New Hybrid Algorithm for Solving the Capacitated Production Planning with Stochastic Demand

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Abstract

Optimal production planning has a significant effect on product price. In this research, a production planning problem with stochastic demand has been studied. This problem has set up cost, Inventory holding cost and lost sale cost. In this problem only a certain type of product is produced. Also production capacity of each period is limited. Horizon planning is finite and it is made of N periods. Inventory of the beginning of the first period equals zero. Inventory of the end of planning horizon will be sold. Demand of each period is an arbitrary continuous random variable with known probability density function. First, the stochastic model has been transformed in to a deterministic model. Then, the deterministic model has been solved. The obtained solution has been used as a primal feasible solution in the new proposed hybridalgorithm. The proposed algorithm is created by combining analytical method andsimulation, with cyclic coordinate method. Finally, four examples have been solved using the proposed method. The solutions have been compared to exact solutions. In three examples out of four, the proposed method could successfully obtain the exact solutions.

Keywords—*capacitated production planning problem; stochastic demand; simulation; cyclic coordinate method.*

I. INTRODUCTION

Production planning is one of the most influential factors in obtaining benefits in various industries. In other words, an optimal production plan can decrease the total production cost in horizon planning. Total cost is the sum of constant and variable cost of production, holding cost, backorder cost and lost sale cost. A production plan defines the amount of production in each period when the planning horizon is finite. It is obvious that demand of customers must be satisfied properly but in the real world problems, demand is uncertain. Wagner and Whitin considered the problem of unconstrained-resource, a single level production planning[1]. Ramsay and Rardin suggested some heuristics for multistage production planning problems [2].Absi and Safia studied multi production planning assuming that the production capacity is limited [3]. Drexl and Kimms checked production planning deterministic model and dynamic model, assuming that the production capacity is limited [4].FatemiGhomi and Hashemin solved the problem by transforming the stochastic problem to a deterministic problem [5].Hashemin proposed that demand, which is a random variable, can be replaced with a constant value [6]. There are studies conducted on the cyclic coordinate method in N-dimensional space for unrestricted problems [7].In this paper, it is assumed that the planning horizon is finite. In other words, the amount of production must be defined in each period out of total N period of planning horizon.It is supposed that, the holding cost and lost sales cost is known. Also, demand of each period is a random variable with known probability density function. A new hybrid algorithm is developed by combining an analytical method, Monte Carlo simulation and a cyclic coordinate method. Some examples have been solved using the newly proposed method.

II. DETERMINISTIC MODEL

In this section, it is assumed that the demand is deterministic in each period.

A. *Assumptions*

In the studied model, set up cost, inventory holding cost, variable production cost exist and production capacity of each period are limited.

Also, it is supposed that the inventory in the beginning of planning horizon and in the end of planning horizon is zero.

B. *Notations*

The notations below have been used in this paper:

p_i =unit variable production cost in period i
f_i =set up cost in period i
h_i=unit holding inventory cost in period i
C_i=production capacity in period i
N =number of periods of planning horizon
D_i =deterministic demand quantity in period i
x_i =production quantity in period i
y_i=inventory at the end of period i
z_i=0 if nothing is produced in period i
z_i=1 if anything is produced in period i

C. Mathematical Model

$$\text{Min} \sum_{i=1}^{N} (f_i z_i + p_i x_i + h_i y_i)$$

Subject to:

$y_0 = y_N = 0$

$x_i + y_{i-1} - y_i = D_i$

$x_i - C_i z_i \leq 0 z_i \in \{0, 1\}$

$x_i \geq 0 \quad , \quad y_i \geq 0$

$i = 1,2, \dots, N$

D. Solving Method for Deterministic Model

1) Ramsay and Rardin Method

This method transforms the deterministic model into a shortest path problem. Then, it solves the shortest path problem[2].

2) FatemiGhomi and Hashemin Model

By splitting a period to two or more periods, artificial nodes were created. Then by solving the newly generated shortest path problem, it is shown that the solution of Ramsay and Rardin method can be non-optimal. This method is not recommended for solving the problem that described in section C.Purpose was only to show that the previous method sometimes cannotobtain the optimal solution [5].

In many production planning problems, variables x_i may be integer. Since the mathematical model of these problems is an integer linear programming with zero-one variables, solving these problems requires too much computational effort. In these problems,feasible sets $\{z_1, z_2, \dots, z_N\}$ can be defined. It should be noted that $C_1 \geq D_1$ and $z_1 = 1$, in all feasible sets.

By replacing the values of $z_i$in mathematical model, the modelis transformed to a linear programming model. Then, this modelmust be solved for each feasible set.

The assumptions which may exist in many problems are as follows:

I. Same variable production cost for all periods

II. Same holding inventory cost for all periods

Subject to the above assumptions, the objective function of the problem would be as follows:

$$\text{Min} \left(\sum_{i=1}^{N} f_i z_i + p \sum_{i=1}^{N} x_i + h \sum_{i=1}^{N} y_i \right)$$

Since the set of values $\{z_1, z_2, \dots, z_N\}$ is known for each feasible production plan, $\sum_{i=1}^{N} f_i z_i$ is constant for each production plan.

Also due to $y_0 = y_N = 0$, there would be $\sum_{i=1}^{N} x_i = \sum_{i=1}^{N} D_i$. So, the next term of objective function, $p \sum_{i=1}^{N} x_i$, would also be constant; and $\sum_{i=1}^{N} y_i$should be minimized for each feasible production plan. To achieve this goal, the constraints can be divided into the two following groups:

$$\text{Group 1} \begin{cases} x_i + y_{i-1} - y_i = D_i \\ x_i \leq C_i \end{cases} \quad \forall \ z_i = 1$$

$$\text{Group 2} \begin{cases} y_{i-1} - y_i = D_i \\ x_i = 0 \end{cases} \quad \forall \ z_i = 0$$

The last constraint for $i = N$is as follows:

$x_N = y_{N-1} = D_N$

Since$x_i$are notexist in the objective function, the efforts have been exerted to determine the value of x_N in order to minimize y_{N-1}. Therefore, if $z_N = 1$ then

$x_N = \text{Min}\{C_N, D_N\} \quad , \quad y_{N-1} = \text{Max}\{0, D_N - C_N\}$

And if $z_N = 0$, the $y_{N-1} = D_N$. The $(N-1)$ th constraint belongs to either group 1 constraints or group 2 constraints.

I. If $(N-1)$th constraint belongs to group 1, then$x_{N-1} + y_{N-2} - y_{N-1} = D_{N-1}$.

In the latter constraint, y_{N-1} and D_{N-1} are known; so the value of x_N can be determinedin order to minimize y_{N-2}. Therefore,

and consequently

$x_{N-1} = \text{Min}\{D_{N-1} + y_{N-1}, C_{N-1}\}$

$y_{N-2} = \text{Max}\{0, D_{N-1} + y_{N-1} - C_{N-1}\}$

So, in general if$z_i = 1$, then

$y_i = \text{Max}\{0, D_{i+1} + y_{i+1} - C_{i+1}\}$

II. If $(N-1)$ th constraint belongs to group 2, then and consequently

$y_{N-2} - y_{N-1} = D_{N-1}$

$y_{N-2} = D_{N-1} + y_{N-1}$

In general if $z_i = 0$, the

$y_{i-2} - y_{i-1} = D_{i-1}$

And consequently

$y_{i-2} = D_{i-1} + y_{i-1}$

III.MATHEMATICAL MODEL WITH STOCHASTIC DEMAND

A. Assumptions

It is supposed that there is a single level production planning problem with finite horizon. This horizon comprises N periods. In each periodwe can have set up cost, inventory holding cost, lost sale cost and variable production cost.

Also production capacity of each period is limited. Demand of each period is an arbitrary continuous random variable with known probability density function.

It is assumed that the inventory in the beginning of planning horizon is zero.

B. Notations

p_i =unit variable production cost in period i

f_i =set up cost in period i

h_i =unit holding inventory cost in period i

C_i =production capacity in period i

N =Number of periods of planning horizon

D_i =Deterministic demand quantity in period i

S_N =sale price of one unit of product at the end of planning horizon
cc_i =unit cost of lost sale

Decision variables of the model are defined as below:
x_i=production quantity in period i
y_i=inventory at the end of period i
z_i =0 if nothing is produced in period i
z_i =1 if anything is producedin period i

E. Mathematical model

$$\mathrm{MinZ} = \mathrm{E}\left[\sum_{i=1}^{N} f_i z_i + \sum_{i=1}^{N} p_i x_i + \sum_{i=1}^{N} h_i \mathrm{Max}\{0, x_i + y_{i-1} - D_i\} + \sum_{i=1}^{N} cc_i \,\mathrm{Max}\{0, D_i - x_i - y_{i-1}\} - S_N y_N\right]$$

Subject to:
$y_0 = 0$
$x_i - C_i z_i \le 0 \qquad z_i \in \{0,1\}$
$x_i \ge 0 \quad , \quad y_i \ge 0$
$i = 1,2, \dots, N$

F. SolvingMethod forStochastic Model

3) Hashemin Method

It is proposed that demand random variable is replaced with constant value, if lost sale is big and inventory cost of period is small. It is preferred for $D_i$to be replaced with a value bigger than$\bar{D}_i$.

By this replacement, the stochastic model can be transformed to a deterministic model. Then All feasible sets of z_i are defined. By using the new heuristic method, optimum values of other decision variables are computed for each feasible set so,the value of objective function of the model can be calculated for each feasible set. Finally, optimum solution of the model is obtained.

Since this solution is obtained by solving the deterministic model, the above mentioned optimum solution is not the optimum solution for stochastic model. However we can compute the expected value of objective function for that solution (solution of transformed model) using the simulation method. It is evident that in stochastic case, $x_i + y_{i-1} - y_i = D_i$ may not be satisfied.

IV. CYCLIC COORDINATE METHOD

Cyclic coordinate method solves the optimization problem by performing approximate minimization a long coordinate direction. Cyclic coordinate method is an iterative method, this method in each iteration tries to improve the objective function.

Suppose that X_0 is aninitial point for the method. Also, suppose that

$$e_i = \begin{bmatrix} e_{i1} = 0 \\ \vdots \\ e_{ii} = 1 \\ \vdots \\ e_{in} = 0 \end{bmatrix} \quad \text{and} \quad X_0 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

in this method coordinate axes are moved as much as α until the optimal point is obtained.
If $f(X_0)$is the objective function at the X_0then $f(X_0 + \alpha e_i)$will be the expected value of the above mentioned function at the new point.
Steps of the coordinate method is shown as an algorithm as below:
Step1: define X_0, ε and set$i = 1$
Step2: if $f(X_0 + \alpha e_i) < f(X_0)$then X_0=$X_0 + \alpha e_i$ go to step3, otherwise if $f(X_0 - \alpha e_i) < f(X_0)$ then X_0=$X_0 - \alpha e_i$ go to step3, otherwise go to step 3
Step3: if $i = N$ then set $i = 1$ and go to step2, otherwise $i = i + 1$ and go to step2.
Stop criteria in step2
$[f(X_0 - \alpha e_i) < f(X_0)orf(X_0 + \alpha e_i) < f(X_0)]$ and $|f(X_0 + \alpha e_i) - f(X_0)| < \varepsilon$

V. A PROPOSED HYBRID ALGORITHM

By replacing the expected value of demand random variable, the stochastic model is converted to deterministic model.
The created deterministic model can be solved by the FatemiGhomi and Hashemin method which was described in section D. so, X_i, $i = 1, \dots, N$ can be obtained.

It is supposed that X_0=$\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$. Also, it is assumed that $\alpha = 1$,$f(X_0)$shows the expected value of total cost which can be estimated by simulation.
Steps of the proposed hybrid algorithm are as follows:

Step1:suppose that theinitial point of C.C.M. isX_0. Then estimate $f(X_0)$,set $i = 0$, and define ε.
Step2:put $i = i + 1$. If $i > N$ put $i = 1$ and go to step3, otherwise without changing i go to step3.
Step3:if$x_i + \alpha \le C_i$ go to step4, otherwise go to step5.
Step4:if $f(X_0 + \alpha e_i) < f(X_0)$put $\quad x_i = x_i + \alpha$. otherwise go to step5. If $|f(X_0 + \alpha e_i) - f(X_0)| < \varepsilon$stop. The final answer is the X_0point. Otherwise go to step2.
Step5:if $x_i - \alpha \ge 0$ go to step6, otherwise go to step7.
Step6:if $f(X_0 - \alpha e_i) < f(X_0)$ put $x_i = x_i - \alpha$. otherwise go to step7. If$|f(X_0 - \alpha e_i) - f(X_0)| < \varepsilon$ stop.
The final answer is the X_0 point. Otherwise go to step2.
Step7:ifX_0 does not change go to step2.

VI. EXAMPLES

Some computer programs have been written in MATLAB 2014 for different methods. These programs are executed on a computer with below specifications:

CPU 1.8 GHZ, two core.

A. *Example1*

In this problem, there are 4 periods. In all periods, production capacity, holding inventory cost for one unit of product in one period, lost sale cost for one unit of product in one period, unit variable production cost, sale price of one product at the end of planning horizon and set up costs are as follows:

$c_i = 100\ \ h_i = 5\ \ cc_i = 20\ \ p_i = 7\ \ s_N = 5\ \ f_i = 40$

Information of demands has been shown in tableI.

Example1 has been solved using different methods. Results of each method have been shown in tableII.

Table (I) Information of demands in example1

Period (i)	Distribution function of demand	parameters
1	uniform	a=15 , b=35
2	uniform	a=2.5 , b=7.5
3	uniform	a=40 , b=52
4	uniform	a=50 , b=58

Table(II)Answers of different methods

Period (i)	Amount of production by Hashemin method	Amount of production by the new hybrid algorithm	Optimal amount of production in period i
1	30	29	28
2	0	0	0
3	46	45	45
4	54	54	53
Total cost	1165.8	1162.5	1161.8

B. *Example2*

In this problem, there are 6 periods. In all periods, production capacity, holding inventory cost for one unit of product in one period, lost sale cost for one unit of product in one period, unit variable production cost, sale price of one product at the end of planning horizon and setup costs are as follows:

$c_i = 32\ \ h_i = 0.5\ \ cc_i = 5\ \ p_i = 1\ \ s_N = 0.5\ \ f_i = 50$

Information of demands has been shown in table III.

Example2 has been solved using different methods. Results of each method have been shown in table IV.

Table (III)Information of demands in example2

Period (i)	Distribution function of demand	Parameters	Cut point
1	uniform	a=2 , b=6	-
2	Cut exponential	$\lambda = \frac{1}{20}$	40
3	Cut exponential	$\lambda = \frac{1}{10}$	20
4	Cut exponential	$\lambda = \frac{1}{30}$	50
5	uniform	a=5, b=9	-
6	triangular	a=2, b=4, c=6	-

Table (IV)Answers of different methods

Period (i)	Amount of production by Hashemin method	Amount of production by the new hybrid algorithm	Optimal amount of production
1	28	30	30
2	0	0	0
3	0	0	0
4	32	24	24
5	0	0	0
6	0	0	0
Total cost	233.7958	230.7647	230.7647

C. *Example3*

In this problem, there are9 periods. In all periods, production capacity, holding inventory cost for one unit of product in one period, lost sale cost for one unit of product in one period, unit variable production cost, sale price of one product at the end of planning horizon and set up costs are as follows:

$c_i = 11\ \ h_i = 0.7\ \ cc_i = 4\ \ p_i = 2\ \ s_N = 0.4\ \ f_i = 30$

Information of demands has been shown in table V. Example3 has been solved using different methods. Results of each method have been shown in tableVI.

Table (V)Information of demands in example3

Period (i)	Distribution function of demand	Parameters	Cut point
1	uniform	a=0.5,b=3.5	-
2	triangular	a=3,b=6,c=9	-
3	Cut exponential	$\lambda = \frac{1}{4}$	30
4	uniform	a=1.5, b=4.5	-
5	triangular	a=5,b=10,c=15	-
6	Cut exponential	$\lambda = \frac{1}{2}$	20
7	uniform	a=2, b=6	-
8	Cut exponential	$\lambda = \frac{1}{10}$	20
9	triangular	a=2,b=4,c=6	-

Table (VI)Answers of different methods

Period (i)	Amount of production by Hashemin method	Amount of production by the new hybrid algorithm	Optimal amount of production
1	9	8	8
2	0	0	0
3	11	6	6
4	0	0	0
5	11	11	11
6	0	0	0
7	0	0	0
8	11	7	7
9	0	0	0
Total cost	250.3083	238.9467	238.9467

D. Example4

In this problem, there are 12 periods. In all periods, production capacity, holding inventory cost for one unit of product in one period, lost sale cost for one unit of product in one period, unit variable production cost, sale price of one product at the end of planning horizon and set up costs are as follows:

$c_i = 12 \ h_i = 0.5 \ cc_i = 5 \ p_i = 1 \ s_N = 0.3 \ f_i = 40$

Information of demands has been shown in table VII.

Example4 has been solved using different methods. Results of each method have been shown in table VIII.

Table (VII)Information of demands in example2

Period (i)	Distribution function of demand	parameters	Cut point
1	uniform	a=4, b=6	-
2	triangular	a=4, b=5, c=6	-
3	Cut exponential	$\lambda = \frac{1}{10}$	20
4	uniform	a=1, b=3	-
5	triangular	a=1, b=2, c=3	-
6	Cut exponential	$\lambda = \frac{1}{2}$	20
7	uniform	a=3, b=6	-
8	Cut exponential	$\lambda = \frac{1}{10}$	20
9	uniform	a=2, b=8	-
10	triangular	a=3, b=5, c=7	-
11	Cut exponential	$\lambda = \frac{1}{5}$	30
12	triangular	a=1, b=2, c=3	-

Table (VIII)Answers of different methods

Period (i)	Amount of production by Hashemin method	Amount of production by the new hybrid algorithm	Optimal amount of production
1	10	11	11
2	0	0	0
3	11	10	10
4	0	0	0
5	0	0	0
6	8	8	8
7	0	0	0
8	12	11	11
9	0	0	0
10	12	9	9
11	0	0	0
12	0	0	0
Total cost	311.8840	309.6054	309.6054

VII.CONCLUSIONS

In this paper, a production-planning problem has been studied when the planning horizon is finiteand demands of each period are arbitrary random variables. Also, capacity of production in each period is limited and setup cost, variable production cost, holding cost and lost sales cost are present. A new hybrid algorithm has been developed by combining analytical method, simulation and cyclic coordinate method. It has been shown that the new hybrid algorithm outperforms previous methods.

In future researches, subject to the same assumptions, the study of problem is suggested when the number of products are two or more. Also, this problem can be studied when the steps of production in each period is two or more.

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