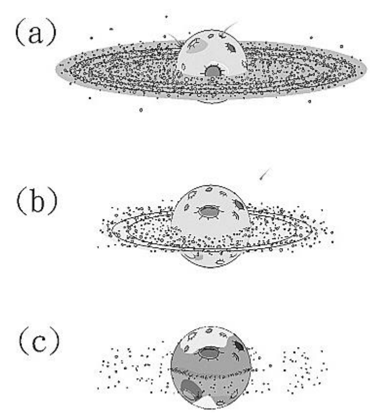

Fig. 10 the circum-iapetian disk of dust and ice

2.6. Iapetus – A Regular Saturinian Moon

Three features of Iapetus make it a class apart among the Saturnian moons.

These are its present spin period of 79.3 days, the present oblate spheroid shape corresponds to the equilibrium figure of a hydrostatic body rotating with a period of 16 hours [32] and its equatorial ridge[33]. Iapetus has the largest non-hydrostatic anomaly. Our moon is a distant second [34,35]. Levison et al. (2011) [19] put forward the hypothesis of impact generated circum-iapetian disk of dust and ice.

This gave birth to sub-satellites beyond Roche's Limit. This hypothesis explains the three features mentioned above.

2.7. Enceladus- Saturn

Enceladus, a tiny satellite of Saturn, remains an enigma. Its south pole gives fountains of water. Enceladus produce a water plume large enough to drench the whole Saturnian system.

2.8. Titan-Saturn

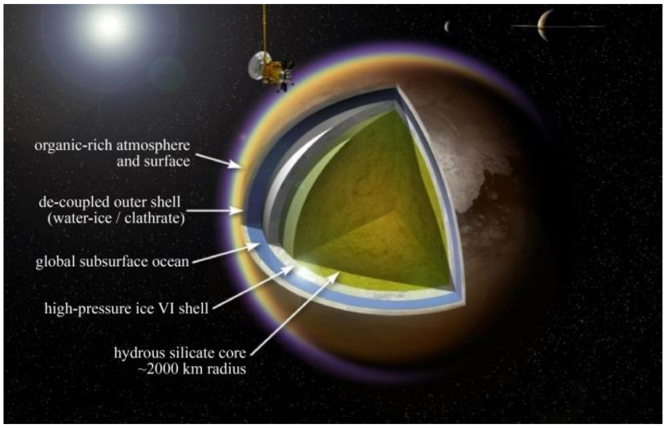

Fig. 11 Layered view of Titan

Cassini's study has concluded that Titan is a close cousin of Earth but with its own characteristic idiosyncrasies.

Titan's atmosphere and surface behave like Earth- with clouds, rainfall, river valleys and lakes.

But instead of water, we have hydrocarbon. Titan seasons change unexpectedly very unlike that on Earth.

2.9. Hubble Image of Heavily Tilted Uranus

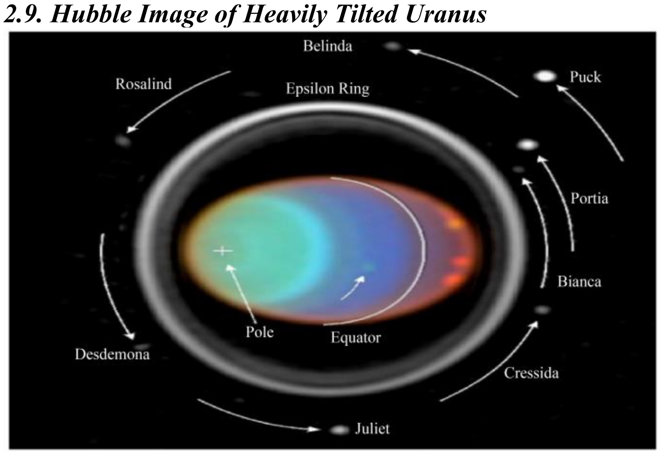

Fig. 12 Hubble image of heavily tilted Uranus

Figure 12 shows the rings and moons lying within Laplace Plane Transition orbit $r_L = 1.4\times10^6$Km are constrained by the oblate Uranus into its equatorial plane.

The moons Desdemona (62,660 Km), Juliet (64,360Km), Cressida (61,770Km), Bianca (59.160Km), Portia (66,200 Km), Puck (86,000Km), Belinda (75,260 Km) and Rosalind (69,930 Km) orbiting in the near-vertical equatorial plane of heavily tilted Uranus are seen in Figure 10. Epsilon Ring has a radius of 50,000Km.

2.10. Irregular Satellites of Uranus

Sycorx($12{,}213\times10^6$Km) and Caliban(7.169×10^6Km) are both the most outlying moons in Uranus, which has Hill Radius($70{,}129.4\times10^6$Km) and Laplace Plane Transition orbit r_L(1.4×10^6Km).

Hence both these outlying moons are in stable orbit around Uranus as they are deep inside the Hill radius.

But both these moons are beyond the Laplace Plane Transition; hence, their orbits are strongly dominated by solar perturbation, and they are constrained to remain nearer to the heliocentric plane, namely the ecliptic plane.

2.11. Neptune-Triton

Goldreich et al. (1989)[36] give a more detailed picture of Triton capture.

It was a collisional capture with a regular moon of Neptune, which resulted in a highly eccentric orbit.

Eccentricity resulted in tidal dissipation in Triton which resulted in circularization of Triton orbit.

Today it is nearly circular in 1 billion years. Triton was molten during tidal evolution, and it cannibalized the regular satellites and perturbed Nereid.

This perturbation caused a highly eccentric orbit (0.758) and highly inclined orbit (27.6°) of Nereid. The regular satellites within 5RN survived and were constrained to inclined orbits.

2.12. Irregular Satellites of Neptune

In the outer part of the Neptunian system, there is a population of satellites with various processes of origin. These are irregular satellites and are characterized by wide orbits, large inclinations with respect to the equatorial plane of Neptune, large eccentricities and long orbital periods. Most probably, Neptune did not capture this large number of small satellites through gas-drag capture mechanisms[6,7,8].

These irregular satellites may have been acquired through the following mechanisms: Collisional capture[9];

Chaos-assisted capture from low energy orbit[10] Or various binary capture scenarios [11,12,13.14,15].

2.13. Pluto

New Horizon has shown that Pluto resembles Titan in terms of landscape.

As already discussed its four small moon system is very strange and inexplicable.

2.14. Kozai Resonance

Kozai resonance has proven to be an important orbit-altering mechanism that can bring an outer satellite within the inner part of a Planet's Hill radius (RH). Alternatively it can take an irregular satellite outside the Hill sphere and make it free of planetocentric orbit. As a direct consequence of Kozai resonance, very few satellites have orbits beyond Neptunian Laplace Plane Transition (rL = 1.8×106Km) at an inclination with respect to the Ecliptic between 50° and 140°. The orbital configuration of the massive inner satellites directly influences the size of the Kozai resonance zone in the Hill sphere and is known as a given Planet.

The Trojans in Planet's orbital path and co-orbital with the respective planet. Sun, planet and satellite form the Centrally Restricted Three Body Problem, which has five fixed point solutions, namely L1, L2, L3, L4 and L5. The satellites trapped at L4 and L5 are co-orbital with the respective planet and are known as Trojans. Practically all eight planets have their respective Trojans.

Planet, its respective natural satellite and a test particle also constitute a CRTBP and it has its corresponding 5 Lagrange Points.

The table containing the Trojans of various CRTBP systems is given in Appendix VII.

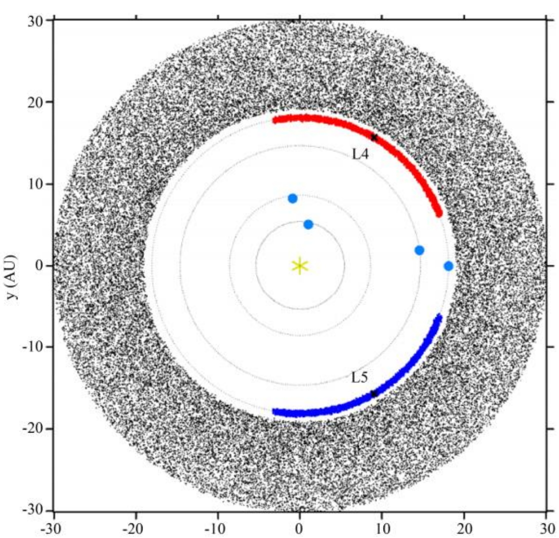

Fig. 13 Trojans of Neptune

In Figure 13, the orbital paths of Jupiter, Saturn, Uranus and Neptune are shown, and Planets are represented by green spheres. In the orbital path of Neptune, the outermost orbit, the Trojans are shown. There are 13 Trojans at L4, and 13 Trojans are at L5.

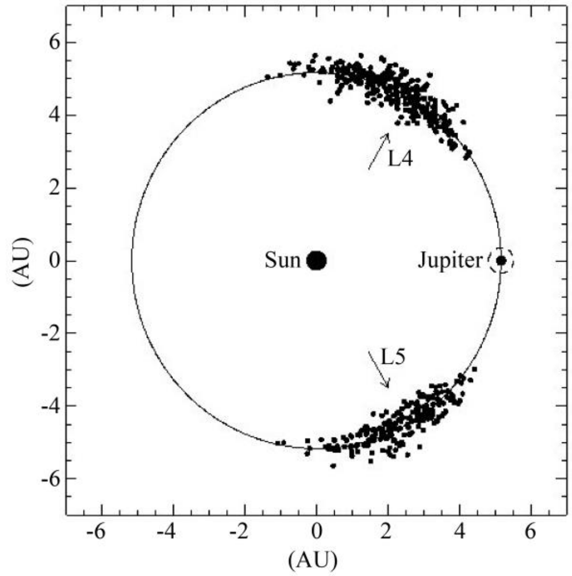

Fig. 14 Trojans of Jupiter

3. Conclusion

The moons and ring systems of the 8 planets and dwarf planets have a mathematical basis for their orbital configurations, but much remains to be understood. Here, we have given some broad principles.

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Appendix I

Table of Data-set of Earth, Jupiter, Saturn, Uranus and Neptune

Planets	Earth	Jupiter	Saturn	Uranus	Neptune	Comments
Mass($\times 10^{24}$ Kg)	5.9723	1898.19	568.34	86.813	102.413	
Radius(Km)	6371	69911	58232	25362	24622	
†J2 ($\times 10^{-6}$)	1082.63	14736	16298	3343.43	3411	
‘a’($\times 10^{6}$ Km)	149.6	778.57	1433.53	2872.46	4495.06	
R_H($\times 10^{6}$ Km)	0.01AU= 1.496280	53.1531	65.4727	70.1294	115.959	respective Hill Radius
‘r_L’($\times 10^{6}$ Km)	0.0615555	2.3	2.5	1.4	1.8	
‘r_L’($\times R_{Planet}$)	$10R_E$ ($17R_E$)*	$33R_J$	$42.5R_S$	$53R_U$	$73.45R_N$	.
Laplace Plane Transition becomes particularly important for planets having high obliquity, namely Earth(23.44°), Mars(25.19°), Saturn(26.73°), Uranus(98°), Neptune(28.31°) & Pluto(122.53°).						

Appendix II

Table of Jupiter Satellites

Jup.Sat.	Ganymede (Galilean)	Callisto (Galilean)	Io (Galilean)	Europa (Galilean)
Mass(Kg)	1.48×10^{23}	1.08×10^{23}	8.94×10^{22}	4.8×10^{22}
Rad.(Km)	2631	2400	1815	1569
(ρ) (gm/cc)	1.94	1.86	3.55	3.01
‘a’($\times 10^{6}$) Km	1.07	1.883	0.4216	0.6709
P_{spin}(d)	7.154553	16.68902	1.769138	3.551181
P_{orbit}(d)	7.154553	16.68902	1.769138	3.551181

Appendix III

Table of Saturinian Satellites

Saturn Satellites	Titan	Rhea	Iapetus	Dione	Tethys	Enceladus	Mimas
Mass (Kg)	1.35×10^{23}	2.49×10^{21}	1.88×10^{21}	1.05×10^{21}	7.55×10^{20}	8.40×10^{19}	3.8×10^{19}
Rad. (Km)	2575	765	730	560	530	250	196
(ρ) (gm/cc)	1.88	1.33	1.21	1.43	1.21	1.24	1.17
‘a’($\times 10^6$) Km	1.221850	0.527040	3.5613	0.3774	0.29466	0.23802	0.18552
P_{spin} (d)	15.94542	4.517500	79.33018	2.736915	1.887802	1.370218	0.942422
P_{orbit} (d)	15.94542	4.517500	79.33018	2.736915	1.887802	1.370218	0.942422

Appendix IV

Table of Uranus Satellites

Uranus Satellites	Titania	Oberon	Umbriel	Ariel	Miranda	Puck
Mass(Kg)	3.49×10^{21}	3.03×10^{21}	1.27×10^{21}	1.27×10^{21}	6.33×10^{19}	-
Rad. (Km)	788.9	761.4	584.7	528.9	235.8	77
(ρ) (gm/cc)	1.7	1.64	1.52	1.56	1.15	-
‘a’($\times 10^6$) Km	4.35840	5.826	2.6597	1.9124	1.2978	0.86
P_{spin} (d)	8.705892	13.46324	4.144177	2.520379	1.413479	-
P_{orbit} (d)	8.705892	13.46324	4.144177	2.520379	1.413479	0.761832

Appendix V

Table of Neptunian Satellites

Neptunian Satellites	Triton	Proteus	Nereid	Larrisa	Galatea	Despina
Mass (Kg)	2.14×10^{22}					
Rad. (Km)	1350	200	170	104×89	79	74
(ρ) (gm/cc)	2.07					
‘a’($\times 10^6$) Km	0.3548	0.1176	5.5134	0.0736	0.062	0.0525
P_{spin} (d)	-5.87685					
P_{orbit} (d)	-5.87685	1.122315	360.1362	0.554654	0.428745	0.334655

Appendix VI

Table of Plutonian Satellites

Plutonian Satellites	Pluto	Charon	Styx	Nix	Kerberos	Hydra
Mass(Kg)	1.27×10^{22}	1.9×10^{21}	-	-	-	-
Rad. (Km)	1137	586	5	20	6	20
		1 spin per orbit	6.22 spin per orbit	13.6 spin Per orbit	6.04 spin Per orbit	88.9 spin Per orbit
‘a’($\times 10^6$) Km	5913.52 from Sun	0.019640 From Pluto	0.042656 From Pluto	0.048694 From Pluto	0.057783 From Pluto	0.064783 From Pluto
P_{spin} (d)	6.38725	6.38725	3.239	1.829	5.33	0.4295
P_{orbit} (d)	248.54y Around Sun	6.38725	20.162	24.85	32.168	38.202

Appendix VII

Table of the Trojans

Primary-Secondary	L4	L5	Comments
Sun-Earth	Asteroid2010TK7	Asteroid2010S016	Trojans of Earth in Earth’s orbit
Earth-Moon	Kordylewski cloud		Trojan of Moon in Moon’s orbit
Sun-Jupiter	Dozen asteroids	Dozen asteroids	Trojans of Jupiter in Jupiter’s orbit.
Saturn-Tethys	Telesto	Calypso	Tethys, leading Telesto and lagging Calypso all three are co-orbital, synchronous, orbital period 1.88d
Saturn-Dione’s	Helene	Polydeuces	Dione’s, Helene and Polydeuces are co-orbital
Sun-Neptune	13Trojans	13 trojans	
Sun-Uranus	2011QF99		
Sun-Mars	7 Trojans	7 Trojans	