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Material Implication, Paradox of Material Implication and its Criticism

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Abstract *- At first, George Boole introduced conditional propositions as the basis for the better development of propositional logic. A compound statement composed of two atomic statements (antecedent and consequent) by the connective 'if—then' is called propositional logic. For example: "If it is raining, then the roads are wet". Here the first statement of this conditional proposition, 'It is raining,' is called the antecedent, and the second, 'the roads are wet,' is called the consequent. The material conditional in which conditional is false only when the antecedent is true and the consequent is false, but it is always true in all other cases. So, a definition of a material implication is – a conditional proposition is true when either the antecedent is false, or the consequent is true. The conditional is false when the antecedent is true and the consequent is false. Whereas, if there is no relation between the antecedent and the consequent, then the truth-functional definition of material implication leads to a paradox called the paradox of material implication. Basically, the twin paradox arises here. For example, the conditionals 'if the earth is a star, then Socrates is a philosopher and 'if the earth is a star, then Socrates is not a philosopher are both true because the antecedent is false. Secondly, the conditionals 'if Islamabad is the capital of Pakistan, then New Delhi is the capital of India' and 'if Islamabad is not the capital of Pakistan, then New Delhi is the capital of India' are both true because the consequent is true. C.I. Lewis clearly establishes these paradoxes in 1912 '(I) a false proposition implies any proposition; and (II) a true proposition is implied by any proposition'.*

Keywords *- Conditional statement, Truth value, Truth function, Material implication, Implier, Implicans, Paradox of material implication.*

1. Introduction

Before we discuss the truth-functional connective material implication, we have to clarify the idea about the conditional statement. In ordinary discourse, we have different types of conditional statements. From the term `conditional', it is clear that something is stated with some condition in a conditional statement. For example, addressing the participant of a meeting, a person tells that Mr. X will participate in the meeting if we give him the presidential post. Here, it is clear that the participation of Mr. X at the meeting is conditioned by the situation to give him the presidential post. That is, to give him a presidential post implies Mr. X's participation at the meeting. From this example, what is clear about a conditional statement is that it consists of two statements, One: in which the condition is stated; Two: the statement conditioned by the first. The first is called the implier statement, and the second is implied. So a conditional statement is also called a hypothetical or an implication.

2. Conditional Statements

A conditional statement asserts that its consequent is implied by its antecedent. It does not assert that its antecedent is true. But when its antecedent is true, its consequent must be true. Otherwise, the statement is regarded as false. Only the important thing to note about a conditional statement is that the relation of implication is asserted between its antecedent and consequent.

In ordinary discourse, we find different types of conditional statements in which the notion of implication differs from each other. First, we may speak of an implication that holds a logical relationship. For example, to make a decision about someone's mortality, we usually argue that if all men are mortal and Mr. X is a man, then Mr. X is also mortal. This argument shows that 'if' is followed by the first statement, ' all men are mortal' and 'Mr. X is a man' and the word 'then' is followed by the third statement ', Mr. X is mortal'. This third statement follows logically from the first two. So the relation of the implication between the first two and the third is logical. Second, we speak of a definition of implication where the implied statement follows from the implier as a definition. For example, if Mr. X is a human being, then Mr. X is rational. Philosophers defined the term 'man' as a rational animal. So rationality and animality are a definition of the human being. Third, we use the implicative relationship between cause and effect to state the causal relationship. So we say, if we cut off the head of my finger, then blood will come out. And finally, we get an implicative relationship that depends on our decisions. For example, a supporter of a particular football team may say that if my team loses the game, I shall cut down my ear. Here, we defined that the statement reports a decision of the speaker to behave in a certain way under certain circumstances.

3. If—then (Material Conditions)

What we find about the four types of a conditional statements is that they differ from each other in meaning. But, have they any identifiable common meaning, any partial meaning common to these admittedly different types of implication? Seemingly there is no such common meaning among them. All four types of conditional statements are true under different conditions. So, we may consider in what condition(s) they are being false. We shall see that all these four types of conditional statements are false if their consequent is false with the true antecedent. That is, any conditional statement 'if p then q' is known to be false in case the conjunction 'p.~ q' is known to be true. So a conditional statement to be true, the indicated conjunction must be false. Its negations ~ (p . ~ q) must be true. It is and then is regarded as a part of the meaning of any conditional statement.

Extracting this common meaning of conditional statements from ordinary language, symbolic logic uses the symbol '⊃' (horseshoe) between p and q as an abbreviation of ~ (p. ~ q). It is the partial meaning of implication, not bearing full of imports.

The horseshoe is a truth-functional connective, and the following truth table indicates its exact significance:

p	q	~q	p . ~q	~ (p . ~ q)	p ⊃ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Here, the first two columns represent all possible truth values for the component statements p and q, and the third, fourth and fifth represent successive stages in determining the truth values of the compound statement ~ (p . ~ q) in each case. The sixth column is identical to the fifth since the formulas that head them are defined to express the same proposition. The horseshoe symbol must not be thought of as representing the meaning of 'if – then' or the relation of implication, but rather symbolizes a common partial factor of the various implications signified by the 'if – then' phrase.

We can regard the horseshoe as symbolizing a special, extremely weak kind of implication. It is expedient for us to do so since the convenient way to read 'p ⊃ q' are 'if p then q', 'p implies q' or 'p only if q'. The weak implication symbolized by '⊃' is called a material implication. Its special name indicates that it is a special concept and not to be confused with the other, more usual kinds of implications, for example, 'if Russia is a democracy, then I am a Dutchman'. It is clear that the implication expressed here is neither logical nor definitional. No 'real connection' is alleged to hold between what the antecedent states and what is stated by the consequent. This conditional is ordinarily intended as an emphatic or humongous method of denying the truth of its antecedent, for it typically contains a notoriously or ridiculously false statement as its consequent. Any such assertion about truth values is adequately symbolized using the truth-functional connective '⊃'.

4. Different Structure of Conditional Statements

Conditional statements can be expressed in various ways. A statement of the form 'if p then q' could equally well be expressed as 'if p, q', 'q if p', 'that p implies that q', 'that p entails that q', 'p only if q', 'that p is a sufficient condition that q' or as 'that q is a necessary condition that p' and any of these formulations will be symbolized a 'p ⊃ q'.

It is easily proved that 'p ⊃ (q ⊃ p)' and '~ p ⊃ (p ⊃q)' is tautologies. They seem rather strange when expressed in English as 'A true statement is implied by any statement whatever' and as 'A false statement is implies any statement whatever'. They have been called some writers the paradoxes of material implication. When it is kept in mind that the horseshoe symbol is a truth-functional connective that stands for material implication rather than either 'implication in general' or more usual kinds of implications such as logical or casual, then the tautologous statement forms in question are not at all surprising. And when the misleading English formulations are corrected by inserting the word 'materially' before 'implied' and 'implies', the air of paradox vanishes. Material implication is a special, technical concept that the logician introduces and uses because it simplifies his task of discriminating valid from invalid arguments.

5. The paradox of Material Implication

We shall now enter the material implication problem. In logic, we find that the statement of the form 'if p then q' sometimes reaches paradoxically. In his book 'Logic' (part-I), Johnson discusses the material implication of paradoxes, and consequently, he tries to remove this paradox from logic.

Johnson uses the name 'implicative' for what is generally called hypothetical or conditional proposition -- proposition of the form 'if p then q'. Again, he calls the antecedent of the implicative 'implicans' and the consequent of the implicative 'implicate'. Here it should also be noted that Johnson uses the name 'implication' in the sense of 'material implication'.

An implicative proposition is false if and only if its antecedent is true and the consequent is false; otherwise, the implicative is true. It means that:

1) If p is false, then
If p, then q
If p, then ~ q

both propositions are true. Whatever the proposition q is, 'if p then q' is true if p is false.

And. II) If q is true, then
If p then q
If ~ p then q

both propositions are true. Whatever the proposition p is, 'if p then q' is true if q is true.

Johnson says these two consequences of the uncritical acceptance of the traditional formulas are often expressed paradoxically thus:

I) A false proposition implies any proposition.
And
II) Any proposition implies a true proposition.

What is said in (I) and (II) are sometimes called paradoxes of material implication.

Johnson thinks that these paradoxes of material implication are not due to any unnatural use of the term 'implication'. They follow it accepted logical rules. These logical rules permit us to derive a less determinate proposition from a more determinate one. For example, we may show the following derivations.

Suppose it is asserted that p is false.

So,
1. ~ p
2. ~ p v q 1, Add.
3. p ⊃ q 2, Def of 'or'

Now, if it is asserted that q is true, then we have the following valid derivations:

1. q
2. q v ~ p 1, Add.
3. ~ p v q 2, Com.
4. p ⊃ q 3, Def of 'or'

And,

1. q
2. q v p 1, Add.
3. p v q 2, Com.
4. ~ ~ p v q 3, D.N.
5. ~ p ⊃ q 4, Def of ‘or’

These derivations clearly show how the paradoxes of implication logically follow from accepted logical rules. So Johnson says the implicative propositions that are reached thus paradoxically may be claimed to be true on the strength of the falsity of the implicans or the truth of the implicate. They are not peculiar logic, nor are they due to any arbitrary use of the terms on the logician's part. So, we may validly say, "If 2+3=7, then it will rain tomorrow", and "it will rain tomorrow" would imply that "2+3=5".

6. Conclusion

Johnson tries to remove this paradox in logic. The ordinary purpose for which are implicative proposition is put inference. So much so that most people would hesitate to assert the relation expressed in an implication proposition unless they were prepared to use it for inference. That is, implicative is ordinarily tantamount to potential inference. Now, Johnson says, if an implicative is reached paradoxically, it cannot be used in further inference. If we try to use it in a further inference, either fallacy of circularity or the fallacy of contradiction arises. Only the implicative, whose implicans and implicative are entertained hypothetically, can be used in the process of further inference. Now, if the proposition 'if p then q' is inferred from the affirmation of 'q', it cannot be used in a further inference for its consequent or implicate is adopted assertorically. Suppose we proceed from this implicative proposition conjoined with p to infer q. In that case, it involves the fallacy of circularity, for the first premise is inferred from the conclusion. Again, if we proceed from this implicative proposition conjoined with the denial of q to infer the denial of p, the fallacy of contradiction causes for, in the second premiss, we have asserted the denial of q, whose affirmation has already been asserted in the first premiss.

Thus we may give a list of invalid inferences, where in each case, either the fallacy of circularity or the fallacy of contradiction arises if a paradoxically reached implicative is used as a premiss. In each of these inferences, we shall use the notation 'H' to mention 'Hypothetically entertained' and '|-' to mention 'Categorical asserted'.

The list is as follows:

I)	1. If	p then |-	q H	(Suppose this first premiss is inferred from the denial of p)
	2.	p |-		
∴	3.	q |-		

This involves the fallacy of contradiction.

II) 1. If p then q (Suppose this first premiss is inferred from the denial of p)
|- H
2. ~ q
|-
∴ 3. ~ p
|-

This involves the fallacy of circularity.

III) 1. If p then ~ q (Suppose this first premiss is inferred from the denial of p)
|- H
2. p
|-
∴ 3. ~ q
|-

This involves the fallacy of contradiction.

IV) 1. If p then ~ q (Suppose this first premiss is inferred from the denial of p)
|- H
2. q
|-
∴ 3. ~ p
|-

This involves the fallacy of circularity.

Suppose, 'if p then q' is inferred from the affirmation of q, then we have the following invalid inferences.

I) 1. If p then q
H |-
2. p
|-
∴ 3. q
|-

This involves the fallacy of circularity.

II) 1. If p then q
H |-
2. ~ q
|-
∴ 3. ~ p
|-

This involves the fallacy of contradiction.

Suppose, 'if ~ p then q' is inferred from the affirmation of q, then we have the following invalid inferences.

I) 1. If ~ p then q
H |-
2. ~ p
|-
∴ 3. q
|-

This involves the fallacy of circularity.

II) 1. If ~ p then q
H |-
2. ~ q
|-
∴ 3. p
|-

This involves the fallacy of contradiction.

So, the Johnson solution to the paradoxes of implication is found in the consideration that though an implicative may be rightly inferred from the denial of its implications or the affirmation of its implicate, it cannot be used in a further inference without committing a logical fallacy, either of contradiction or circularity.

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