Chi Square Method for Testing the Inter-Arrival and Service Patterns

Satish Verma[1], Dr. Sridhar. K[2], Ankit Kashyap[3]
[1,2,3]*(Mechanical Department, C.S.I.T Durg, C.S.V.T.U Bhilai, India)*

Abstract:

Chi-square (X^2) test is a nonparametric statistical analyzing method often used in experimental work where the data consists in frequencies or 'counts' For construction of any simulation model and improvisation needed to be made for any banking service most important task is to understand the pattern which the arrival or service pattern follows. Testing of distribution pattern following or not is a difficult task as it is needed to check its feasibility. In this paper usage of Chi-Square test method is being done to check the nature of arrival and service rate for the concerned bank of study by checking observed pattern with Null-Hypothesis, Poisson's Distribution and Various other Patterns.

Keywords – *Poisson's Distribution, Null-Hypothesis, Critical Values, Degree of Freedom, Chi-Square.*

I. INTRODUCTION

Queue is an undesirable but truth of life. Everyone has to face this queue in one or the other form; it is always desirable to reduce the length of the queue and waiting time for customers to be reduced as low as possible considering all the costs associated with it. For performing any analysis of experiment the basic requirement is the analysis of the pattern of queue which it follows, after that it becomes easy to understand the nature of the queue and in response to that required steps can be taken for its improvement. Hence it became necessary to understand the pattern followed by the queue of our interest. There are many methods available for the testing of the pattern, but out of it Chi-Square test is considered as the most efficient and easy method for testing the nature of Queue and the service provider. This study of pattern of arrival and service is not only useful for Banking Industry but also for every sector where there is a generation of the queue waiting to be served. Some of the examples are in manufacturing industries, in railways for purchasing tickets, in hospital to consult the doctor and in colleges and school for academic works and many more situations. Hence it is clear that everyone has to face this unwanted wastage of time in the queue, and for reducing such waiting time there is a great need of understanding the pattern which can be done with the help of Chi-Square Test. For studying and testing the basic elements of Queuing theory have to be studied, they are:

QUEUE: Queue is defined as the collection of customers in a systemized ways which are waiting for their turn to be served by a service provider or service station. The method of arranging and managing such customers in a well defined and organized way is termed as queuing.

QUEUING SYSTEM: Depending upon the number of customers served by any system, there are two defined types of system. (1) Finite System or closed system and (2) Infinite System or Open System. In Finite system the customers are limited or fixed in number where as there is no such limitations in customers in case of the Infinite or Open system. The bank system which was of concern regarding the paper is Open or Infinite system as the number of customers is unpredictable and always varies and the customer arrives in a random fashion which cannot be determined exactly.

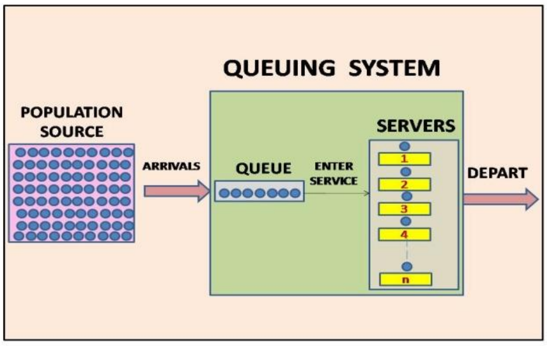

Fig-1: Basic Queuing Model

SERVICE: Service is defined as an activity requested by the customers from the service providers. The rate at which customer arrives per unit time in the service station (Bank) is defined as the arrival rate of customers, it is denoted by a symbol 'lambda (λ)'. The rate at which the service being provided to the customers per unit time is defined as the service rate of the Service provider (Bank), it is denoted by a symbol 'mu (μ)'.

SCHEDULING: The process of organizing queue of customers so as to serve them in a systematic way is termed as scheduling. There are mainly three types of scheduling in a queue management system used by the service provider.

(i) **FCFS (First Come First Served)**: In this scheduling process the customers are being served as per the order of their arrival. The customer which came first will be served first and the one who came last will be served last;

(ii) **RSS (Random Selection for Service)**: In this scheduling process the customers are selected in a random fashion and being served. In this type of Scheduling, the priority among the customers cannot be determined.

(iii) **PRI (Priority Rating Index)**: In this scheduling process the priority is being given to the customers as per the class of service they requested. The Customers which require Higher Class of service is given highest priority and are being served first.

(iv) **SPF (Shortest Processing First)**: In this type of scheduling process the priority is given to the customers depending upon the type of their service i.e. time required to perform that service. The customer requires less processing time will get higher priority and the customer having highest processing or service time got the least priority. The priority is being given irrespective of their arrival.

In Bank Queuing System, depending upon the number of service stations or counters there are mainly three types of Queuing Model.

(1) **Single Queue System:** In this type of system the customer enters into a single queue and waiting for their turn to be coming to get served as they don't have any other option available to them.

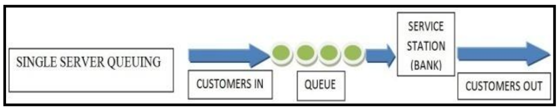

Fig-3: Single Queue System

(2) **Multiple Queue System:** In multiple queue system as there are number of counters available to the customers, the customers on their self-judgment enters the queue whose length is least and thus reducing the waiting time of the queue. The customers can also renege in such type of Queuing system in an effort to reduce the waiting time of the queue.

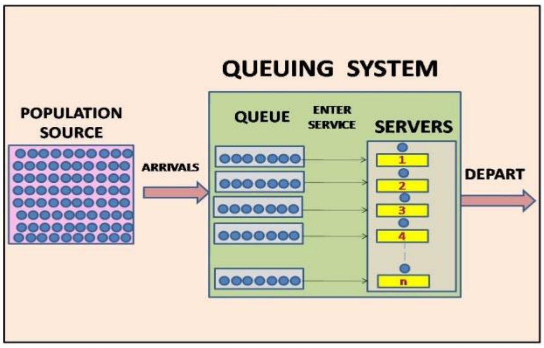

Fig-2: Multiple Queue System

The bank, which we are considering for our study purpose comprises of a single server and single queue system.

<u>**Types of Arrival or Service Patterns**</u>: The arrival or Service pattern can follow one or none of the Theoretical defined Patterns. It was sometimes assumed that arrival pattern follows the Poisson's distribution and Service Providers follows Negative-Exponential pattern, sometimes it was assumed that they follow Null-Hypothesis or Normal or Binomial Distribution pattern, hence it becomes necessary to study all the parameters involved, they are:

a) Arrival Rate (λ): It is defined as the number of customers arrives in the service station to get served with respect to time. It can be of any units depending upon the requirement either number of customers per second or per minute or per hour or per day.

b) Service Rate (µ) : It is defined as the numbers of customers being served per unit time, it has also many different units as per the requirement like Number of Customers per second or per minute or per day etc.

Statistical Hypothesis Testing of Collected Data to Determine the Distribution Pattern.

A Statistical Hypothesis is a scientific hypothesis used to test a data which are either collected from observations or modeled on the basis of random numbers generated. Statistical Hypothesis Testing is used to determine whether the collected data or modeled data are statistically significant or not, over a pre-specified level of significance. Hypothesis tests are used to determine what outcomes of a study would to the rejection of the null-hypothesis for a pre-specified level of significance. Statistical Hypothesis sometimes also calls as Confirmatory Data Analysis as it is used to confirm whether the data following pre-defined distribution pattern or not. There are various Statistical Hypothesis Testing Method available to test the data, some of them are 't-Test', 'z-Test', 'f-Test', 'Chi-Square test' etc. All of the tests have some significance and some limitations; we are using Chi-Square test for our Statistical test for checking of Null Hypothesis and Poisson's Distribution Tests.

<u>**Null – Hypothesis**</u>: According to the null hypothesis, testing it was assumed that the distribution of frequency is expected to be uniform, and same frequency of occurrence is expected for every class of interval. Null-Hypothesis is considered to be true until test, verify other distribution. In Statistics it is denoted as 'H_O' read as "H-nought", "H-null", or "H-zero".

Ho: Distribution does not follow the tested Distribution pattern.

Ha: Distribution does not follow the tested Distribution pattern.

Degree of Freedom: Degree of Freedom is defined as the number of different ways through which the dynamic system can change without affecting the coordinates belonging to that value. The greater the Degree of Freedom, the less is the variation and greater is the accuracy obtained. For a single variable data the value of Degree of Freedom in (n-1) and for multiple Variable data Degree of Freedom = (n-1)(m-1). Where n = number of rows and m = numbers of columns.

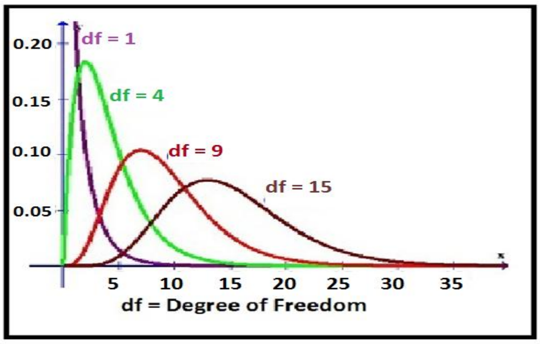

Fig-4: Degree of Freedom for various Curves

Level Of Significance: It represents the level of accuracy associated with the testing procedure, greater the level of significance, greater is the accuracy as lesser is the range of acceptance of variations in the collected data or the Simulated model. For Calculation purpose, we have considered Level of Significance as 1% valued to 0.01. With the help of value corresponding to the respective value of degree of freedom and level of significance also called as 'Alpha Value', the Critical value of Chi-Square Test is obtained.

Critical Value – It is defined as the maximum value of variance to be accepted in Chi-Square Testing, if the value of calculated Chi-Square is less than the Critical value, then we can say that modeled data or collected data is following the tested distribution pattern and the value is not-significant but if not so then it is insignificant. The values of Chi-Square corresponding to level of significance and degree of freedom is shown below in Table-1

Table – 1: Critical values of Chi-Square

	P					
DF	**0.975**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
1	0.000982	1.642	2.706	3.841	5.412	6.635
2	0.0506	3.219	4.605	5.991	7.824	9.210
3	0.216	4.642	6.251	7.815	9.837	11.345
4	0.484	5.989	7.779	9.488	11.668	13.277
5	0.831	7.289	9.236	11.070	13.388	15.086
6	1.237	8.558	10.645	12.592	15.033	16.812
7	1.690	9.803	12.017	14.067	16.622	18.475
8	2.180	11.030	13.362	15.507	18.168	20.090
9	2.700	12.242	14.684	16.919	19.679	21.666
10	3.247	13.442	15.987	18.307	21.161	23.209
11	3.816	14.631	17.275	19.675	22.618	24.725
12	4.404	15.812	18.549	21.026	24.054	26.217
13	5.009	16.985	19.812	22.362	25.472	27.688
14	5.629	18.151	21.064	23.685	26.873	29.141
15	6.262	19.311	22.307	24.996	28.259	30.578
16	6.908	20.465	23.542	26.296	29.633	32.000
17	7.564	21.615	24.769	27.587	30.995	33.409
18	8.231	22.760	25.989	28.869	32.346	34.805
19	8.907	23.900	27.204	30.144	33.687	36.191
20	9.591	25.038	28.412	31.410	35.020	37.566
21	10.283	26.171	29.615	32.671	36.343	38.932
22	10.982	27.301	30.813	33.924	37.659	40.289
23	11.689	28.429	32.007	35.172	38.968	41.638
24	12.401	29.553	33.196	36.415	40.270	42.980
25	13.120	30.675	34.382	37.652	41.566	44.314

Chi-Square Test: 'Chi-Square' pronounced as 'Ki-Square' is a testing method used to validate the Modeled or Observed data. The chi square test is used to calculate the Goodness of Fit of distribution pattern and also it is used to test many other statistical needs. Chi-Square test needs the expected value as per the distribution pattern to be tested, the degree of freedom of the system of concern, level of significance to test the Goodness of Fit.

Expected Value: The distribution pattern to be tested follow some rules for its distribution of frequency for the corresponding value or class interval. Thus, this expected value is compared with the value as per observed or modeled distribution pattern, and the extent of variance is tested among the theoretical and observed or modeled vale, to check the acceptance for Goodness of Fit.

II. LITERATURE REVIEW

Mohammad Shyfur Rahman chowdhury et. al (2013) analyzed different common queuing situations and developed mathematical models for analyzing waiting lines with certain assumptions. Those assumptions are that (A) Arrivals are from an infinite source or a very large population, (B) Arrivals are following Poisson's distribution, (C) Arrivals are considered on a FCFS basis and there is a condition of not to consider balking or reneging, (D) Service times follows the negative exponential distribution and (E) The mean service rate is always greater than the average arrival rate [1]. Toshiba Sheikh et al. (2013) focuses on the M/M/Z/∞: FCFS model is converted into M/M/1/∞:FCFS to know which one of the above mentioned two is more efficient, a line or more lines [2]. Donald Hammond et. al (1995) In this paper an attempt was made to arrive bank teller management policies for providing quality service levels at optimal cost [3]. Dr. Prashant Makwana and Gopalkrushna Patel studied the nature of arrival rate and service rate of customers in a restaurant and thereby studied the queue length, waiting time and utilization factor of the restaurant [4]. Dr. Ahmed and S. A. AL-Jumaily et. al (2007) studied the nature of queue and build the automatic queuing system which automatically switches between the various algorithms available for queuing system as per the requirement based upon service rate and arrival rate [5]. “Probability, Random Variables and Stochastic Processes” by Athanasios Papoulis and S. Unnikrishna Pillai gives the probability distribution pattern for arrival and service time and methods to test distribution pattern.

III. PROBLEM IDENTIFICATION

From the Review of the past work done on this topic, it was found that the observed pattern does not follow the theoretical distribution pattern, because of the following factors present in the real condition, which are required to be removed. Thus making decisions in such cases will become difficulty, thus there arises a need to check whether the observed distribution pattern is following the theoretical model or not. The factors or problems involved in the system are:

a) **Jockeying**: Jockeying is defined as a situation in Queuing Theory where the customer first enters into a queue and when finding other queues waiting time to be less due to higher service rate or less number of customers, customer decides to leave the queue in which he/she waiting to be served and switches to that queue where waiting time is less in comparison. These types of barriers are seen only where there is more than one number of servers and the scheduling does not use the diffused queue method. Due to jockeying it became difficult to calculate the correct value of length of customers and waiting time.

b) **Reneging**: In Queuing Theory Reneging is defined as a situation where customer joined the queue for being served, but depending upon priority if there is some other important work for him/her person decides to leave the queue. Many times long waiting time and length of the queue is a major reason for a person who is jockeying.

c) **Inappropriate Number of Service Stations**: This type of problem can exist at anytime and anywhere, this type of situation arises when the calculated value of required number of service station differs from the actual required number of service stations required.

d) **Throughput time of Service Provider**: This type of barrier is arrived due to a fact that efficiency or working speed differs from person to person, hence sometimes it happens that the estimated value of service rate assumed during calculation is greater than the actual service rate of the person providing service, this leads to increase in waiting time and length of the queue.

e) **Illiteracy of Customers**: Another true and unavoidable fact is that we can only improve the performance for the task which is in our perspective, but we can’t improve the efficiency level of other person. One such situation is arrived when we compare a literate person with an illiterate person, the difference in literacy level results to difference in service time. It is observed that serving a literate person quite easy and time saving when compared to an illiterate person.

f) **Technical Errors**: Technical errors are always a part of the barrier when we consider Queuing model in the banking industry, some of the errors can be removed to an extent when taken precautions and ready for that situation, but there are some of the technical errors also where we are unable to do anything for avoiding such errors. For situations like power cut we can prevent this by making backups for electricity, but if there is a case of server fails, then we can’t solve that typical situation.

g) **Wrong Forecast**: Forecasting plays an important role, as it is a part of managerial decision, much of the Queuing model depends upon the forecast of the situation, if the forecast is perfect, then many of the barriers vanish itself, but it has a negative side too, if the forecast is wrong and there will be a long gap between the actual and the forecasted, then all the barriers get amplified and results in a big error.

h) **Seasonal or Off-seasonal Factors**: Sometime it is required to forecast and change as per the requirement, similar condition arises when there is a sudden change in the arrival rate of the customers normally saw during festival season or any crop season, for interacting with those conditions, proper planning and change is needed to be adopted for facing such errors, otherwise it will lead to a sudden increase in waiting time and queue length.

i) **Balking**: The situation when a person decided not to join on watching the length of the queue itself is termed as Balking. Balking is of great concern when we consider for Queuing system in a bank because it leads to loss of customers to the bank thereby affecting the revenue of the bank. Balking happens due to the one or combination of many such factors creating barriers, the ultimate goal is to remove the balking and increase the revenue of the bank.

j) **Collusion:** Collusion is termed as an agreement between two or more persons or parties or groups either legally or illegally, it is done to limit open competition by misleading others and taking unfair advantage of competition. In Banking or Queuing Theory, Collusion is defined as a situation where a large number of people join or leave a Queue in a bulk, at the same time. Also, it sometimes happens that with due agreement among persons, work of many persons was allotted to a small number of people, which violates the rule of Queuing Theory. It creates an uneven change in Length of Queue.

k) **Cyclic Factors:** Cyclic Factors refers to those factors which comes regularly after a certain period of time, like monthly salary time at first week of the month and crop season for farmers. It is a major factor of concern when considering Queuing Theory, because it is a regular and potential factor where the arrival rate of customer changes after a certain interval due to which requirement of a number of service rates varies over a cyclic period of time.

IV. METHODOLOGY

The method of testing the distribution pattern for inter-arrival and service rate proceeds in a sequential manner mentioned below:

1. Collection of observed data for testing distribution pattern

Collection of data from the bank about the arrival pattern of customers and the service time required by service provider to serve different customers and the system by which scheduling process is done in that bank. The bank of our concern consists of a single server (counter) for customers and the bank adopts First In First Out (FIFO) scheduling for serving the customers. Large amount of data were taken to precisely study the nature of queue in the bank, some part of the data collected was shown below as sample of data collected in Table 1.

Table 2: Collection of data for testing of distribution pattern

A	B	C	D	E	F	G	H	I	J
1.		0	00:00	0		00:00	00:02	2	0
2.		2	00:02	0		00:02	00:04	2	0
3.		2	00:04	0		00:04	00:06	2	0
4.		1	00:05	1		00:06	00:08	2	3
5.		0	00:05	3		00:08	00:09	1	2

NOTATIONS USED IN TABLE 1&4

A – Customer Number
B – Random Number For Arrivals Time Gap
C - Time Gap Between Arrivals
D - Clock Reading
E - Waiting Time
F – Random Number for Service Time
G - Service Start Time
H - Service End Time
I - Service Time
J - Queue Length

2. Calculation of mean for inter-arrival time and service time.

Calculation of mean is very important for calculation of probability as per poisson's distribution, mean can be calculated by using the formula mentioned below;

$$m = \frac{\sum F(x).E(x)}{\sum F(x)},$$

Where;

F(x) = Frequency of occurrence of inter-arrival time gap or Service Time.
E(x) = Inter-arrival time gap or Service Time

Calculating Mean for Inter-arrival Time Gap

Table 3 : Mean for Inter-Arrival Time

Inter-Arrival Time [E(x)](minutes)	0	1	2	3	4	5	6	7	8	9	12	13	15	23	Total
Frequency [F(x)]	18	11	16	7	5	5	4	4	3	5	2	2	2	1	85

$$\text{Mean(m)} = \frac{333}{85} = 3.92 \text{ min}$$

Calculating Mean for Service Time

Table 4 : Mean for Service Time

Service Time [E(x)](minutes)	1	2	3	4	5	6	7	8	9	Total
Frequency [F(x)]	9	31	26	10	2	3	2	1	1	85

$$\text{Mean (m)} = \frac{248}{85} = 2.92$$

3. Calculation of expected frequency as per theoretical distribution pattern

<u>**Expected Value as per Null – Hypothesis**</u>: Null Hypothesis relies on the basis that the frequency of an event is equally distributed among all the Class interval or value.

<u>**Expected Value as per Poisson's Distribution**</u>: As per the poisons Distribution pattern, expected frequency or probability of occurrence of an event can be calculated by using the Poisson's distribution pattern for the corresponding value of arrival rate (λ). The Probability for occurrence of an event is given by;

$$P_m = e^{-\lambda}.\frac{\lambda^m}{m!}.\sum F(x)$$

Where;
m = mean
P_m = Probability of Occurrence of that mean in given interval.
λ = Arrival Rate of Customers
The numerical value of "**e**" is equal to 2.718

4. Calculation Of Chi-Square Value for Testing Goodness of fit and comparing with critical values from table for Goodness of fit,

The Necessary Condition to be taken in the amount when testing the Goodness of fit is that value of observed frequency needed to be greater than or equal to 5, for correct testing, if the value of frequency in observed or modeled distribution is less than 5 then it gets pooled to cross an observed frequency value greater than 5.

Table 5: CHI Square Testing

Time Gap Between Arrivals	Observed Frequency (O)	Expected Frequency (E)	(O-E)^2	(O-E)^2/E
0	18	6.071	142.32	23.45
1	11	6.071	24.32	4.01
2	16	6.070	98.60	16.24
3	7	6.070	0.86	0.14
4	5	6.070	1.14	0.19
5	5	6.070	1.14	0.19
6	4	6.070	4.28	0.71
7	4	6.070	4.28	0.71
8	3	6.070	9.42	1.55
9	5	6.070	1.14	0.19
12	2	6.070	16.56	2.73
13	2	6.070	16.56	2.73
15	2	6.070	16.56	2.73
23	1	6.070	25.70	4.23
∑=14	∑=85	∑=85		$x^2 = \sum \frac{(O-E)^2}{E} =$ 59.99

Degree of Freedom = (14-1) = 13
For DOF= 13 and Level of Significance; p=1% = 0.01
Critical Value of $x^2 = 27.688$
59.99 > 27.688; Calculated > Critical. Hence it is significant.
Not Good for Fit.

From the above table we observed that the observed frequency belonging to 12,13,15 and 23 minute of time gap between arrivals is less than 5, therefore all the observed frequency belonging to 12,13,15 and 23 minutes time gap between arrivals are polled up to make observed frequency greater than 5. Similarly; (6,7) and (8,9)Now the data is ready for being tested for distribution test by Chi-Square test.

Table 6: CHI Square Test of Time Gap Between Arrivals For Null-Hypothesis

Time Gap Between Arrivals	Observed Frequency (O)	Expected Frequency (E)	(O-E)^2	(O-E)^2/E
0	18	9.440	73.27	7.76
1	11	9.445	2.43	0.26
2	16	9.445	43.03	4.56
3	7	9.445	5.95	0.63
4	5	9.445	19.71	2.09
5	5	9.445	19.71	2.09
7	8	9.445	2.07	0.22
9	8	9.445	2.07	0.22
15	7	9.445	5.95	0.63
∑=9	∑=85	∑=85		$x^2 = \sum \frac{(O-E)^2}{E} =$ 18.46

Degree of Freedom = (9-1) = 8
For DOF= 8 and Level of Significance; p=1% = 0.01
Critical Value of $x^2 = 20.090$
18.46 < 20.090; Calculated < Critical. Hence it is not-significant.
Holds null Hypothesis and hence Goodness of fit.

Table : 7 : CHI Square Test of Time Gap Between Arrivals For Poisson's Distribution

Time Gap Between Arrivals	Observed Frequency (O)	Expected Frequency (E)	(O-E)^2	(O-E)^2/E
0	18	1.683	267.98	159.22
1	11	6.545	19.85	3.03
2	16	12.920	9.49	0.73
3	7	16.915	98.31	5.81
4	5	16.575	133.98	8.08
5	5	13.005	64.08	4.93
7	8	13.09	25.91	1.98
9	8	3.315	21.95	6.62
15	7	0.952	36.58	38.42
∑=9	∑=85	∑=85		$x^2 = \sum \frac{(O-E)^2}{E} =$ 228.82

Degree of Freedom = (9-1) = 8
For DOF= 8 and Level of Significance; p=1% = 0.01
Critical Value of $x^2 = 16.19$
228.82 > 20.090; Calculated > Critical. Hence it is significant.
Doesn't hold Poisson's distribution and hence does not hold Goodness of fit .

Table 8 : CHI Square Test of Service Time For Null-Hypothesis

Time Gap Between Arrivals	Observed Frequency (O)	Expected Frequency (E)	(O-E)^2	(O-E)^2/E
1	9	17	64	3.765
2	31	17	196	11.53
3	26	17	81	4.765
4	10	17	49	2.88
7	9	17	64	3.76
∑=5	∑=85	∑=85		$x^2 = \sum \frac{(O-E)^2}{E} =$ 26.7

Degree of Freedom = (5-1) = 4
For DOF = 4 and Level of Significance = 1% = 0.01
Critical Value of $x^2 = 13.277$
26.70 > 13.277; Calculated > Critical. Hence it is significant.
Doesn't Holds null Hypothesis and does not hold Goodness of fit.

Table 9 : CHI Square Test of Service Time For Poisson's Distribution

Time Gap Between Arrivals	Observed Frequency (O)	Expected Frequency (E)	$(O-E)^2$	$(O-E)^2/E$
1	9	13.345	19.45	1.457
2	31	19.465	311.70	15.867
3	26	19.041	41.60	2.185
4	10	13.855	81.72	5.898
7	9	19.239	362.90	18.863
∑=5	**∑=85**	**∑=85**		$x^2 = \sum\frac{(O-E)^2}{E} =$ 44.27

Degree of Freedom = (5-1) = 4
For DOF= 4 and Level of Significance; p=1% = 0.01
Critical Value of x^2 = 13.277
44.27 > 13.277; Calculated > Critical. Hence it is significant.
Doesn't holds Poisson's Distribution and hence does not Goodness of fit.

V. RESULTS

From testing of the collected data as per Chi-Square method, it was found that observed distribution pattern for Inter-arrival time follows only null-hypothesis, whereas the distribution pattern for service time does not follow and theoretical distribution pattern, this difference is present due to the irregular arrival and service time for exceptional cases which differs the observed pattern from the theoretical distribution pattern.

VI. CONCLUSIONS

Chi-Square testing was found to be an easy and reliable method to test for distribution pattern. The difference in the observed pattern from the theoretical distribution pattern can be easily understood by observing the Chi-Square test table, and some work can be performed to remove such exceptions, if it is needed to make the observed pattern as the theoretical distribution pattern.

REFERENCES

[1] Mohammad Shyfur Rahman Chowdhury, Mohammad Toufiqur Rahman and Mohammad Rokibul Kabir, (Jun-2013), " Solving Of Waiting Lines Models in the Bank Using Queuing Theory Model the Practice Case: Islami Bank Bangladesh Limited, Chawkbazar Branch, Chittagong" IOSR Journal of Business and Management (IOSR-JBM) Volume 10, Issue 1 (May. - Jun. 2013), PP 22-29.

[2] Toshiba Sheikh, Sanjay Kumar Singh, Anil Kumar Kashyap (2013), "Application of Queuing theory for the Improvement of Bank Service" International Journal of Advanced Computational Engineering and Networking, ISSN: 2320-2106 Volume- 1, Issue- 4, June-2013.

[3] Donald Hammond and Sathi Mahesh, (1995), "Proceedings of the 1995 Winter Simulation Conference.

[4] Dr. Prashant Makwana and Gopalkrushna Patel "Minimizing The Waiting Time For Service With Queuing Model".

[5] Dr. Ahmed S. and A. AL-Jumaily "Automatic Queuing Model for Banking Applications" (IJACSA) International Journal of Advanced Computer Science and Applications,Vol. 2, No. 7, 2011.

[6] "Probability, Random Variables and Stochastic Processes" by Athanasios Papoulis and S.Unnikrishna Pillai 4th-Edition.