Constrained Consumable Resource Allocation in Stochastic Metagraphs with Discrete Random Times

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Abstract

This paper presents a new heuristic algorithm for allocation of constrained consumable resource in stochastic metagraphs. The duration of each edge is a discrete random variable with known probability function and also depends on the amount of resource allocated to it. It is assumed that a certain type of consumable resource is needed to execute each activity of the project. The amount of resource which can be allocated to each activity is constrained to specific values. So, we must allocate the constrained resource to activities optimally. The problem is to maximize the probability of completion of stochastic metagraph before the due date of the project. Solving this problem using the analytical method is very time consuming. Therefore, we have developed a new heuristic algorithm in order to solve the problem.

Keywords: *Consumable resource, Stochastic metagraph, Constrained resource allocation, Project management, Project completion time.*

I. INTRODUCTION

Networks provide a powerful tool to model objects and relationships among them. In other words, the purpose of the network is to assist the user's intuition in understanding important relationships among data elements, aggregates, etc. One of the various network models which have attracted the attention of the researchers in recent years is metagraph. Metagraph is a new type of graphical structure which was introduced recently by Basu and Blanning [1],[2]. The concept of metagraph and some of its applications have been described in [3]. The metagraph can be used to model problems in many areas such as decision support systems [4], [5], workflow systems [6],[7],etc.

In this paper, we have considered the methagraphs as a tool for modeling and analyzing project management systems. The study of metagraph as a tool for project planning and control begins by Basu and Blanning [7].

In several studies, metagraphs are considered as tools for project planning and control. A non deterministic project with non deterministic characters has been shown as a fuzzy metagraph or stochastic metagraph. Computation of the completion time for fuzzy metagraphs has been described in [8]. Time cost trade-off in fuzzy metagraphs has been studied in [9].Various algorithms for constrained resource allocation in fuzzy metagraphs have been presented in [10]–[12]. Computation of the completion time of stochastic metagraphs has been presented in [13]. Allocation of limited resource has been performed with renewable resource [14].

Thus, in this paper, we consider a new variant of the constrained consumable resource allocation in metagraphs, where the activity duration of project is stochastic. For solving this problem, the current paper proposes a new heuristic algorithm**.** Different examples have been designed for examining the proposed algorithm.

II. BASIC DEFINITIONS

A. Generating Set

The generating set of a metagraph is the set of elements $X = \{x_1, x_2, \dots, x_n\}$, which represent variables of interest, and which occur in the edges of the metagraph [3].

B. Edge

An edge e in a metagraph is a pair $e = \langle V_e, W_e \rangle \in E$ (where E is the set of edges) consisting of an invertex $V_e \subset X$ and an outvertex $W_e \subset X$, each of which may contain any number of elements. The different elements in the invertex (outvertex) are coinputs (cooutputs) of each other [3].

C. Metagraph

A metagraph $S = \langle X, E \rangle$ is then a graphical construct specified by its generating set X and a set of edges E defined on the generating set [3].

D. Simple Path

An element $x \in X$ is connected to element $x' \in X$ if the sequence of edges $(e'_k, k = 1,2,\dots,K')$ exists such that, $x \in V'_1$, $x' \in W'_{K'}$ and $\forall k = 1,2,\dots,K'-1$, $W'_k \cap V'_{k+1} \neq \emptyset$. This sequence of edges is called a simple path from x to x'. x is called source and x' is called target. K' is called the length of simple path [10].

E. Stochastic Metagraph

A stochastic metagraph is identified with $F(X, E, T)$. $X = \{x_i, 1,2, \dots, I\}$ is called the generating set. x_i is called the element of X. $E = \{e_k, k = 1,2, \dots, N\}$ is the set of edges. Each edge is an ordered pair as (V_k, W_k). $V_k \subset X$ is called the invertex of e_k and $W_k \subset X$ is called the outvertex of e_k such that $\forall k, V_k \cap W_k = \emptyset$. It is supposed that the duration of edge e_k is a known discrete random variable and is represented by t_k such that $t_k \in T$ [13].

III. DESCRIPTION OF THE PROBLEM

Assume a non deterministic project formulated as a stochastic metagraph. The duration time of each edge is a discrete random variable with a given probability function. Probability function of duration of each activity depends on the amount of consumable resource allocated to it. Clearly, the amount of resource which can be allocated to each activity is limited to specific values. Here, the objective function is the probability of completion of stochastic metagraph before the due date of project. We assumed that the due date of the project is known and definite. It is evident that we must maximize the objective function.

We assume that:

- The metagraph of project has a source invertex and a target outvertex.
- The completion time of each edge is a discrete random variable with a given probability function.
- Activity implementation requires only a certain type of consumable (non-renewable) resource.
- Probability function of activity durations is dependent on the amount of resource allocated to the activity
- The due date of the project is a known and constant and value.
- The amount of resource allocated to each activity is limited to some specific values.
- Difference between levels of allocable resource to each activity is considered a unit.

We have used the following notations:

e_k	*k*th activity(edge), $k = 1,2, \dots, N$
N	Number of activities (edges)
t_k	Duration time random variable of *k*th activity
s_{l_k}	The amount of resource allocated to *k*th activity, $l_k = 1,2, \dots, z_k$
$f_k(s_{l_k}, t_k)$	Probability function of *k*th activity completion time, when the allocated resource to this activity is s_{l_k}
D	Due date of project
t	Completion time random variable of project
Rs	The available amount of limited resource
Z	The maximum amount of limited resource which can be allocated to project
P_r	*r*th path of the metagraph which starts from source invertex and terminates in target outvertex
P	The set of the paths of the metagraph, $P = \{P_r, r = 1,2, \dots, U\}$
W	The amount of difference between Z and Rs
$E_k(s_{l_k})$	Average of completion time of *k*th activity, when the allocated resource to that is s_{l_k}
$Q^{(i)}$	n tuple ordered of allocated resource to activities 1 to N in *i*th iteration of algorithm, $Q^{(i)} = (s_{l_1}^{(i)}, \dots, s_{l_N}^{(i)})$
M_k	Number of paths that *k*th activity lies on them
$\Delta E_k(s_{l_k}, s_{l_k - 1})$	The amount of difference between $E_k(s_{l_k - 1})$ and $E_k(s_{l_k})$
α_k	Effective time coefficient of *k*th activity

IV. PROPOSED METHOD

Step 1. Compute the Z and W.

Step 2. Set $i = 0, l^{(i)} = (l_1 = z_1, l_2 = z_2, \dots, l_N = z_N)$ and $Q^{(i)} = (s_{l_1}^{(i)}, s_{l_2}^{(i)} \dots, s_{l_N}^{(i)})$. Consider $Q^{(0)}$ as the initial allocation, so that ($\sum_{k=1}^{N} s_{l_k}^{(0)} = Z$).

Step 3. Calculate the mean of completion time for all the paths of the metagraph, when the allocated resource is $Q^{(i)}$. Select the path with minimum completion time mean. If there are two or more paths with equal completion time mean, choose the path with minimum S.D. (standard deviation).

Step 4. Calculate the α_k for all the activities of the selected path in the step 3.

Step 5. Among the activities of chosen path in step 3, select the activity with minimum α_k. If there are two or more activities with equal α_k, select the activity according to the following prioritization:

(a) Select the activity that lies on few effective paths.

(b) Select the activity with minimum $\Delta E_k(s_{l_k}, s_{l_k - 1})$.

Step 6. Choose l_k of the selected activity in step 5 and set $l_k = l_k - 1$ and $i = i + 1$. According to l_k, determine $l^{(i)}$ and $Q^{(i)}$. If $i < W$ then return to step 3 otherwise stop and go to the step 7.

Step 7. It is obvious that $Q^{(i)}$ specifies final allocation. According to $Q^{(i)}$, obtain the $P = (t \leq D \mid Rs)$ using the simulation method.

V. EXAMPLE

Consider the metagraph in Figure 1. The amount of limited resource is 17 (RS = 17) and the due date of project is 7 (D = 7). The probability function of activities depends on the resources allocated to them, and it has been shown in TABLE I.

Table I. Information Of Activity Durations Of Example

l_1	s_l	$f_1(s_{l_1}, t_1)$	l_2	s_l	$f_2(s_{l_2}, t_2)$	l_3	s_l	$f_3(s_{l_3}, t_3)$
1	3	$=\frac{1}{2}$ $t_1=1$ $=\frac{1}{2}$ $t_1=2$	1	3	$=\frac{1}{2}$ $t_2=3$ $=\frac{1}{2}$ $t_2=4$	1	2	$=\frac{1}{3}$ $t_3=3$ $=\frac{2}{3}$ $t_3=4$
2	4	$=\frac{3}{4}$ $t_1=1$ $=\frac{1}{4}$ $t_1=2$	2	4	$=\frac{1}{3}$ $t_2=2$ $=\frac{2}{3}$ $t_2=3$	2	3	$=\frac{1}{5}$ $t_3=2$ $=\frac{4}{5}$ $t_3=3$

l	s	$f_4(s_{l_4}, t_4)$			$f_5(s_{l_5}, t_5)$
1	2	$=\frac{1}{4}$ $t_4=2$ $=\frac{3}{4}$ $t_4=3$	1	4	$=\frac{1}{2}$ $t_5=1$ $=\frac{1}{2}$ $t_5=2$
2	3	$=\frac{6}{7}$ $t_4=2$ $=\frac{1}{7}$ $t_4=3$	2	5	$=\frac{2}{3}$ $t_5=1$ $=\frac{1}{3}$ $t_5=2$

Steps of our proposed method for solving the example, are as follows:

Step 1. Compute the Z and W.

Z = 4+4+3+3+5=19

W= 19-17=2

Step 2. $i = 0$, $l^{(i)} = (l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2,\ l_5 = 2)$, $Q^{(i)} = (s_{l_1}^{(i)}, s_{l_2}^{(i)}, \ldots, s_{l_N}^{(i)})$ and consider $Q^{(0)} = (4, 4, 3, 3, 5)$ as initial allocation.

Step 3. Calculate the mean of completion time for all the paths of the metagraph, when the allocated resource is $Q^{(0)}$. Choose the shortest path. The results of calculations of step 3 have been shown in TABLE II.

Step 4. The α_k of all activities of the selected path in step 3 was calculated and the results have been shown in TABLE II.

Table II. Calculations of Step 3 and 4 in 1th Iteration

Activity	Increasable paths			α_k
	e	e		
			–	–
				0
			—	—
			2	0.3333
Completion time mean of path	5.5		Selected path: $e_1 - e_2 - e_5$ Selected activity : e_5	

Step 5. According to the results gained from implementation of steps 3 and 4, path $e_1 - e_2 - e_5$ has the minimum completion time mean and activity e_1 on the this path has the minimum α_k. Thus one unit is reduced from the amount of resource allocated to this activity .

Step 6. $l_1 = l_1 - 1 = 1$ and $i = i + 1 = 1$. $l^{(1)} = (l_1 = 1, l_2 = 2, l_3 = 2,\ l_4 = 2,\ l_5 = 2,)$ and $Q^{(1)} = (3, 4, 3, 3, 5)$. Since $i < 2$ then go to step 3.

Step 3. Calculate the mean of completion time for all the paths of the metagraph, when the allocated resource is $Q^{(1)}$. Choose the shortest path. The results of calculations of step 3 have been shown in TABLE III.

Step 4. The α_k of all activities of selected path in step 3 was calculated and the results have been shown in TABLE III.

Table III. Calculations Of Steps 3 And 4 In 2th Iteration

Activity	Increasable paths		M_k	α_k
	e_1 -e_2 -e_5	e_3 –e_4 -e_5		
e_1			–	–
e_2			1	0.8333
e_3			—	—
e_4				
e_5			2	0.3333
Completion time mean of path	5.5		Selected path: $e_1 - e_2 - e_5$ Selected activity : e_5	

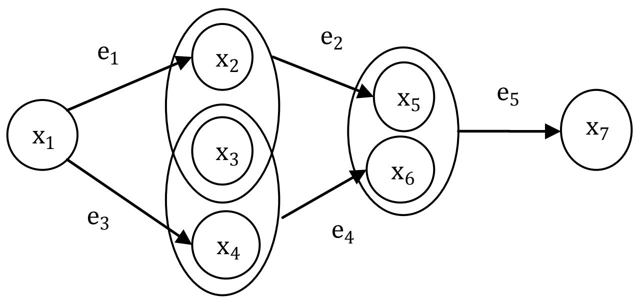

Figure 1: Metagraph of a project with 5 activities

Step 5. According to results obtaied from implementation of steps 3 and 4, path $e_1 - e_2 - e_5$ has the minimum completion time mean and activity e_5 on the this path has the minimum α_k. Thus one unit is reduced from the amount of resource allocated to this activity .

Step 6. $l_5 = l_5 - 1 = 1$ and $i = i + 1 = 2$. $l^{(2)} = (l_1 = 1, \quad l_2 = 2, l_3 = 2, l_4 = 2, l_5 = 1)$ and $Q^{(2)} = (3,4,3,3,4)$. Since $i = 2$ then stop and go to the step 7.

Step 7. The $Q^{(2)} = (3,4,3,3,4)$ is allocation of available resource to project activities that was obtained by proposed method. According to $Q^{(2)}$, the amount of $P = (t \leq 7 \mid Rs = 17)$ has been calculated by simulation method:

$$P = (t \leq 7 \mid Rs = 17) = 0.9429$$

VI. EVALUATION OF PROPOSED METHOD

In this section, some test problems are designed for evaluating the performance of the proposed method. Also, all feasible resource allocations are generated by computer and evaluated by simulation programs. The obtained solutions from both methods (new algorithm and simulation method) have been compared to each other. The results can be seen in TABLE IV , V and VI.

In some of the test problems, new heuristic method didn't achieve the optimal resource allocation, but the probability of completion time of their metagraph, is close to optimal probability of completion time of their metagraph according to the obtained resource allocation. This error was less than 0.01 in some case and in one case it was 0.2.

In order to compare simulation method and proposed algorithm, consider example 14 according to information shown in TABLE IV and VI. In this example, the number of all resource allocation states is 4096 among which, 495 states are feasible. In other words, in these feasible states we have ($S_1 + \cdots + S_{12} = 40$). For solving this example by using simulation method, 495 states should be investigated and finally, one state that maximizes the $P = (t \leq 14 \mid Rs = 40)$ should be chosen as the optimal resource allocation. However, if we want to solve this example using the proposed algorithm, the solution can be obtained by four iterations ($Z - Rs = 44 - 40 = 4$) . It is obvious that solving the above mentioned example by means of the simulation method, requires relatively long calculation time. But using the proposed algorithm, calculation time will be reduced significantly. It is evident that proposed method reduces the computational efforts significantly. TABLE VI depicts this fact.

Table IV. The Results of the Calculations of the Test Problems Using the Proposed Method

Example number	Number of activity	Rs	D	Obtained allocation by proposed algorithm	P (t ≤ D| Rs) related to obtained allocation by proposed algorithm
1	5	18	6	(3,3,3,4,5)	0.7985
2	5	17	7	(3,4,3,3,4)	0.9429
3	5	16	8	(4,3,2,4,3)	0.9001
4	6	20	7	(3,3,4,3,5,2)	0.75
5	6	23	8	(4,2,5,3,5,4)	0.5446
6	8	30	17	(6,2,1,3,5,6,3,4)	0.9587
7	8	29	15	(2,4,4,5,3,2,4,5)	0.9315
8	8	28	13	(2,4,3,4,2,6,3,4)	0.9484
9	10	38	11	(5,3,1,4,6,1,3,4,7,4)	0.8278
10	10	51	14	(5,4,3,9,9,6,1,6,3,5)	0.9267
11	12	40	15	(3,2,4,4,3,4,1,5,3,3,2,6)	0.9861
12	12	42	14	(2,3,2,4,5,5,3,5,2,3,6,2)	0.9644
13	12	43	13	(2,2,3,5,5,4,1,7,3,4,4,3)	0.5677
14	12	40	14	(3,3,4,5,5,2,4,3,3,3,2,3)	0.8709

Table V. The Results Of The Calculations Of The Test Problems Using The Simulation Method

Example number	Obtained optimal allocation by simulation method	P (t ≤ D | Rs) related to obtained allocation by simulation method
1	(4,3,3,3,5)	0.8071
2	(3,4,3,3,4)	0.9431
3	(4,3,2,4,3)	0.8999
4	(3,3,3,3,5,3)	0.8572
5	(4,2,5,2,6,4)	0.5902
6	(6,2,1,3,5,6,3,4)	0.9588
7	(3,3,4,4,3,3,4,5)	0.9796
8	(2,4,3,4,2,6,3,4)	0.9484
9	(5,3,2,4,5,2,2,4,7,4)	0.9248
10	(5,4,3,9,9,6,2,6,2,5)	0.9636
11	(3,2,4,4,3,4,1,5,3,3,2,6)	0.9862
12	(2,3,2,4,5,5,3,5,2,3,6,2)	0.9643
13	(2,2,3,5,5,4,1,7,4,4,3,3)	0.7770
14	(4,3,4,5,5,2,4,3,2,3,2,3)	0.9175

Table VI. Volume Of Calculations In Simulation Method And Proposed Method

Example Number	Total number of allocation states	Number of feasible allocation states	Z	Number of iterations of the proposed algorithm
1	32	18	21	3
2	32	10	19	2
3	32	10	18	2
4	64	15	22	2
5	64	15	25	2
6	256	56	33	3
7	256	56	32	3
8	256	8	29	1

9	1024	120	41	3
10	1024	120	54	3
11	4096	495	44	4
12	4096	495	46	4
13	4096	792	48	5
14	4096	495	44	4

VII. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this paper, we have presented a new heuristic algorithm for allocation of constrained consumable resource to edges of a stochastic metagraph with the aim of maximizing the probability of completion time of stochastic metagraph before the due date of the project. The results of test problems indicated that in most problems the proposed algorithm is successful.

In simulation method, the number of feasible allocations can be very great. Evaluation of all of these allocations requires tedious computations. The proposed algorithm allows us to achieve the acceptable or best results without having to evaluate all of the feasible solutions. Therefore, computing time is reduced significantly.

For further research, the following extensions are recommended:

1) This research can be conducted with continuous random times.
2) Allocation of limited resource to the activities can be an interesting subject to study when we have several types of limited resources (renewable and consumable).
3) This research can be extended to more than one kind of non-renewable resource.

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