Direct Numerical Simulation of Turbulent Flow Around an Impulsively Started Circular Cylinder by Using Mesh-Free Vortex Method

Amirreza Mohammadian[#1], Suresh Akella [*2], A.M.K Prasad [#3]
Mechanical Engineering Department, Osmania University
Hyderabad, 500007, India

Abstract

Direct numerical simulation of turbulent flow, wake flow, using the random vortex method involving vortex blobs is presented and implemented in this paper. The method is used to perform high-resolution simulations of incompressible two-dimensional, Navier-Stokes fluid flows. In the random vortex method, vorticity of the flow field is the primary variable. After generation on the cylinder wall, it is followed in two separate fractional time step in a Lagrangian approach, namely convection and diffusion. In this paper, the developed random vortex method applied to the flow past one impulsively started circular cylinder. the flow simulations are carried out in the high Reynolds number Re=140000. Instantaneous velocity vector field and Instantaneous velocity and position of each vortex element in the flow at same time are plotted. The code has been validated by using experimental data to demonstrate accuracy of produced solution without the effects of grid-based numerical diffusion.

Keywords — *Turbulence, Vortex, Random vortex, mesh-free, Vorticity.*

I. INTRODUCTION

There is an involvement of lot of engineering problems pertaining to gases or liquids over solid bodies. If we consider the following examples i.e. air flow over cars and airplanes, the wind blowing over building and bridges, the slashing of sea waves against an offshore oil rig etc... There isn't often any parallel connectivity between these flows and the contour of solid surface completely but instead away from these flows it creates a wake such as behind a ship. Conventional numerical schemes don't come in handy while operating such separated flows.

For the mesh-based methods as *Re* increases the scales of fluid motion reduce in size. For obtaining adequate solutions, the numerical schemes must model the smallest scales of motion and hence there is an increase in the computational power that is needed to solve the problem with the increase in Reynolds number. The numerical schemes are mostly mesh-based. As mentioned above there is some difficulties with grid-based which avoid in Vortex methods.

Vortex methods are successful and attractive approach for the numerical simulation of incompressible viscose flow at high Reynolds number (Sarpkaya [18], Lorena A. Barba [14] and Leonard [12]) [11]. Vortex methods used a vorticity-velocity formulation .The vorticity field is discretized into a finite number of vortex elements with the specified strengths (rather than specifying it on a fixed grid). The velocity field is obtained from the vorticity field and track of a finite number of vortex elements are kept in a Lagrangian reference of frame. As, the vorticity is tracked in order to displace of individual particles, There is no necessity for a fixed grid on which the governing differential equations and the unknowns are identified. Rvm (random vortex method) is one off simple and completely grid-free vortex method, in which each time step is divided into two steps. In the first time step, the mechanism of diffusion is frozen and displacement of the center of elements which is due to convection of the flow field is calculated by applying fourth order Runge-Kutta scheme and in the second time step, the effect of diffusion is considered using a random walk.

The concept of random vortex and vortex blob is presented by Chorin [5]. The theoretical analysis of the random vortex method for two-dimensional fluid flow with a free-space boundary is investigated by Goodman [9] and Long [13]. There are several advantages of vortex methods are investigated by Puckett, E. G. [16] Beale and Majda [2] Dutta [7]. The mathematical analyses of convergence and accuracy for inviscid fluid flow have been investigated by Anderson and Greengard [1], Hald [10], Cottet et al. [6]. Fogelson and Dillon [8], Roberts [17], Mortazavi et al. [15] and Leonard [12].

II. GOVERNING EQUATIONS

By applying Curl Operator on Navier Stocks equations and then merging it with incompressible continuity equation we can gain vorticity transfer equation in two-dimensions.

$$\frac{D\rho}{Dt}+\rho\nabla.\vec{V}=0\Rightarrow\frac{\partial\rho}{\partial t}+\vec{V}.grad\rho=-\rho\vec{\nabla}.\vec{V}$$
(1)

$$\frac{\partial\vec{V}}{\partial t}+\vec{V}.grad\vec{V}=-\frac{1}{\rho}gradP+\nu\nabla^2\vec{V}$$
(2)

$$\frac{\partial\vec{\omega}}{\partial t}+\left(\vec{V}.\vec{\nabla}\right)\vec{\omega}=\nu\Delta\vec{\omega} \quad (3)$$

As it can be seen in equation (3), for solving the vorticity transfer equation there is no need to calculate the pressure field. By removing dimension from (2) and (3) with regard to the reference velocity, $U_\infty=1m/s$, and cylinder's diameter, D, we get:

$$\frac{\partial\vec{V}^*}{\partial t^*}+\vec{V}^*.grad\vec{V}^*=-\frac{1}{\rho}gradP^*+\nu\nabla^2\vec{V}^*$$

$$\frac{\partial\vec{\omega}^*}{\partial t^*}+\left(\vec{V}^*.\vec{\nabla}^*\right)\vec{\omega}^*=\frac{1}{\mathrm{Re}}\Delta^*\vec{\omega}^*$$

(4) $$\mathrm{Re}=\frac{\rho u_\infty D}{\mu}=\frac{u_\infty D}{\upsilon}$$

$$x^*=\frac{x}{D}\ ,\ y^*=\frac{y}{D}\ ,\ u^*=\frac{u}{u_\infty}\ ,\ v^*=\frac{v}{u_\infty}\ ,$$

$$t^*=\frac{tu_\infty}{D}\ ,\ \omega^*=\frac{D}{u_\infty}\omega$$

Where P^*, V^* and t^* are pressure, velocity and dimensionless time respectively. υ is kinematic viscosity , μ is dynamic viscosity, and ρ represent density.

No-slip condition due to fluid viscosity and no-penetration are boundary conditions and inviscid (Potential) flow throughout the field is initial condition.

$$\vec{U}\left(\vec{r},t\right).\hat{e}_n=\vec{U}_B.\hat{e}_n$$ No-penetration boundary condition

(5)

$$\vec{U}\left(\vec{r},t\right).\hat{e}_t=\vec{U}_B.\hat{e}_t$$ No-slip boundary condition

(6)

$\hat{e}_n$, $\hat{e}_t$ are orthogonal and tangent unit vector respectively, $\vec{U}$ is velocity vector and $\vec{U}_B$ is velocity of body. As mentioned before, equation (3) is solved in two steps. First step related to convection mechanism and the second step related to diffusion mechanism. The equation relating to convection mechanism is in fact Euler equation in the form of vorticity which emphasizes vorticity remain constant for each particle along the direction of motion.

In the convection mechanism we have inviscid flow ($\nu=0$). Therefore by omitting the term $\nu\Delta\vec{\omega}$ from vorticity equation, the equation relating to convection mechanism is:

$$if\ \ \nu=0\Rightarrow\frac{\partial\vec{\omega}}{\partial t}+\vec{V}.grad\,\vec{\omega}=0 \rightarrow\begin{cases}\dfrac{d\vec{r}_p}{dt}=V(\vec{r}_p)\\ \dfrac{D\vec{\omega}_p}{Dt}=0\end{cases}$$

(7)

Diffusion mechanism (second step) is investigated considering the effect of fluid viscosity and Brownian motion of vortices. Diffusion equation is gained by omitting the term $(\vec{V}.\vec{\nabla})\vec{\omega}$, so:

$$\frac{\partial\omega}{\partial t}=\nu\nabla^2\omega\rightarrow\begin{cases}\dfrac{d\vec{r}_p}{dt}=0\\ \dfrac{d\vec{\omega}_p}{dt}=\nu\Delta\vec{\omega}(r_p)\end{cases}$$

(8)

In the convection mechanism (first step) vortices induce velocity on their surrounding and each other. Induce velocity by j^{th} vortex on the point z_i, is as follows:

$$\bar{W}_{vortex}(z_i)=\sum_i\frac{-i\Gamma_j\left|z_i-z_j\right|}{2\pi\times\max(\left|z_i-z_j\right|,\delta)}(\frac{1}{z_i-z_j})$$

(9)

$\delta=\dfrac{h}{2\pi}$ is vortex core or radius of each vortex, N is the number of vortices, Γ_j is circulation of j^{th} vortex, h is the length of segments that must be equal and $\bar{W}=u-iv$ is conjugation of velocity. To eliminate the normal velocity on the surface, we locate sinks and sources on the center of segments. Induced velocity which resulting from sinks and sources located on z_j, on the point z_i is as follows:

$$\bar{W}_{source}(z_i)=\frac{\alpha(j)}{2\pi}(\frac{1}{z_i-z_j})\qquad i\neq j$$

(10)

Where α_j is the strength of j^{th} source or sink that is negative for sinks and positive for sources.

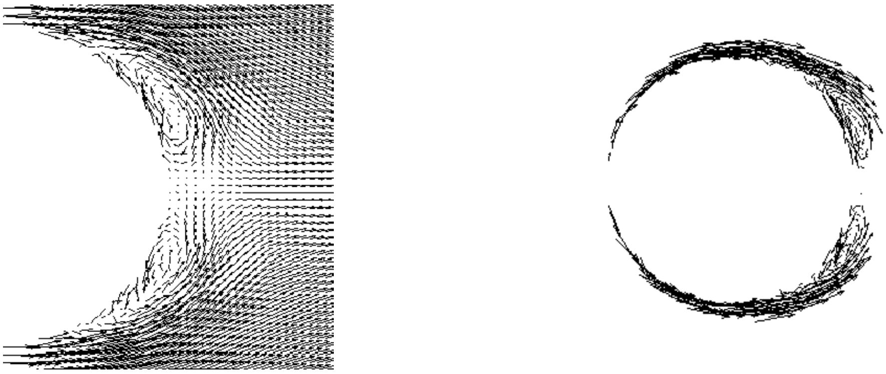
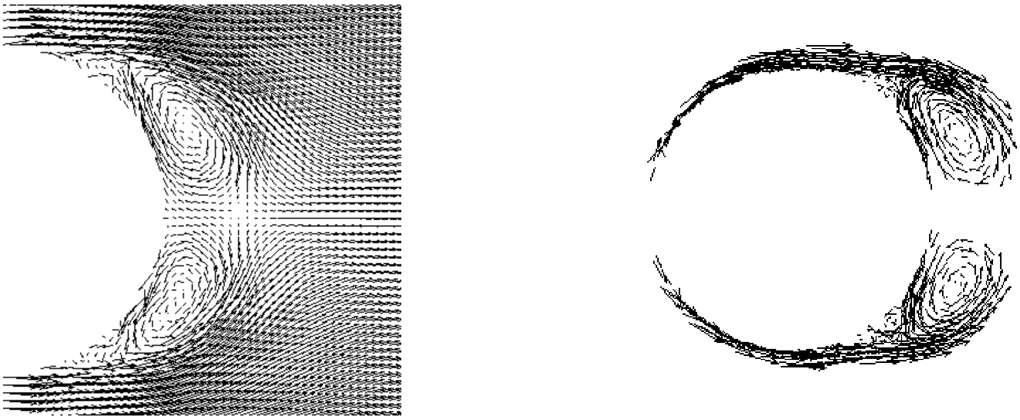
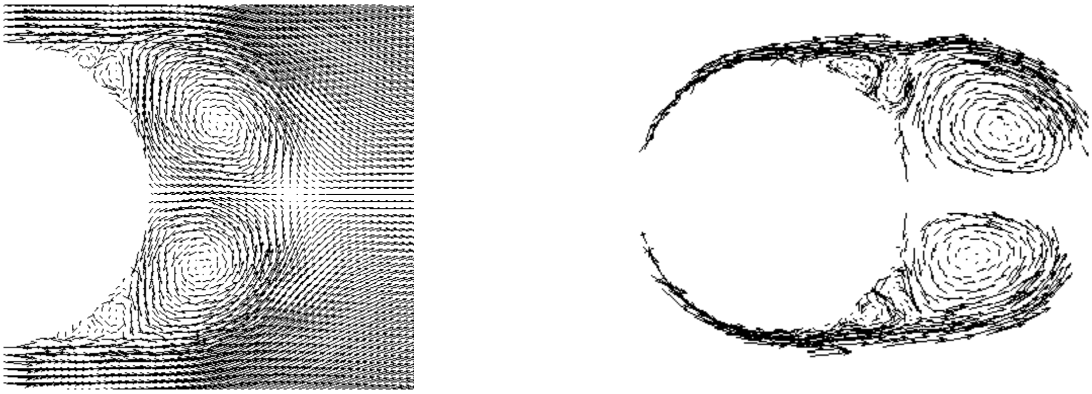


Fig. 1 Flow past a cylinder at *R* = 140000. Plot of the Instantaneous velocity vector field (left). Plot of the Instantaneous velocity and position of each vortex element in the flow at same time (right). *t* = 0.75



a)



b)

Fig. 2 Flow past a cylinder at *R* = 140000. Plot of the Instantaneous velocity vector field(left). Plot of the Instantaneous velocity and position of each vortex element in the flow at same time (right). a) t=1 ,b) t=1.5

Effect of sources and sinks on each other induces an equal velocity, but in the opposite direction of the induced velocity by vortices and satisfying no-slip boundary condition. By solving a series of N equations and N unknowns, strength of sink and source is calculated as follows [8]:

$$A\alpha = b \tag{11}$$

$$\text{A:}\begin{cases} a_{ij} = U_1(ij)n_1 + U_2(ij)n_2 & i \neq j \\ a_{ii} = \frac{1}{2}h^{-1} & i = 1,\dots,N \end{cases} \tag{12}$$

$$U_1(ij) = -\frac{1}{2\pi}\frac{X_j - X_i}{{R_{ij}}^2}\; U_2(ij) = -\frac{1}{2\pi}\frac{Y_j - Y_i}{{R_{ij}}^2} \tag{13}$$

$$ {R_{ij}}^2 = (X_j - X_i)^2 + (Y_j - Y_i)^2 $$

$$b_i = -\vec{W}_{vortex}.n \tag{14}$$

Where A is the coefficients matrix. In the convection mechanism (second step) random motion of vortices takes place based on Gaussian random variable. Relevant equation, (8), is the heat equation and can be solved using Green’s function for (2-D).

$$G\left(\vec{r},t\right)=\frac{1}{4\pi\nu t}\exp\left[-\frac{r^2}{4\nu t}\right] \quad (15)$$

Green's function is equal to the probability density function of Gaussian variable with zero mean and variance σ. Therefore, probability density function of Gaussian variable can be shown as:

$$p(\eta,t)=\sqrt{\frac{1}{2\pi\sigma^2}}\exp\left[-\frac{\eta^2}{2\sigma^2}\right] \quad (16)$$

By comparing (15), (16) we find out that the above function is a random variable Gaussian function with zero mean and variance of $\sigma=\sqrt{\frac{2t}{\mathrm{Re}}}$. So $z_j(t)$ is a position of j^{th} vortex in $T=t$, its position in $T=t+\Delta t$ can be calculated as:

$$Z_j(t+\Delta t)=Z_j(t)+\left[\begin{matrix}\bar{W}_{pot.}(j)+\bar{W}_{vortex}(j,t)\\ +\bar{W}_{source}(j,t)\end{matrix}\right]\Delta t+\eta_j \quad (17)$$

Where T denoted the time.

III. RESULT AND DISCUSSION

By using random vortex method the impulsively started flow past one circular cylinder is studied in considerable detail. In all of the numerical calculations done on the one circular cylinder, the boundary of the circle is divided into M=100 pieces, the length of each piece denoted by $h=2\pi/M$.

The circle has radius r=0.5. The Reynolds number is Re=140000, mainly because simulation in high Reynolds number is the important feature of RVM method.

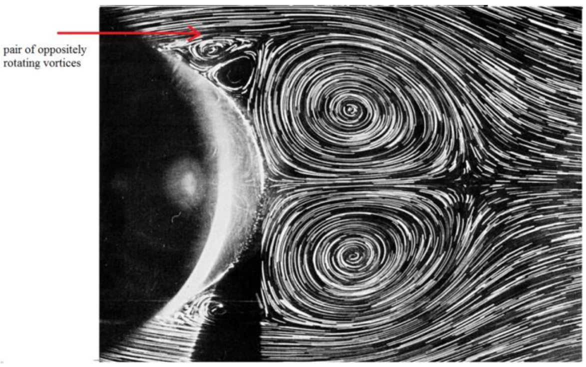


Fig. 3 Experimental Result for one Impulsively Started Cylinder Bouard. R and Coutanceau(1980)[3]

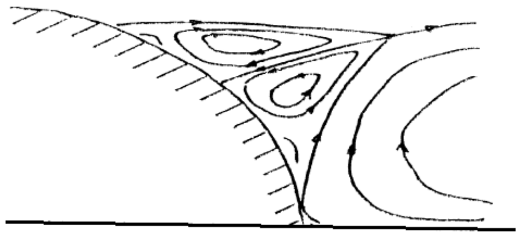


Fig. 4 Structure of α-phenomenon

The cylinder is impulsively set into motion at time t=0, where t is measured in non-dimensional units. Δt is chosen to be 0.2 or 0.05. After some experimentation we chose $\varepsilon_{\max}=0.01$.

A. *Velocity and Vorticity Field*

The Velocity and vorticity field are plotted in fig (1-5). 183 vortex elements are initially generated in order to satisfy the tangential boundary conditions. These vortices by the mechanism of convection and diffusion which drown from random walks method move towards the rear of the cylinder. The length of the vectors in the figures corresponds to the speed of the fluid at that point, and the arrow indicates the direction of the flow.

By evaluation of time variation of vorticity creation on the boundary and distribution of vortices around the cylinder we can understand the mechanism of vorticity transfer and mechanisms of the development of secondary vortices.

At t=0.75 (fig. 1) it can be seen that eddies in rear of the cylinder are just beginning to take form. At t=1(see fig. 2a), a complex flow pattern emerges. The plot shows that there are two distinct regions. The larger of these regions is the main vortex. This vortex has quit strong vorticity and the center of it moves toward downstream. The second region which located close to the separation point is a region of high concentration of vortices.

At t=1.5 (see fig 2.b), the structure of wake consists of three vortices which including the one big vortex in the rear of the cylinder and a pair of oppositely rotating vortices in the secondary vortex region. This structure is called α-phenomenon. The structure of α-phenomenon is showed in (fig. 3) and (fig. 4).

According to (fig. 5) eddies are merging due to diffusion and at time t=4 one of eddies grows larger even larger than the cylinder itself.

B. *U-Velocity for One Cylinder*

U-velocity (Component of the velocity vector along x direction) around one cylinder are obtained for several section which proportional to the flow direction at Re=140000.the results obtained for one cylinder are compared with experimental result of

Cantwell and Coles [4]. It can be seen very good agreement between obtained result and experimental result. Note that the simulation parameter is: Re=140000, $\varepsilon_{max} = 0.01$, $\Delta t = 0.05$. And result obtained for 300 iteration. Fig. 6 is related to U-velocity around one cylinder.

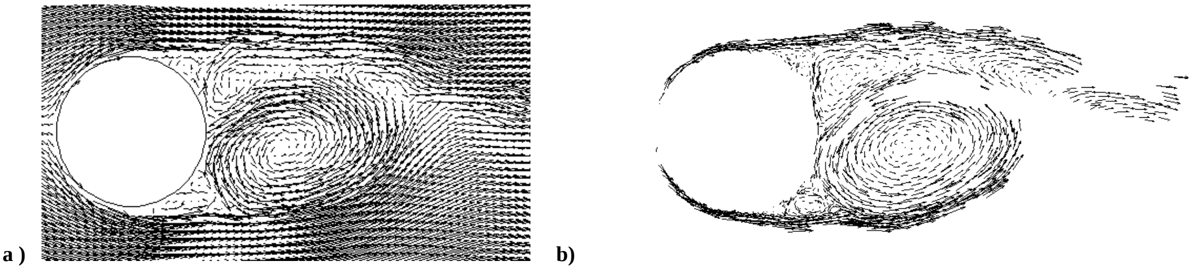


Fig. 5 Flow past a cylinder at *t* = 4 and *R* = 140000. *(a)* Plot of the Instantaneous velocity vector field. *(b)* Plot of the Instantaneous velocity and position of each vortex element in the flow at same time. The flow is asymmetric

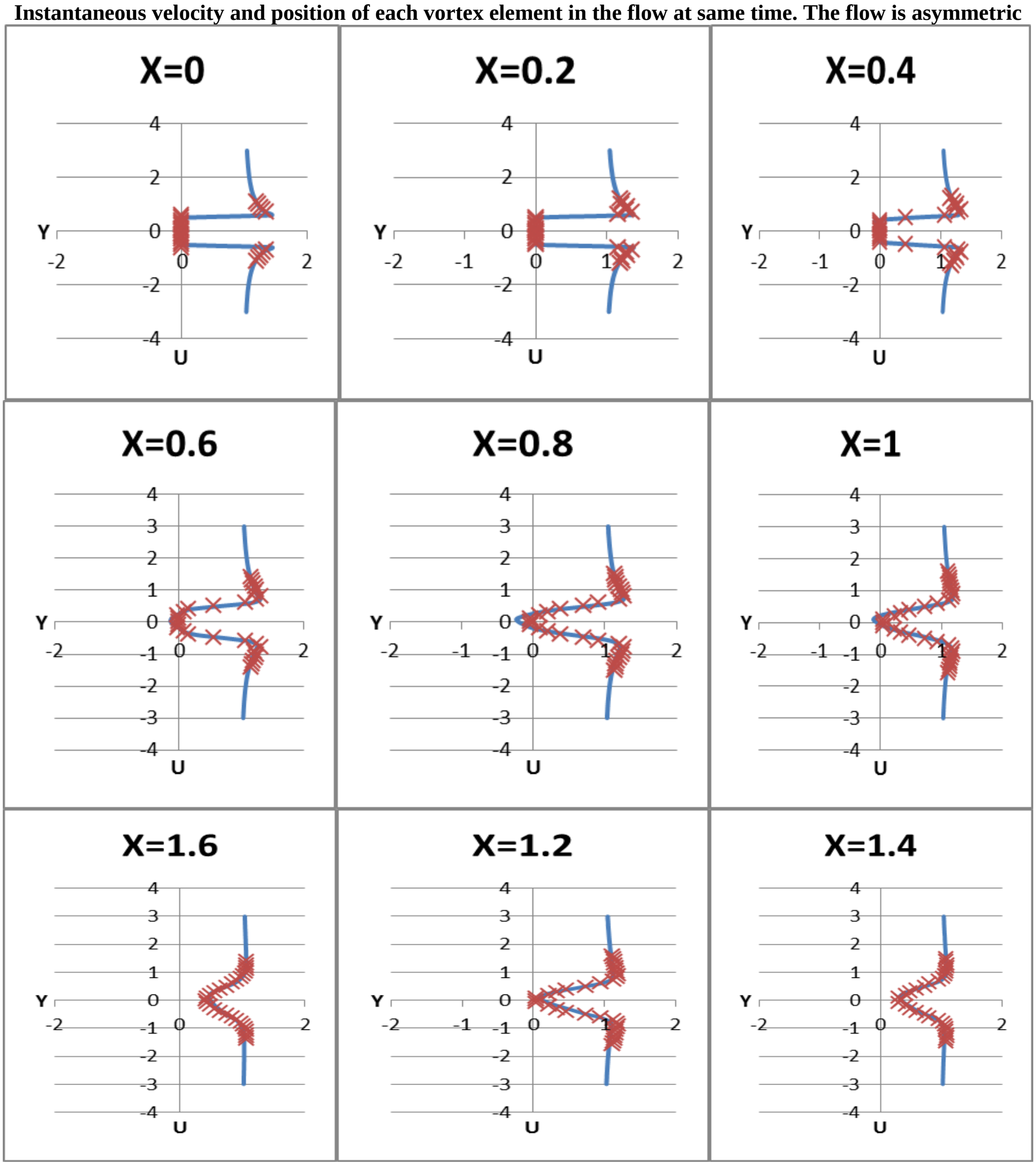


Fig. 6 Obtained results (solid line) for distribution of u-velocity around one cylinder and compared with experimental results (symbol) $\varepsilon_{max} = 0.01$, $\Delta t = 0.05$, $itr = 300$, Re=140000, X=0 to X=1.4

IV. CONCOLUSION

A grid-free random vortex method is used to study the unsteady flow development behind an impulsively started circular cylinder at Reynolds numbers 140000.the flow development is very complex and fast. By tracking the vortex elements we are able to find the relation between the areas of recirculation in the wake and the vortex structures. Our simulation gives detail about the mechanism that controls the development of the wake at very high Reynolds numbers. We indicate that our solution have very good agreement with experimental result. some advantages of vortex methods is as follows :(1) The physical mechanisms in actual flow can be simulated easily by the interactions of computational vortices, (2) the method can be easily used for complex body geometry because it is a grid free method, (3) vortex methods are self- adaptive (4) the method provides an economical simulation of the flow field at high Reynolds numbers when the vorticity is concentrated to narrow regions like wakes and boundary layers, therefore, can be resolved more accurately.

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