Non-Renewable Constrained Resource Allocation in Fuzzy GERT Networks with Exclusive-or, Probabilistic Nodes

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Abstract

Uncertainty in realization of some activities of the project can be shown by GERT networks. In other words, GERT networks are used to describe these projects. Also, in the real world, activity durations are nondeterministic and they are dependent on the amount of resource allocated to them. Due to uncertainty of activity duration, these durations are shown by positive trapezoidal fuzzy numbers. Certain kinds of projects are executed by completing one path out of the network paths. Therefore, in this research, it is assumed that the project can be shown as a GERT network with Exclusive-or, Probabilistic nodes. Here, two new algorithms have been developed for constrained resource allocation. The first algorithm selects eligible activities to increase the allocated resource and the second algorithm attempts to decrease the completion time of the project through pair comparisons. In an example, advantages of both new algorithms have been presented.

Keywords — *Constrained Resource Allocation, GERT Network, Project Completion Time, Fuzzy Trapezoidal Number.*

I. INTRODUCTION

In the real world projects, suitable resource allocation is very valuable. On the other hand, the resources required for project execution are limited. Hence, it is necessary to consider the allocation of resource in the project planning and control. Project resources are divided into two categories of renewable resources such as human resources and non- renewable ones such as materials. In many of real world projects, available resources are limited. So, optimal resource allocation to project activities is very considerable. In some of real projects, realization of activities and duration time of activities are uncertain. These projects can be shown by GERT networks [9]. On the other hand, special kinds of GERT networks can be completed by completing of one and only one path out of network paths. In these networks, there are only exclusive-or, probabilistic nodes. Researchers show the uncertainly of activity durations by random variable or fuzzy numbers.

An innovative model based on genetic algorithm to solve problem of resource allocation has been presented [1]. Completing the project in the shortest time is a substitute for random networks [2].Utilizing trapezoidal fuzzy numbers, a new method for scheduling fuzzy GERT networks, has been presented in research projects [4]. Triangular and trapezoidal fuzzy sets have been applied to show uncertainty [5]. In another research, it has been supposed that GERT networks have Exclusive-or, Probabilistic nodes. A methodology has been prepared to combine analytic and simulation methods in order to allocate limited resource to project activities [7]. Uncertainty of activity duration has been studied by using fuzzy numbers and discrete-event simulation [8]. Constrained consumable resource allocation has been studied in GERT-type networks with exclusive-or, probabilistic nodes when the activity durations are known random variables [3]. However, this research studies non-renewable constrained resource allocation in fuzzy GERT networks with Exclusive-or, probabilistic nodes. To achieve this aim two heuristics have been developed. First algorithm obtains a primal solution that is better than that of Hashemin’s algorithm [6]. The second proposed heuristic can improve the solution of the first heuristic. Some examples have been solved using both heuristics.

II. BASIC ALGORITHM FOR STATIC CONSTRAINED RESOURCE ALLOCATION IN GERT NETWORKS

In this method [5], the value of resources allocated to all activities must be determined before the start of the project. For executing each activity one kind of non-renewable resource is necessary. The main aim is allocating the constrained resource to the activities such that completing time of project decreases.

A. Assumptions

1) The network has a source node and can have multiple end nodes.
2) The network has only Exclusive-or, Probabilistic nodes.
3) The network has no loops.
4) The duration of each activity is a positive trapezoidal number.
5) Allocation of resource must be done statically.
6) Each activity needs one kind of non-renewable resource.
7) Amount of available resource is limited.

8) Allocable resource to each activity is constrained to specific levels

B. Notations

N Number of activities

R Number of paths

P_{ij} Occurrence probability of j th path which terminates in i th sink node

P_k Accomplishment probability of k th activity, given that start node of this activity has been realized.

S_{ij} Activity set of j th path which terminates in i th sink node.

t_k Completion time of k th activity as a trapezoidal fuzzy number (a, b, c, d)

S_{l_k} The amount of resource allocated to k th activity when the level of allocated resource is l_k

$\mu_k(t_k, S_{l_k})$ Membership function of k th activity completion time when resource the allocated to it, is S_{l_k}

$\bar{t}_k(S_{l_k})$ Estimated crisp value for completion time of k th activity when the amount of resource allocated to k th activity is S_{l_k}

R_s The available amount of limited resource

$T(A_{ij})$ Completion time of j th path which terminates in i th sink node.

T Completion time of network

m Number of end nodes

n_i Path number which terminate to i node

M Structural Matrix of the GERT network with R rows and N columns $M = [m_{ij,k}]_{R \times N}$

$m_{ij,k} = 1 \quad if \quad k \in S_{ij}$

$m_{ij,k} = 0 \quad if \quad k \notin S_{ij}$

C. Steps of the Basic Algorithm [5]

step1: Allocate the maximum value for $S_{l_k}, \forall k$

Step2: If $\forall ij$, $\sum_{k \in S_{ij}} S_{l_k} \leq R_s$ then stop, otherwise go to step3.

Step3: Determine the $P_{uv} = \underset{ij}{Min}\{P_{ij} \mid \sum_{k \in S_{ij}} S_{l_k} > R_s\}$ such that $P_{ij} = \prod_{k \in S_{ij}} P_k$

Step4: $W_k = \sum_{ij} (P_{ij} \sum_{ij} \bar{t}_k(S_{l_k}) m_{ij,k}), \quad \forall k \in S_{uv}$

$$W_r = \underset{k \in S_{uv}}{Min}\{W_k\}$$

Such that

$$\bar{t}_k(S_{l_k}) = \frac{\int_{a_k}^{d_k} t_k \mu_k(t_k, S_{l_k}) dt_k}{\int_{a_k}^{d_k} \mu_k(t_k, S_{l_k}) dt_k}$$

Then decrease the resource allocated to activity $r \in S_{uv}$ i.e. $S_{l_r} \leftarrow S_{l_r - 1}$ and go back to step 2.

After executing the algorithm steps we can determine the probability function of the random variable of the network completion time and compute the mean as a fuzzy number. The network completion time formula and the average of completion time are computed as follows:

$$P(T = T(A_{ij})) = P_{ij} \qquad (1)$$

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij}) \qquad (2)$$

$\oplus$ is fuzzy summation and $\bullet$ is scalar multiplication.

III. THE NEW PROPOSED ALGORITHMS

In the proposed algorithms, defined assumptions and notations are not different from the assumptions and notations of the basic algorithm. So, only some new notations are introduced.

A. Proposed Algorithms Notations

M_r r th row of Matrix M

D Each component of vector D represents the amount of resource allocated to an activity

C Each component of vector C is shown by c_{ij} and represents the consumed resource by each path out of all other paths

Q Set of paths that have consumed resources is less than R_s i.e. $Q = \{S_{ij} \mid c_{ij} < R_s\}$

$\overline{Q}$ Set of paths that have consumed resources is R_s i.e. $\overline{Q} = \{S_{ij} \mid c_{ij} = R_s\}$

Z Set of activities that allocated resources of which can be increased one unit

B. Steps of the **First** *Proposed Algorithm*

Step1: By implementing the basic algorithm determine the resource allocated to all activities.

Step2: The multiplication of the vector D in the matrix M $(M \cdot D)$ is the vector C . So, $c_{ij} = \sum_{k \in S_{ij}} S_{l_k}$

If $\{k \in S_{ij} \mid S_{ij} \in Q\} \cap Z = \phi$ stop and go to step 6 otherwise go to step 3.

Step3: Suppose that $Y = \{k \in S_{ij} \mid S_{ij} \in Q\} \cap Z$ and go to step 4.

Step4: Suppose that $\sum_{ij} m_{ij,q} = \underset{k \in Y}{Min} \; \{\sum_{ij} m_{ij,k}\}$ go to step 5

Step5: Set $S_{l_q} \leftarrow S_{l_q + 1}$, update D and set $Z = Z - \{q\}$. Also, update $Q, \overline{Q}$ then, go back to step 2.

Step6: Compute the network completion time as follows:

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij})$$

B. Steps of the Second Proposed Algorithm

Step1: By using the first proposed algorithm, determine the resource allocated to the activities. Suppose that vector D is the vector of resource allocated to the activities in the final step of first proposed algorithm.

Step2: Compute $C = M \cdot D$ Then choose $\{r, q\}$ such that, $r \in S_{ij}, q \in S_{ij}$ for known ij . Then set $S_{l_r} \leftarrow S_{l_r} + 1, S_{l_q} \leftarrow S_{l_q} - 1$ or vice versa. If this new solution is feasible i.e. $\forall ij, \sum_{k \in S_{ij}} S_{l_k} \leq R_s$ evaluate $E(T)$ if it decreases, update D and C and repeat this step otherwise repeat this step with another $\{r, q\}$. If you have examined all possible $\{r, q\}$ go to step 3.

Step4: Compute the network completion time as follows:

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij})$$

IV. EXAMPLE

Consider the network in Fig.1. The amount of limited resource is 15 units.

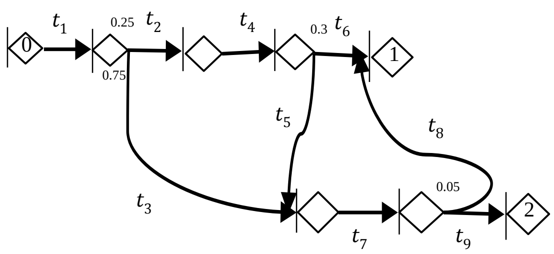

Fig 1: The network of example

Completion time of paths and their realization probabilities are computed as follows:

$t_1 + t_2 + t_4 + t_6$

$P_{11} = (0.25)(0.3) = 0.075$

$t_1 + t_3 + t_7 + t_8$

$P_{12} = (0.75)(0.05) = 0.0375$

$t_1 + t_2 + t_4 + t_5 + t_7 + t_8$

$P_{11} = (0.25)(0.7)(0.05) = 0.00875$

$t_1 + t_3 + t_7 + t_9$

$P_{11} = (0.75)(0.95) = 0.7125$

$t_1 + t_2 + t_4 + t_5 + t_7 + t_9$

$P_{11} = (0.25)(0.7)(0.95) = 0.16625$

Hence, the equivalent network can be shown as Fig.2

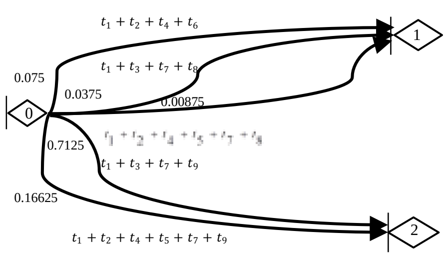

Fig .2 Equivalent network

Table 1 contains the activity completion times.

TABLE I
Activity Completion Times of the Example

k	l_k	S_{l_k}	a_k	b_k	c_k	d_k
1	1	4	2	3	5	6
	2	5	1	2	4	5
2	1	1	2	3	4	5
	2	2	1	2	3	4
3	1	5	3	4	5	6
	2	6	2	3	4	5
4	1	1	2	3	4	5
	2	2	1	2	3	4
5	1	1	3	4	5	6
	2	2	2	3	4	5
6	1	5	3	4	5	6
	2	6	2	3	4	5
7	1	3	2	3	5	6
	2	4	1	2	4	5
8	1	1	2	3	4	5
	2	2	1	2	3	4
9	1	1	2	3	4	5
	2	2	1	2	3	4

By using the basic algorithm, resources allocated to activities are obtained as below:

$S_{l_1} = 5$, $S_{l_2} = 2$, $S_{l_3} = 5$, $S_{l_4} = 1$, $S_{l_5} = 1$,
$S_{l_6} = 6$, $S_{l_7} = 4$, $S_{l_8} = 1$, $S_{l_9} = 1$

Expected value of network completion time is a trapezoidal fuzzy number and can be computed as below:

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij}) =$$

$$0.075 \bullet (6,10,15,19) \oplus$$
$$0.0375 \bullet (7,11,17,21) \oplus$$
$$0.00875 \bullet (10,16,24,30) \oplus$$
$$0.7125 \bullet (7,11,17,21) \oplus$$
$$0.16625 \bullet (10,16,24,30) =$$
$$(7.45, 11.8, 18.075, 22.425)$$

By using the first proposed algorithm, resource allocated to some activities is changed as shown below:

$S_{l_1} = 5$, $S_{l_2} = 2$, $S_{l_3} = 5$, $S_{l_4} = 1$, $S_{l_5} = 2$,
$S_{l_6} = 6$, $S_{l_7} = 4$, $S_{l_8} = 1$, $S_{l_9} = 1$

New resource allocation reduces the expected value of network completion time.

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij}) =$$

$$0.075 \bullet (6,10,15,19) \oplus$$
$$0.0375 \bullet (7,11,17,21) \oplus$$
$$0.00875 \bullet (9,15,23,29) \oplus$$
$$0.7125 \bullet (7,11,17,21) \oplus$$
$$0.16625 \bullet (9,15,23,29) =$$
$$(7.275, 11.625, 17.9, 22.25)$$

By using the first proposed algorithm, resource allocated to some activities is changed as shown below:

$S_{l_1} = 5$, $S_{l_2} = 2$, $S_{l_3} = 5$, $S_{l_4} = 2$, $S_{l_5} = 1$,
$S_{l_6} = 6$, $S_{l_7} = 4$, $S_{l_8} = 1$, $S_{l_9} = 1$

New resource allocation reduces the expected value of network completion time.

$$E(T) = \sum_{i=1}^{m} \oplus \sum_{j=1}^{n_i} \oplus P_{ij} \bullet T(A_{ij}) =$$

$$0.075 \bullet (5,9,14,18) \oplus$$
$$0.0375 \bullet (7,11,17,21) \oplus$$
$$0.00875 \bullet (9,15,23,29) \oplus$$
$$0.7125 \bullet (7,11,17,21) \oplus$$
$$0.16625 \bullet (9,15,23,29) =$$
$$(7.2, 11.55, 17.825, 22.175)$$

Therefore, in this example, the first proposed algorithm has been able to decrease the expected value of network completion time. The second proposed algorithm also reduces the expected value of network completion time.

Five other examples have been solved by basic and new heuristic algorithms. In these examples, one of two new heuristics or both could successfully reduce the expected value of network completion time which were presented by basic algorithms.

V. CONCLUSIONSAND RECOMMENTION

This research has studied the non-renewable resource allocation in special types of GERT networks with exclusive-or, probabilistic nodes and fuzzy activity completion time. To achieve this aim new heuristic algorithm that it is called first algorithm has been developed which can improve the result of the basic algorithm [5]. Then, the second algorithm has been proposed that tries to improve the first algorithm's solution. In this research, sometimes only one of the two algorithms is sufficient while other times, both algorithms can improve the basic algorithm's solution. In this research, the GERT network only contains exclusive-or, probabilistic nodes. Study of this kind of resource allocation can be proposed on GERT networks with other types of nodes in the future. Also, the duration of the project activity is considered as trapezoidal fuzzy numbers. It is suggested that other types of fuzzy numbers be used in future researches and the results be compared with each other.

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