Evaluation of Various Reliability Measures of Three Unit Standby System Consisting of One Standby Unit and One Generator

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Abstract

The paper presents reliability measures of compressor standby system which consist of three compressor units and one generator. Initially two compressor units are in functioning state while third compressor unit and generator are in standby state. For system operation working of at least two compressor units is must. System can interrupt due to failure of compressor unit and halt in electricity .These constraints can be overcome by using standby compressor unit and generator. For practical utility many years real failure and repair data has been collected from Verka milk plant to evaluate various measures of reliability effectiveness such as MTSF, availability ,busy period and profit. All these measures have been computed numerically as well as graphically by using semi Markov process and regenerative point technique.

Keywords — *Regenerative technique ,semi-Markov process, refrigeration system*

INTRODUCTION

In field of reliability , standby units are commonly used for uninterrupted functioning of system to increase its efficiency .Many researchers [1-7] have contributed a lot through their research by keeping in their mind the role of standby unit in system functioning .There are many practical situations available in real life where system functioning can be interrupted due to failure in the operating unit or by other factors needed to keep system in functioning state such as continuous electricity supply . Going on these lines in this paper we have considered unit failure and electricity cut as major issues of the concern in proper functioning of the system.

The present study is the sincere effort of evaluating reliability measures of compressor standby system which consist of three compressor units and one generator. Initially two compressor units are in functioning state while third compressor unit and generator are in standby state. For system operation working of at least two compressor units is must .

It has also been assumed that any recent failure in the compressor unit will be given priority over the previously failed compressor unit. System functioning can be interrupted due to failure of compressor unit and halt in electricity .These interruptions can be overcome with the help of standby compressor unit and generator respectively . For practical utility many years real failure and repair data has been collected from Verka milk plant to evaluate various measures of reliability effectiveness such as MTSF, availability ,busy period and profit. All these measures have been computed numerically as well as graphically by using semi Markov process and regenerative point technique.

Notations

λ_1	Constant failure rate of compressor unit 1
λ_2	Constant failure rate of compressor unit 2
λ_3	Constant failure rate of compressor unit 3
λ	Constant failure rate of electricity
Si	State number i, i=0,1,2,3,4,5,6,7,8,9,10
ß	Operating rate of generator
$G_1(t)$, $g_1(t)$	c.d.f and p.d.f of the repair time of compressor unit 1
$G_2(t)$, $g_2(t)$	c.d.f and p.d.f of the repair time of compressor unit 2
$G_3(t)$, $g_3(t)$	c.d.f and p.d.f of the repair time of compressor unit 3
O_I ,O_{II},O_{III}	Compressor units 1 , 2 and 3 are in operating state
S_{III}	Compressor unit 3 is in standby state
Fr_I,Fr_{II} Fr_{III}	Compressor units 1,2 and 3 are under repair respectively
F_{RI},F_{RII}	Compressor unit 1 and 2 are under repair from previous state respectively
Fwr_I ,Fwr_{II}	Compressor unit 1 and 2 are waiting for repair respectively
G_w	Generator is in operating state
G	Generator is in standby state

Model Description and Assumptions

1) All the random variables are independent.
2) Failure times are assumed to have exponential distribution whereas repair times have general distribution.
3) The system has single repairman facility.
4) The repairman comes immediately as soon as unit fails.

5) After each repair , the system works as good as new one .

Transition Probabilities and Mean Sojourn Times

All the 10 states are regenerative states in the Fig 1 are regenerative states.States S_3,S_5 , S_6 andS_8 are down states, state S_4 and S_7 are halt states. The non zero elements p_{ij} can be represent as below

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2}, p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2}$$

$$p_{10} = g_1^*(\lambda+\lambda_2+\lambda_3), p_{20} = g_2^*(\lambda+\lambda_1+\lambda_3)$$

$$p_{13} = \frac{\lambda_2(1-g_1^*(\lambda+\lambda_2+\lambda_3))}{\lambda+\lambda_2+\lambda_3}, p_{14} = \frac{\lambda(1-g_1^*(\lambda+\lambda_2+\lambda_3))}{\lambda+\lambda_2+\lambda_3}$$

$$p_{15} = \frac{\lambda_3(1-g_1^*(\lambda+\lambda_2+\lambda_3))}{\lambda+\lambda_2+\lambda_3}, p_{26} = \frac{\lambda_1(1-g_2^*(\lambda+\lambda_1+\lambda_3))}{\lambda+\lambda_1+\lambda_3}$$

$$p_{27} = \frac{\lambda(1-g_2^*(\lambda+\lambda_1+\lambda_3))}{\lambda+\lambda_1+\lambda_3}, p_{28} = \frac{\lambda_3(1-g_2^*(\lambda+\lambda_1+\lambda_3))}{\lambda+\lambda_1+\lambda_3}$$

$$p_{90} = g_1^*(\lambda_2+\lambda_3), p_{93} = \frac{\lambda_2}{\lambda_2+\lambda_3}(1-g_1^*(\lambda_2+\lambda_3))$$

$$p_{95} = \frac{\lambda_3}{\lambda_2+\lambda_3}(1-g_1^*(\lambda_2+\lambda_3)), p_{10,0} = g_2^*(\lambda_1+\lambda_3)$$

$$p_{10,6} = \frac{\lambda_1}{\lambda_1+\lambda_3}(1-g_2^*(\lambda_1+\lambda_3)), p_{10,8} = \frac{\lambda_3}{\lambda_1+\lambda_3}(1-g_2^*(\lambda_1+\lambda_3))$$

By above transition probabilities

$$p_{01}+p_{02}=1, p_{10}+p_{13}+p_{14}+p_{15}=1$$
$$p_{20}+p_{26}+p_{27}+p_{28}=1$$
$$p_{90}+p_{93}+p_{95}=1, p_{100}+p_{106}+p_{108}=1$$
$$p_{31}=1, p_{51}=1, p_{62}=1, p_{82}=1, p_{49}=1, p_{710}=1$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2}, \mu_1 = \frac{(1-g_1^*(\lambda+\lambda_2+\lambda_3))}{\lambda+\lambda_2+\lambda_3}$$

$$\mu_2 = \frac{(1-g_2^*(\lambda+\lambda_1+\lambda_3))}{\lambda+\lambda_1+\lambda_3}, \mu_3 = \int_0^\infty \bar{G}_2(t)dt = k_1$$

$$\mu_6 = \int_0^\infty \bar{G}_1(t)dt = k_4, \mu_9 = \frac{1-g_1^*(\lambda_2+\lambda_3)}{\lambda_2+\lambda_3}$$

$$\mu_{10} = \frac{1-g_2^*(\lambda_1+\lambda_3)}{\lambda_1+\lambda_3}, \mu_4 = \frac{1}{\beta} = \mu_7 = k_2$$

$$\mu_5 = \int_0^\infty \bar{G}_3(t)dt = \mu_8 = k_3$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$$

$$m_{01}+m_{02} = \mu_0, m_{10}+m_{13}+m_{14}+m_{15} = \mu_1,$$
$$m_{90}+m_{93}+m_{95} = \mu_9, m_{10,0}+m_{10,6}+m_{10,8} = \mu_{10}, m_{62} = k_4$$
$$m_{7,10} = k_2, m_{82} = k_3, m_{49} = k_2, m_{51} = k_3, m_{31} = \mu_3, m_{49} = k_2$$
$$m_{20}+m_{26}+m_{27}+m_{28} = \mu_2,$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of given system , we regard the failed states of the system as absorbing states and the mean time to system failure (MTSF) when the system starts from state S_0 is

$$MTSF = T_0 = \lim_{S\to 0} \frac{1-\phi_0^{**}(s)}{s}$$

Using L ' Hospital rule and putting the value of ø_0^{**}(s) we have

$$T_0 = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 p_{01}$$
$$D = 1 - p_{10}p_{01} - p_{20}p_{02}$$

Availability Analysis

Using the arguments of the theory of regenerative processes In steady state availability of the system is

$$A_0 = \lim_{S\to 0}(sA_0^*(s)) = \frac{N_1}{D_1}$$

where

$$\begin{aligned}
N_1 = & -p_{02}(-1+p_{13}p_{31}+p_{15}p_{51})(\mu_2+\mu_{10}p_{27}p_{710}) - \mu_1 p_{01} \\
& (-1+p_{26}p_{62}+p_{10,6}p_{27}p_{62}p_{7,10}+p_{28}p_{82}+p_{10,8}p_{27}p_{82}p_{710}) \\
& +\mu_0(-1+p_{26}p_{62}+p_{10,6}p_{27}p_{62}p_{7,10}+p_{28}p_{82}+p_{10,8}p_{27}p_{82}p_{710}) \\
& (-1+p_{13}p_{31}+p_{15}p_{51}+p_{14}p_{31}p_{49}p_{93}+p_{14}p_{51}p_{49}p_{95}) \\
& -p_{14}p_{49}(\mu_9 p_{01}(-1+p_{26}p_{62}+p_{10,6}p_{27}p_{62}p_{710}+p_{28}p_{82} \\
& +p_{10,8}p_{27}p_{82}p_{7,10}) + p_{02}(\mu_2+\mu_{10}p_{27}p_{710})(p_{13}p_{93}+p_{51}p_{95}))
\end{aligned}$$

$$\begin{aligned}
D_1 = & \mu_0(-1+p_{13}p_{31}+p_{15}p_{51}+p_{14}p_{31}p_{49}p_{93}+p_{14}p_{51}p_{49}p_{95}) \\
& (-1+p_{26}p_{62}+p_{10,6}p_{27}p_{62}p_{710}+p_{28}p_{82}+p_{10,8}p_{27}p_{82}p_{7,10}) \\
& +(-1+p_{26}p_{62}+p_{10,6}p_{27}p_{62}p_{710}+p_{28}p_{82}+p_{10,D8}p_{27}p_{82}p_{710}) \\
& (-\mu_1 - p_{13}\mu_3 - p_{15}k_3 - p_{14}\mu_3 p_{49}p_{93} - p_{14}p_{31}k_2 p_{93} - p_{14}p_{31}p_{49}\mu_9 \\
& -p_{14}k_2 p_{90} + p_{14}k_2 p_{90})
\end{aligned}$$

Proceeding in the similar fashion as above following measures in steady state have also been obtained

Busy period Analysis of Repairman

$$B_0 = N_2 / D_1$$

Expected Number of Visits by Repairman

$$V_0 = N_3 / D_1$$

where

$$N_2 = -p_{02}(-1 + p_{13}p_{31} + p_{15}p_{51} + p_{14}p_{31}p_{49}p_{93} + p_{14}p_{49}p_{51}p_{95})$$
$$(\mu_2 + p_{26}k_4 + p_{28}k_3 + p_{27}(k_2 + p_{10,6}k_4 + p_{10,8}k_3))$$
$$-p_{01}(-1 + p_{26}p_{62} + p_{28}p_{82} + p_{10,6}p_{27}p_{62}p_{7,10} + p_{10,8}p_{27}p_{82}p_{7,10})$$
$$(\mu_1 + p_{13}k_1 + p_{15}k_3 + p_{14})(k_2 + p_{49} + (p_{93}k_1 + p_{95}k_3 + \mu_9)))$$
$$N_3 = (1 - p_{26}p_{62} - p_{106}p_{27}p_{62}p_{710} - p_{28}p_{82} - p_{108}p_{27}p_{82}p_{710})$$
$$(1 - p_{13}p_{31} - p_{15}p_{51} - p_{14}p_{31}p_{49}p_{93} - p_{14}p_{51}p_{49}p_{95})$$

Cost Benefit Analysis

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2V_0$$

Where

C_0 = Revenue per unit up time of the system

C_1=Cost per unit time for which the repairman is busy

C_2=Cost per visit of the repairman

Particular Case

For graphical representation , let us suppose that

$$g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t}, g_3(t) = \alpha_3 e^{-\alpha_3 t}$$

Using the particular case, the following values are estimated as

$$\lambda_1 = 0.000042935, \lambda_2 = .0000477627$$
$$\lambda_3 = 0.001020147, \lambda = 0.01434491$$
$$\alpha_1 = 0.04522219, \alpha_2 = 0.003294625$$
$$\alpha_3 = 0.010416666, \beta = 0.10597367$$
$$C_0 = 22090, C_1 = 20007.71428, C_2 = 800$$

Graph Between MTSF and λ_1 (variation in λ_3) fig 2

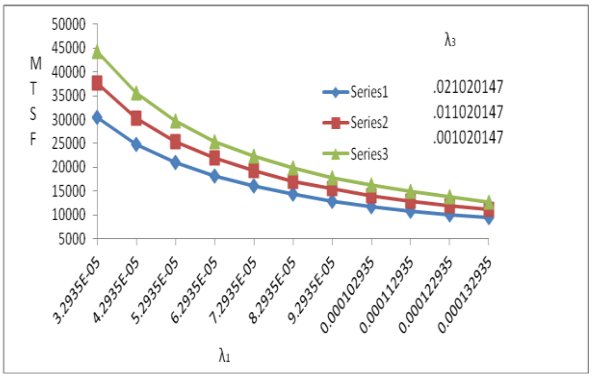
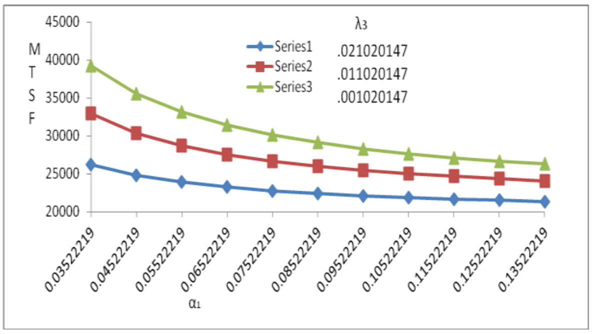


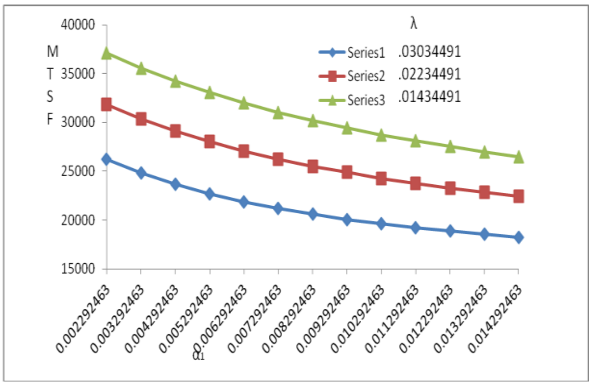
fig. 2 represents the behaviour of the Mean Time to System Failure(MTSF) with respect to the failure rate λ_1.It can be concluded from the graph that the MTSF decreases as the failure rate λ_1 and variation in λ_3 increases.

Graph Between MTSF and α_1 (variation in λ_3) fig 3



Graph in fig 3 represents the behaviour of MTSF and repair rate α_1 with variation in λ_3. It is clear that as repair rate α_1 increases MTSF decreases. As variation is taken in failure rate λ_3 for MTSF , It can be concluded as the failure rate λ_3 increases MTSF decreases.

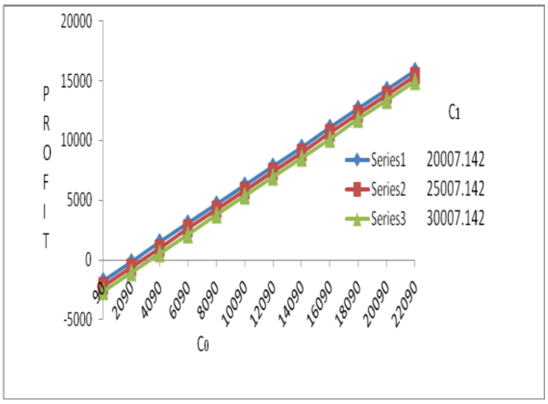
Graph Between MTSF and α_1(variation in λ) fig 4



Graph in fig 4 reveals the behaviour of MTSF and repair rate α_1 for different values of λ. It can be interpreted MTSF decreases as repair rate α_1 increases. It is clear from fig4 MTSF decreases as failure rate λ increases.

Graph Between Profit and Revenue C_0 (variation in C_1) fig 5

It can be interpreted from graph(fig 5) that profit increases with increase in values of revenue per unit up time (C_0).It can also be noticed that if C_1=20007.142 , then P>or=or<0 according as C_0 >or =or<2259 . So for C_1=20007.142, revenue per unit up time should be fixed greater than 2259.Similarly for C_1 =25007.142 and 30007.142 , the revenue per unit up time should be greater than 2824 and 3388 respectively.



CONCLUSION

Three compressors standby system in refrigeration system of verka milk plant have been studied where two units must be in operative state for functioning of system and working of standby unit depend upon the requirement of the system operation .In present paper measures of system effectiveness have been obtained by using semi Markov process and regenerative point techniques. These measures are

1. Steady state probabilities
2. Mean sojourn time
3. Mean time to system failure
4. Availability
5. Busy period of repairman
6. Expected number of visits by repairman
7. Profit analysis
8. Graphs concerning to MTSF with respect to failure rate /repair rate have been plotted
9. Profit graph has been plotted for better understanding of system behaviour. Also by using cut off points one can easily decide about the values of the various parameters for which profit is positive.

In brief , conclusion has basically drawn on the original data from Verka milk plant. Our model can be utilized by anyone using similar system by putting the values of his /her interest in the general expressions calculated by us. Profit Analysis for system has been obtained to increase uptime and reduce the cost occurred in the system.

MTSF =24798.625 hrs.

Availability =0.798864

Busy period of repairman =0.090188 hrs

Expected number of visits =0.000063 hrs

REFERENCES

[1] D.P.Gaver, “Time to failure and availability of parallel systems with repair”, IEEE Trans. Reliab., 12, 30-38,1963.

[2] R.Narang and U. Sharma , “Analysis of two unit standby oil delivering system with two types of repair facility and priority is given to partially failed unit with provision of switching over to another system”, International journal of engineering sciences & research technology , Vol. 3, No. 06, 117-123 ,June2014

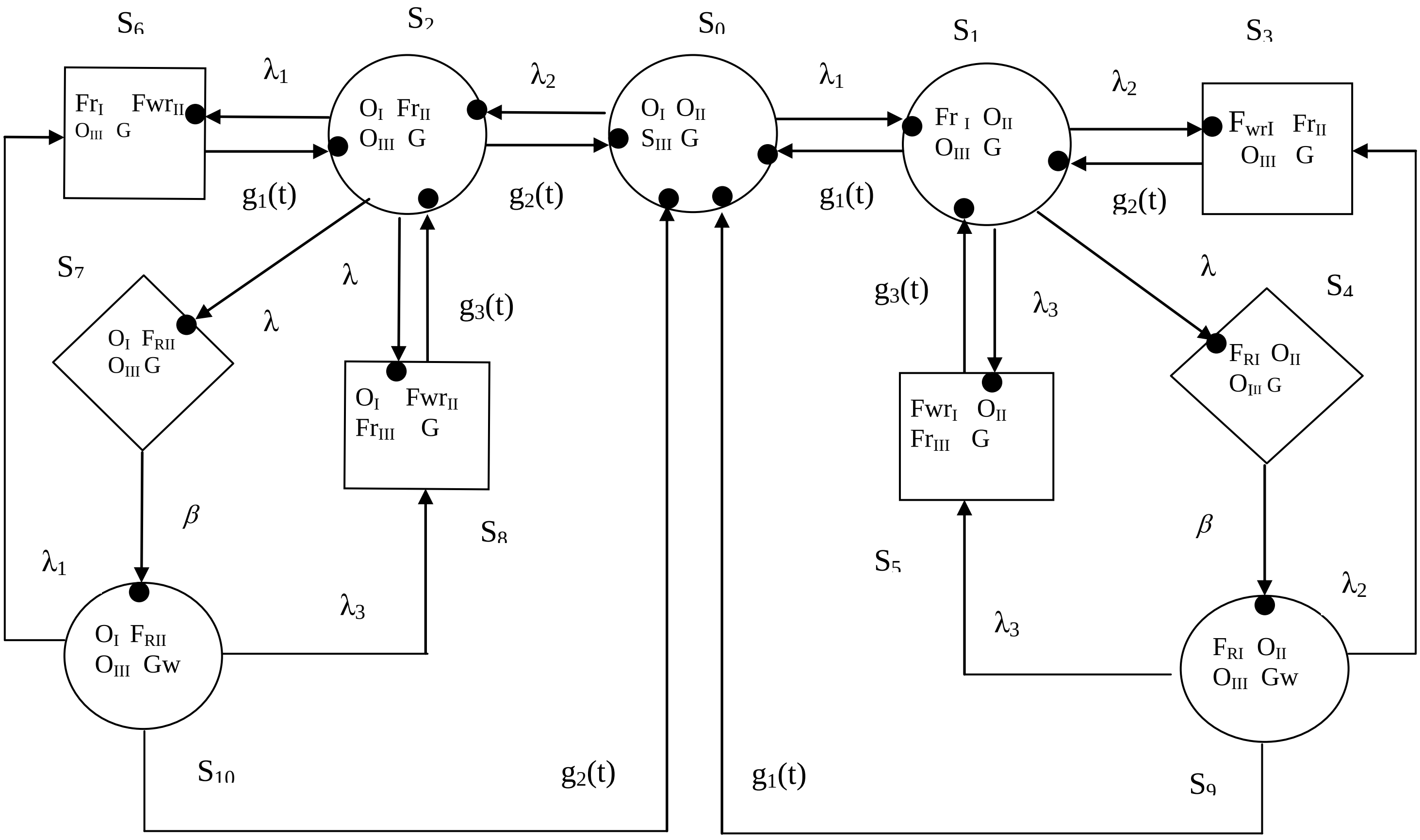
[3] M.C.Rander, A.Kumar and R.K.Tuteja, R, “A two-unit cold standby system with major and minor failures and preparation time in the case of major failure”. Microelectron. Reliab., *32*,1199-1204, 1991

[4] U.Sharma and J. kaur , “Profit evaluation of three units compressor standby system”,International journal of advanced research trends in engineering and technology, vol 3 , no 5 ,26-30, May 2016

[5] R.K. Tuteja and G.Taneja, “Profit analysis of a one-server one-unit system with partial failure subject to random inspection”, Microelectron. Reliab., *33*,319-322, 1993.

[6] M.Yamashem , “A repairable multiple state device with general repair time”, *IEEE*, Trans. Reliab., *R-29*, 276, 1980

[7] Z.Zhang ,W. Gao and Y.Zhou Y , “Reliability modelling and maintenance optimization of the diesel system in locomotives”, Maintenance and reliability , Vol 14,No 4, 302-311 , 2012



State Transition Diagram Fig 1

Where