Constrained Renewable Resource Allocation in Stochastic Metagraphs

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Abstract

In this paper, it is supposed that a project can be presented as a Metagraph. In real World's projects, activity completion time is generally indefinite. No determinism in the activity completion time can be expressed with random variables or fuzzy numbers. Here, it is assumed that the activity implementation times are continuous random variables. Also it is assumed that, one kind of renewable resource is necessary for activity implementation, but the amounts of this resource is definite and limited. In this research, the critical path computations and one of the common criteria of the constrained renewable resource allocation have been generalized. The main aim is reduction of project completion time by suitable scheduling of metagraph activities. Finally, we have estimated the cumulative distribution function of the project completion time. One example has been solved by using the developed method.

Keywords—*Stochastic Metagraph, Project completion time, Constrained renewable resource allocation, Cumulative distribution function.*

I. INTRODUCTION

In recent years, Metagraphs have been used as a tool for analysing the engineering systems. Metagraph and some of its applications have been described in [1]. Metagraph and its specifications have been described in [2],[3]. Applications of metagraph in decision support system have been shown in [4],[5]. Metagraphs are used in workflow management [6],[7],[8]. Enterprise modelling can be executed by using the Metagraphs [9]. A fuzzy project can be shown as a metagraph with fuzzy characters. Forward and backward computation and determination of critical path and critical edges in fuzzy Metagraphs have been described in [10]. Constrained resource allocation in fuzzy Metagraphs has been studied in [11]. Constrained resource allocation in fuzzy Metagraphs via Min-slack has been presented in [12]. Time cost Trade-off in uncertain Metagraphs with trapezoidal fuzzy edge time, has been studied in [13]. A new method for allocation of constrained non-renewable resource in fuzzy Metagraphs is shown in [14]. This new method decreases the computational efforts to achieve the suitable allocation of constrained non-renewable resource. By using the conditional Monte-Carlo simulation, the method is provided for calculating the completion time distribution function of stochastic Metagraphs in [15].

In this paper, the critical path computations and one of the common criteria of the constrained renewable resource allocation have been generalized for the stochastic Metagraphs. By using the generalized method, activity scheduling is defined. Finally, cumulative distribution function of project completion time is computed in due date.

II. DEFINITIONS

In this section, some basic definitions of the Metagraphs that have been cited in [12],[15] are introduced.

A. Definition 1

$X = \{x_i, j = 1,2,\ldots n\}$ is called the generating set. x_i is called the element of X. $E = \{e_j, j = 1,2,\ldots n\}$ is the set of edges. Each edge is an ordered pair as (V_j, W_j). $V_j \subset X$ is called the invertex of e_j and $W_j \subset X$ is called the outvertex of e_j such that $\forall j, V_j \bigcap W_j = \phi$. Figure1 shows a metagraph. In this metagraph, $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}, E = \{e_1, e_2, e_3\}$

$e_1 = (\{x_1\}, \{x_3, x_4\})$, $e_2 = (\{x_4, x_3\}, \{x_5, x_6\})$, $e_3 = (\{x_3, x_4\}, \{x_6\})$.
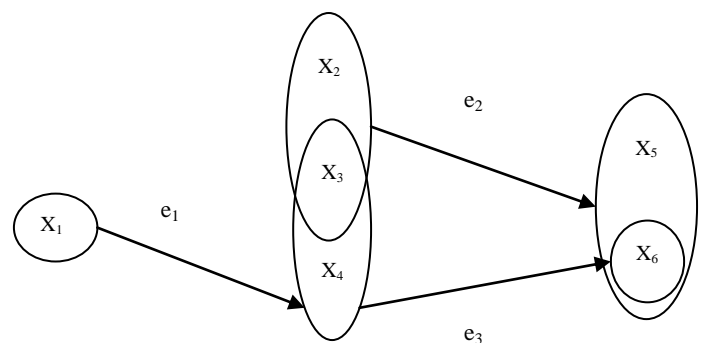

Figure 1: A Metagraph

B. Definition 2

An element $x \in X$ will be connected to element $x' \in X$ if a sequence of edge $(e_k, k = 1, 2, \ldots, K)$ exists such that $x \in V_1'$, $x' \in W_K'$ and $W_k' \cap V_{k+1}' \neq \phi$, $\forall k = 1, \ldots, k-1$ .This sequence of elements is called a simple path from x to x' and K is path length.

C. Definition 3

A Metapath is the set of edges that is shown with $M = (B, C)$. B is the source set and C is the Target set. B is the set of invertex elements which are not outvertex elements. C is the set of outvertex elements which are not invertex elements.

D. Definition 4

Matrix $M_{n \times n}$ is a matrix corresponding to a metagraph with n activity. Components of this matrix are zero or one based on following rule:

$$M_{n \times n} = \left[m_{ij} \right]_{n \times n} \quad , \; m_{ij} = \begin{cases} 1 \\ 0 \end{cases}$$

$1 \rightarrow$ If activity i is prerequisite for activity j .

$0 \rightarrow$ If activity i is not a prerequisite for activity j .

III. CRITICAL PATH METHOD IN DETERMINISTIC METAGRAPH

Suppose that the duration of edge e_j is deterministic and known. That is shown by d_j . Then, by using the following algorithm, completion time of metagraph can be calculated.

A. Forward computations

For each element $x_i \in B$ set $Q_i = 0$ and mark x_i . Let $Q_i = 0$ for all other elements. Let $E_0 = M(B, C)$ .While $E_0 \neq \phi$ for each edge e_j in E_0 ,such that all elements in the invertex of e_j are mark set $ES_j = \max\{Q_i\}, X_i \in V_j$, such that ES_j is the earliest start time of edge e_j .Then for each $x_k \in W_j$ set $Q_k = \max\{Q_k, (ES_j + d_j)\}$,and mark it. Set $E_0 = E_0 \setminus \{e_j\}$, means $E_0 \leftarrow E_0 - \{e_j\}$, repeat the above operations while $E_0 = \phi$ .Then, T^e (earliest completion time of metagraph) is obtained. Set $T^e = T^l = \max Q_i, \forall x_i \in X$.

B. Backward computations

For each element $x_i \in C$ set $L_i = T^l$ and mark it. Let $L_i = T^l$ for all other elements. While $E_0 = M(B, C) \neq \phi$ for each edge e_j in E_0 such that all elements in the outvertex of e_j are marked set $LF_j = \min_{x_i \in w_j} \{L_i\}$,such that LF_j is the latest finish time of e_j .Then, for each $x_k \in V_k$ set $L_k = \min\{L_k, (LF_j - d_j)\}$ and mark it , $x_i \in w_j$ Set $E_0 = E_0 \setminus \{e_j\}$, means $E_0 \leftarrow E_0 - \{e_j\}$,repeat the above operations while $E_0 = \phi$ .So, ordered pair (Q_i, L_i) can be obtained for each element of metagraph.

IV. STOCHASTIC METAGRAPH

Suppose that each activity time is a known continuous random variable. In this case, by generalizing the critical path method in stochastic metagraphs, forward and backward computations are done by simulation. Then by using these computations, we can obtain the average of earliest and latest start time and earliest and latest finish times and averages of them. Also, we can compute the completion time of stochastic metagraph. Details of this computation have been explained in first algorithm.

V. CONSTRAINED RENEWABLE RESOURCE ALLOCATION IN THE STOCHASTIC METAGRAPHS

In this section, allocation method in random Metagraphs is described when the resource is constrained and renewable. Suppose that the metagraph consists of n activity. The necessary duration time for accomplishment of each activity is a continuous random variable. For the implementation of each activity we need a certain amount of renewable resource. The amounts of total constrained resource already have been determined. The purpose of this allocation is to reduce project completion time and to obtain the estimation of project completion time distribution function.

VI. THE STEPS OF THE PROPOSED ALGORITHM

A. First Algorithm ($\overline{Ls}_j, \forall j = 1, \ldots, n$)

Step1: set $K = 1$, $\forall j \quad SLs_j = 0$

Step2: based on cumulative distribution function of each activity time, generate a random value.

Step3: do the forward and backward computations and compute the $\forall j \quad Ls_j$. Suppose that $Ls_j^{\,k}$ is the value of Ls_j in iteration k.

Step4: $\qquad SLs_j = SLs_j + Ls_j^{\,k}$

Step5: if $K = m$ (m is the number of simulation iterations) then, stop and go to the step 7, otherwise go to the step 6.

Step6: set $k \leftarrow k + 1$ and go to step 2.

Step7: $\qquad \overline{Ls_j} = \dfrac{\sum_{k=1}^{m} Ls_j^{\,k}}{m} = \dfrac{SLs_j}{m}$

B. Second Algorithm (Constrained Renewable Resource Allocation)

Step1: set $K = 1$

Step2: set $T^k = 0$. Suppose that the available resource is Rs. Based on cumulative distribution function of each activity time, generate a random value.

Step3: determine the matrix M.

Step4: determine the EAS (eligible activity set) by using the matrix M. This set includes activities which don't have prerequisites or their prerequisites have already been completed.

Step5: order the members of EAS ascending based on the $\overline{Ls_j}$ criteria. The ordered set is called OSS.

Step6: allocate the available resource to OSS activities respectively .In this case, all activities of OSS will be executed or some of activities of OSS will be executed if the available resource is not enough to implementation of all activities of OSS. So, the available amount of resource after this step is calculated as follows:

$$Rs \leftarrow Rs - \sum_{j \in EOSS} SL_j$$

EOSS is subset of OSS that available resource is enough for implementation of activities of this subset.

SL_j is the amount of required resource for implementation of activity e_j.

Step7: if $EOSS \neq \phi$ then, go to the step 8; otherwise go to the step 9.

Step8: $d_u = \min_{j \in EOSS} \left\{ d_j \right\}$

$\qquad T^k \leftarrow T^k + d_u$

$\qquad Rs \leftarrow Rs + SL_u$

Then go to the step 10.

Step 9: $d_u = \min_{j \in XOSS} \left\{ d_j \right\}$

$\qquad T^k \leftarrow T^k + d_u$

$\qquad Rs \leftarrow Rs + SL_u$

XOSS is the set of all running activities that belong to EOSS set.

Then go to the step 10.

Step10: update the matrix M.

Step11: update the EAS, by using the matrix M of step 10. if the EAS is empty, stop and register the metatagraph's completion time ($T^{(k)}$), then go to step 12, otherwise go back to step 5.

Step12: if $k = m$ then stop, otherwise set $k \leftarrow k + 1$

And go back to step2.

C. Third Algorithm

Step1: set $k = 1$, $z = 0$

Step2: if $T^{(k)} \leq t$, set $k \leftarrow k + 1$ and $z \leftarrow z + 1$, otherwise $k \leftarrow k + 1$.

Step3: if $k = m$ then stop and go to the step 4.

Step4: calculate $P(T \leq t) = {z}/{m}$.

VII. EXAMPLE

Assume that the Metagraph of figure 2 shows a project that consists of 5 activities. Activities duration time is a continuous random variable and related information are given in table 1.
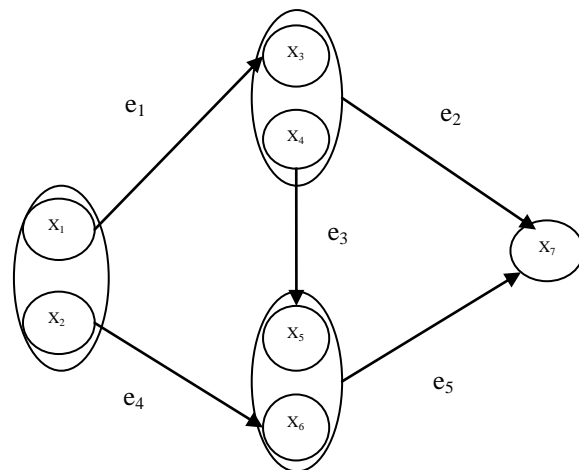

Fig2: Metagraph of Example

Table I. Information of Example

activity	d_j : random variable of activity completion time	SL_j
e_1	d$_1$:uniform distribution function $[a,b] = [4,5]$	3
e_2	d$_2$:Gamma distribution function $(\alpha, \beta) = (5,1)$	1
e_3	d$_3$: Weibull distribution	2

	function $(\lambda, \alpha) = (0.4,3)$	
e_4	d$_4$:Normal distribution function $(\mu, \sigma) = (8,1.5)$	2
e_5	d$_5$:Exponential distribution function $\lambda = 2$	4

By using the proposed algorithms, we have computed the value of cumulative distribution function of completion time of metagraph in different due dates when the constrained resource is 4. Results of computations have been in table 2.

Table2. The Result for example when the constrained Resource is Rs=4

t	$P(T \le t \mid Rs)$	t	$P(T \le t \mid Rs)$
18	0.5337	27	0.9964
19	0.6777	28	0.9983
20	0.7815	29	0.9995
21	0.8625	30	0.9998
22	0.9183	31	0.9999
23	0.9532	32	0.9999
24	0.9724	33	0.9999
25	0.9849	$t \ge 34$	1
26	0.9925		

In other case, we can compute the cumulative distribution function of completion time of metagraph for constant due date (23) and different amounts of constrained resource. Results of these computations are shown in table 3.

Table 3. The Result Calculations Of Example 1 When The Due Date Is T=23.

Rs	$P(T \le t \mid Rs)$	Rs	$P(T \le t \mid Rs)$
4	0.9532	10	1
5	0.9999	11	1
6	1	$Rs \ge 12$	
7	1		
8	1		
9	1		

$F_T(Rs,t) = P(T \le t \mid Rs)$ is the cumulative distribution function of completion time of metagraph when the due date is t and available resource is Rs .

Finally, It is obvious that we can combine the table 2 and 3 .We have shown the expanded table as table 4. Unknown values of table 4 can be computed by similar manners which were described previously.

Table 4.Result of the Calculations Of Example

Rs \ t	4	5	6	7	8	9	10	11	12
18	0.5337	?	?	?	?	?	?	?	?
19	0.6777	?	?	?	?	?	?	?	?
20	0.7815	?	?	?	?	?	?	?	?
21	0.8625	?	?	?	?	?	?	?	?
22	0.9183	?	?	?	?	?	?	?	?
23	0.9532	0.999	1	1	1	1	1	1	1
24	0.9724	?	?	?	?	?	?	?	?
25	0.9849	?	?	?	?	?	?	?	?
26	0.9925	?	?	?	?	?	?	?	?
27	0.9964	?	?	?	?	?	?	?	?
28	0.9983	?	?	?	?	?	?	?	?
29	0.9995	?	?	?	?	?	?	?	?
30	0.9998	?	?	?	?	?	?	?	?
31	0.9999	?	?	?	?	?	?	?	?
32	0.9999	?	?	?	?	?	?	?	?
33	0.9999	?	?	?	?	?	?	?	?
$t \ge 34$	1	?	?	?	?	?	?	?	?

The charts 1 and 2 demonstrate the metagraph completion time cumulative distribution function of the example 1 for the results which were shown in the tables 2 and 3.

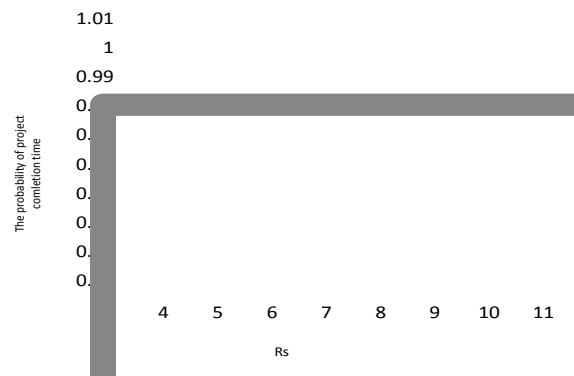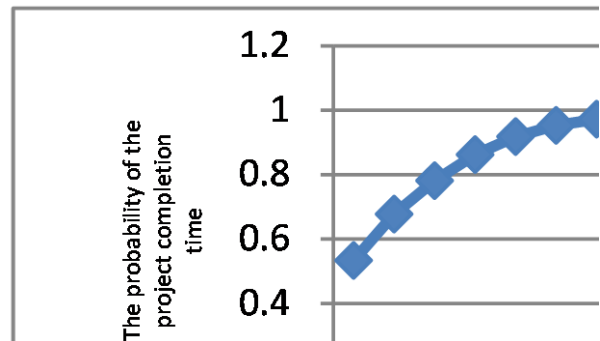**Chart1.metagraph competition time cumulative Distribution function of the example1 for Rs=4**

Chart2.metagraph competition cumulative distribution function of the example1 for t=23

VIII. CONCLUSION

This paper developed a new method for constrained renewable resource allocation in metagraphs when the activities implementation time are continuous random variables. An algorithm has been developed for this purpose, $Min\overline{LS}_j$ criteria has been generalized for this algorithm. The developed algorithm can allocate the renewable constrained resource to metagraph activities such that the delay of the project is decreased. Also, this algorithm can compute the values of cumulative distribution function of metagraph completion time for different values of due dates and constrained resources.

For future researches, the similar problem can be studied when we have two or more kinds of resources. Also, this problem may be studied when some of the resources are renewable and others are consumable.

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