# One Degree of Freedom Manipulator Modelling and Control

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#### Abstract

This paper presents a traditional PID control applied to a robotic manipulator arm with one DOF. Using MATLAB, the motion control simulation of this system was implemented in preparation for actual experiments. The results of the simulation demonstrated that the response time of the system was in a reasonable range, the motion behaviour of the platform was stable and acceptable. Furthermore, the parameters of the controllers were optimized using MATLAB and better results for the time response were obtained. Finally, the experimental results demonstrate that the proposed controller can work probably in limited range of system parameters change.

**Keywords:** *PID control, Dynamic model, Robotic, manipulator arm.* 

### I. INTRODUCTION

The first step in control system design is to obtain the mathematical model, which, describe the dynamics of the plant to be controlled. More accurate dynamic model of the plant led to better control system performance. However in some cases in order to simplify the model, some assumptions must be made including linearization of some components.

Traditionally, system identification relied on a priori knowledge such that mechanistic models, which completely describe the system can be constructed, empirical data is then used to validate these identified models. Cai [1], defined three basic levels of model synthesis:

## 1) White Box

A complete mechanistic model is constructed from a priori knowledge and physical insight. Here, system observations are not used during model identification and are only used for validation. Complete system knowledge of this kind is very rare, and usually some aspects of the system's behaviour are unknown.

## 2) Grey Box

An incomplete model is constructed from a priori knowledge and physical insight; their empirical data is available to infer several unknown parameters. Generally the form of the model can be specified and observational data is used to find specific physical parameters.

#### 3) Black Box

No physical knowledge is used to construct the model. The model structure is chosen as a flexible parameterised function, which is used to fit the observations.

These different approaches to system identification are not as distinct as this classification suggests, and a general rule of thumb is to employ as much a priori knowledge and physical insight as possible. However, the a priori knowledge utilised above is of a mechanistic nature, describing the physics of the system, which is typically unavailable or at least very time consuming to acquire. A priori knowledge can come in many different forms. As an example, an engineer familiar with a system may be able to concisely describe its operation linguistically, with knowledge of the appropriate inputs and only a very rudimentary understanding of the underlying mechanics. This type of knowledge can be of a great benefit to the modelling process. In the coming part the linear model of the single actuator system is developed.

#### **II. SYSTEM DESCRIPTION**

The experimental layout of the electrohydraulic position control system is shown in Figure 1. The control signal is sent from the computer and is put into the servo valve through the D/A converter. The oil flow can be regulated through the valve. The pressure difference between the cylinder chambers is built up and then the position of the cylinder can be under control. The position of the cylinder is measured by a pulse scale, which output is sent to the computer as a feedback signal.

The convenient form of the equation of the servo valve is, [2-5]:

$$\frac{d^2 x_v}{dt^2} + 2\zeta_v \omega_v \frac{dx_v}{dt} + \omega_v^2 x_v = \omega_v^2 k_v k_a u$$
(1)

where the saturation of the servo valve amplifier is considered,

$$u = \begin{cases} u_{sat} & \text{if } u \ge u_{sat} \\ u & \text{if } -u_{sat} \le u \le u_{sat} \\ -u_{sat} & \text{if } u \le -u_{sat} \end{cases}$$
$$q_f = c_d w x_v \operatorname{sgn}(P_s - p_f) \sqrt{\frac{2}{\rho} |P_s - p_f|} ,$$





Figure 1 Schematic Diagram of the Electro-Hydraulic System (one DOF Manipulator)

For  $x_v \leq 0$ 

$$q_{f} = c_{d} w x_{v} \operatorname{sgn}(p_{f}) \sqrt{\frac{2}{\rho}} |p_{f}|,$$
$$q_{n} = c_{d} w x_{v} \operatorname{sgn}(P_{s} - p_{n}) \sqrt{\frac{2}{\rho}} |P_{s} - p_{n}|$$
(3)

The flow equations of the actuator are given by:

$$q_{f} = A_{f} \dot{x}_{p} + \frac{V_{f}}{B} \dot{p}_{f}$$
$$q_{n} = A_{n} \dot{x}_{p} - \frac{V_{n}}{B} \dot{p}_{n}, \quad (4)$$

where

$$V_n = V_{no} - A_n x_p$$

It is assumed that the loading point may be treated as a mass-damper system. The equation for the force developed by the actuator on the loading point can be written as

 $V_f = V_{fo} + A_f x_p$ 

$$M \ddot{x}_{p} + B_{l} \dot{x}_{p} + \operatorname{sgn}(x_{p}) F_{r} = A_{f} p_{f} - A_{n} p_{n} \quad (5)$$

## A. Some Nonlinearities of the Model

The servo-valve possesses several nonlinearities. They are principally, the change in flow gain near null, the nonlinear nature of the pressure-flow relationship, torque-motor hysterics, flow saturation and valve spool friction. It will be assumed that the electronic control system is designed to work below the signal saturation limits therefore the saturation in this stage can be neglected, [6].

### **B.** Nonlinear Flow Gain Characteristic

In Figure 2 the change of flow near null can be attributed to manufacturing tolerances, [5]. If the width of the spool land is smaller than the port in the valve sleeve where the spool is at neutral, the valve is said to be under-lapped. A zero-lapped valve has a spool land identical to the port width. It is also showed that within the under lap region, the underloped valve may have a flow gain twice that of the zero-lapped case, [7-10]. For an overlapped valve, the theoretical flow in the lapped region should be zero resulting in a dead band. However because of practical limitation, minute radial clearances do occur, resulting in some leakage flow leaving the load ports. This gives the reduced flow gain at null (one half of that of zero-lapped valve).



Figure 2 Flow gain characteristic



Figure 3 Leakage-Flow Characteristic

The leakage dominates the performance of the valve at null. A typical curve of leakage flow against valve current under block line condition is shown in Figure 3. The leakage flow curve consists of two parts. One of them is independent of the spool displacement whereas the other is largely dependent on the spool displacement.

The former is mainly due to leakage in the first-stage of the servo valve. This leakage represents a direct power loss but has no other significance. The latter is second-stage null leakage and this affects the damping of the drive at null, [10].

#### C. Nonlinear Pressure-Flow Characteristic

Typical plots of the normalized valve pressure flow curves ate given in Figure 4. The effect of the null leakage on the load pressure is shown in Figure 5. Therefore care should be taken when interpreting the curves in Figure 2.5 within the null region.

and



Figure 4 Normalised Valve Pressure-Flow Curves



Figure 5 Load Pressure-Input Current

#### **D.** Friction Nonlinearities

The nonlinear friction characteristics for an electrohydraulic actuator are very complex due to the interaction of such factors as the bearing design, e.g. the type of lubricant used, the contact stiffness and the quality of slide-way alignment. No general model has yet been developed to predict the performance of this non-linearity.

When motion has started the friction level only changes over to the coulomb level when a certain velocity, called the critical velocity, is reached. The velocity range between plus and minus the critical velocity, is termed the stick-slip region. Once outside this region, stick does not re-appear until the velocity falls to zero.

The static and coulomb friction can be summarised as follows

$$F_{f} = F_{\mu} \operatorname{sgn}(X_{p}) \quad for |X_{p}(s)| \ge V_{cr}$$

$$F_{f} = F_{st} \quad for |X_{p}(s)| < V_{cr}$$

$$(6)$$

where  $F_f$  is nonlinear friction force,  $F_{\mu}$  is coulomb friction,  $F_{st}$  is static friction,  $V_{cr}$  is critical velocity

The first difficult problem encountered in controller tuning is to define what is "good" control, and this unfortunately differs from one to another. The most common practise employed is to adjust the controller to meet the following main performance criteria:

- 1- Accuracy
- 2- Speed of response
- 3- Robustness

The controller parameters are to be selected to minimise the error between the controlled variable

and the set point. With the advent of digital computers, various methods of optimisation may be applied to choose the controller constants to reach the optimal performance.

PID controllers are widely used in industry due to their simple structure for many systems. 90% of the control loops in industrial control are PID controllers, [11-13].

#### **III. TUNING OF PID CONTROL**

PID controllers has remind as an art rather than an exact science. The reasons for this are simple: nobody really knows what the settings should be, since all criteria are qualitative in nature and, because of the large self regulatory capacity of the most process systems, the margin of tolerance is high. The first step in the design strategy is to install and tune a PID controller. The ideal continuous PID controller

$$u = K_{p} \left( e + \frac{1}{T_{i}} \int_{0}^{t} e^{*} d\tau + T_{d} \frac{de}{dt} \right)$$
(7)

returns the controller output u, the constant  $K_p$  is the proportional gain,  $T_i$  is the integral time,  $T_d$  the derivative time, and e is the error between the reference and the process output y, (e=r-y). We are concerned with digital control, and for small sampling periods  $T_s$ , the equation may be approximated by a discrete approximation. Replacing the derivative term by a backward difference and the integral by a sum using rectangular integration, an approximation is

$$u_{k} = K_{p} \left( e_{k} + \frac{1}{T_{i}} \sum_{j=1}^{k} e_{j} T_{s} + T_{d} \frac{e_{k} - e_{k-1}}{T_{s}} \right)$$
(8)

Index k refers to the time instant. By tuning we shall mean the activity of adjusting the parameters  $K_p$ ,  $T_i$ , and  $T_d$ .  $K_P$  should be moderate, otherwise the system will be too sensitive to noise. If the process dynamics are considered, the closed loop system will normally be unstable if  $K_P$  is high. Obviously the setting of  $K_P$  is a balance between the control objectives: stability, noise sensitivity, and load regulation.

Just as in continuous systems, there are three basic types of control: Proportional, Integral, and Derivative, hence the name, PID. In the design examples so far, it has been using the discrete equivalent of lead compensation, which is essentially a combination of proportional and derivative control. Let us now review these three controls as they pertain to a discrete implementation. The term PID is widely used because there are commercially available modules that have knobs for the user to turn and set the values of each of the three control types, which expressed by:

$$D(z) = K_p \left( 1 + \frac{T_z}{T_i(z-1)} + \frac{T_d(z-1)}{T_z} \right)$$
(9)

The previous transfer function of PID controller is conve

| Table 2 Step input response results |                |            |           |       |
|-------------------------------------|----------------|------------|-----------|-------|
| Controller                          | T <sub>d</sub> | Ts         | Tr        | for   |
| PID <sub>Th</sub>                   | 0.05 sec.      | 0.124 sec. | 0.15 sec. | imple |
| PID <sub>P</sub>                    | 0.056 sec.     | 0.123 sec. | 0.16 sec. | ment  |
|                                     |                |            |           | ation |

into the digital computer. These equations can be directly evaluated by computer provided that one past value of the input and the output have been saved.

## **IV. RESULTS**

A software program utilizing C++ language is developed based on Rung Kuta fifth order with tolerance = 0.0001 to perform the difference equation through the PC.

In many practical cases, the desired performance characteristics of control systems are specified in terms of time domine quantities.

The transient response of a system to a step input specify the following:

- Delay time: the time required for the response to reach half the final value the very first time.
- Rise time: the time required for the response to rise from 0% to 100% of the final value (for under damped system).
- Settling time: The time required for the response curve to reach and stay within 5% of the final value and it is related to the largest time.
- Steady state error

The real time implementation was carried out using the same sampling time (3msec) and the same reference step input. The control algorithm is implemented by a personal computer, based on a 533 MHz Celeron processor.

The simulation and practical results to step reference based on PID controller are shown in Figures 6. The summary of comparison is shown in Table 1 based on the delay time  $(T_d)$ , the rise time  $(T_r)$  and the settling time  $(T_s)$ .

The PID control action is shown in Figure 7, the controller is respond to the error, derivative of the error and its integral to minimise all of them to zero, and then keep constant value.

 $Th.=Theoretical,\ P=Practical$ 

#### Table 1 Rules of Thumb for Tuning PID Controller.

| Action           | <b>Rise</b> time | Overshoot | Stability  |
|------------------|------------------|-----------|------------|
| Increase $K_p$   | faster           | increases | gets worse |
| Increase $T_d$   | slower           | decreases | improves   |
| Increase $1/T_i$ | faster           | increases | gets worse |



Figure 6 System Step Response Based On PID Control



Figure 7 Controller Signal For Step Input Based On PID

Next, the time responses of two tests were carried out with the same reference step input. One is with a spring  $(7 \times 10^5 \text{ N/m})$ , which was added in the half stroke of the cylinder as a disturbance for the hydraulic system as shown in Figure 8, and the other is without spring.

Figures 9 show that with the traditional PID control in Figure 8, the delay time difference between the case with spring and without spring was 0.023 sec. and the error was 1.4% at time of 0.5 sec. But when the integral gain was duplicated, the error recovered at time of 0.6sec.

We noticed also the response with PID controller affected by the spring action, so the advanced controller for the complex and highly nonlinear system is strongly required.

The control problem of electro hydraulic cylinder is studied in this paper. A PID controller has been given. It is shown that the control algorithm is simple but it need price model, there is a difference between the simulation and the practical work according to the fact that PID is linear control and the used plant actually is not linear, but it has many nonlinearity as explained before and finally with the load variation there is response variations, this mean the controller is not robust.



Figure 8 Schematic Diagram of the Combination of The Spring with the System.



Figure 9 System Step Response Based on PID Control With and without Spring

#### V. CONCLUSION

The control problem of electro hydraulic cylinder is studied in this paper. A PID controller has been given. It is shown The main advantage of the proposed controller:

- The control algorithm is simple
- The algorithm is easy to implement even on small microprocessor systems.
- Through the simulations and experiments using the servo valve controlled cylinder system, the effectiveness of the PID control was confirmed.

The main disadvantage of the PID:

- It need very price model
- There is a difference between the simulation and the practical work according to the fact that PID is linear control and the used plant actually is not linear, but it has many nonlinearity as explained before.

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