Dynamically Spring Balanced Slider-Crank Mechanism for Reciprocating Machines

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Abstract
Spring balancing is a method commonly applied for statically balanced mechanism. Its field covers applications where inertia forces are relative small compared to the weight of the components. Spring balancing is also a lighter alternative to balancing with counterweights. This type of balancing can be often found in mechanical applications, biomechanics and robotics. One of the most acknowledged models of spring balanced mechanisms is the Anglepoise Lamp. This method is well known in the field of mechanisms balancing, but it has not yet been applied for balancing of the slider-crank mechanism. Nevertheless elastic components have previously been considered. In this paper, the dynamic balancing of the slider-crank mechanism by means of springs is proposed. The method aims to balance the shaking forces without the addition of masses to the mechanism. A series of multibody simulations will illustrate the effects of the proposed balancing method.

Keywords — *Dynamic balancing, spring balancing, shaking force.*

I. INTRODUCTION

Current applications of reciprocating mechanisms include heat pumps, auxiliary power units (internal combustion engines), pneumatic motors, etc. Those applications mostly use single cylinder designs with low gas pressure force relative to the inertia of the moving components. In these cases the main source of shaking forces and shaking moment comes from the motion of the slider-crank mechanism components. The most common mean to balance these mechanisms is by using counter masses on the crankshaft, by having optimal distribution of the masses along the components of the mechanism or by using counter-rotating masses [1]-[3]. In all cases the counter masses only partially balances shaking forces and this happens at the cost of additional mass of the devices on which they operate. The increased mass requires additional power and generates additional wear and stress that can lead to fatigue of the components. Current legislation enforces efficiency labeling for household appliances, internal combustion engines, and other applications [4], [5]. These restrictions compel manufacturers to come with energy efficient alternative means of balancing.

Along the efficiency labeling devices such as air compressors for refrigerators and commercial electric current generators are required to display noise ratings [6], [7]. In these cases the vibration of the reciprocating machine accounts for part of the generated noise. Elastic mounts are often used to connect the reciprocating machines to frames. This improves noise ratings but increases the amount of space required for installation. Thus well balanced equipment can result in improved efficiency and improved comfort. Another way to improve the behavior of reciprocating machines is to predict the efficiency and specific losses for a particular design at a specific set of working parameters. Such a prediction can help the development process and thus design a reciprocating machine that operates at its optimum efficiency [8].

The spring balanced slider-crank mechanism aims to improve both criteria without the addition of the conventional counter mass.

A mathematical model of the spring balanced slider crank mechanism will be built. The model contains the equations that describe the motion of the slider-crank mechanism as well as spring forces.

II. MODEL

The developed model is based on the slider-crank mechanism dynamics. As it can be seen in Figure 1, the forces can be split in two directions. One direction is considered parallel with the piston axis, y, and the other one is perpendicular on it, x. All acting forces will be considered relative to these axes [9].

To have a simplified model the links will be reduced to point masses, so the slider-crank mechanism will be reduced to two point masses. The first mass point is considered at the intersection of the piston axis and the bolt axis. This point has the mass of the piston assembly and the reduced mass from the upper part of the connecting rod. The second point is considered at the center of the crank joint. The point has the reduced mass of the lower part of the connecting rod and the reduced mass of the crank. The coordinates of the points relative to the axis of the crankshaft are:

$$y_1 = r\cdot\cos(\varphi)+l\cdot\sqrt{1-n^2\cdot\sin^2(\varphi)},$$
(1)

$$x_1 = 0,$$
(2)

$$y_2 = r\cdot\cos(\varphi),$$
(3)

$$x_2 = r\cdot\sin(\varphi),$$
(4)

where y_1 is the vertical displacement of the piston, r is the crank radius, φ is the angle between the crank and piston axis, l is the connecting rod length, x_2 is the horizontal displacement of the crank, y_2 is the vertical displacement of the crank and n is the ratio between the connecting rod length and crank radius.

There is only one excitation acting on horizontal direction. This is given by the moving mass of the second point. The inertia force is given by the point's mass and the second derivative of y_2 with respect to time. Therefore the horizontal excitation is a simple harmonic function:

$$F_x = -m_{cr}\cdot\omega^2\cdot r\cdot\sin(\varphi),$$
(5)

where ω is the crankshaft speed. The crankshaft speed is considered constant.

Equation (5) has null values at $\varphi_1=0$ and $\varphi_2=\pi$.

There are two vertical excitations, F_{yp} and F_{ycr}. F_{yp} is the caused by the first mass point and F_{ycr} is caused by the second point mass. Both vertical excitations depend on φ.

$$F_{yp} = -m_p\cdot\omega^2\cdot r\cdot\left(\cos(\varphi)+\frac{(n^2-1)\cdot\cos(2\varphi)+\cos^4(\varphi)}{n^3\cdot\left[1-n^{-2}\cdot\sin^2(\varphi)\right]^{1.5}}\right),$$
(6)

$$F_{ycr} = -m_{cr}\cdot\omega^2\cdot r\cdot\cos(\varphi), \quad (7)$$

$$F_y = F_{yp}+F_{ycr}. \quad (8)$$

The vertical excitation has a fundamental harmonic component and a second order harmonic component. Most of the reciprocating machines have a counter mass on the crankshaft which balances F_{ycr}. The method suggested in this paper does not contain a counter mass on the crankshaft.

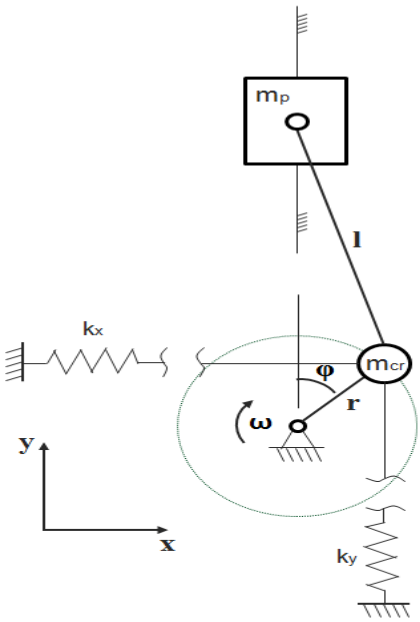

Figure 1 Spring Balanced Slider-Crank Mechanism

Two springs are required in order to counteract the horizontal and vertical excitations. Figure 1 illustrates a possibility to connect the crank joint to springs. The distance between the spring support and the slider-crank mechanism is considered long enough that the springs remain parallel to the x and y axis during operation.

III.SPRING CALCULATION

The shaking forces must be calculated. As previously stated, the coordinates of the crank and piston are determined relative to the axis of the crank. For perfect balancing the spring reactions must have values equal the excitation forces. Thus:

$$k_x\cdot\delta_x = F_x,$$
(9)

$$k_y\cdot\delta_y = F_y,$$
(10)

where k_x is the spring rate of the horizontal spring, δ_x is the deflection of the horizontal spring, k_y is the spring rate of the vertical spring and δ_y is the deflection of the vertical spring.

At φ_1 and φ_2 (fig. 2) the horizontal spring is in a relaxed state.

$$\delta_x(\varphi_1) = \delta_x(\varphi_2) = 0,$$
(11)

where $\delta_x(\varphi_1)$ is the horizontal spring deflection at the crankshaft angle φ_1 and $\delta_x(\varphi_2)$ is the horizontal spring deflection at the crankshaft angle φ_2.

At φ_3 and φ_4 the vertical spring is in a relaxed state.

$$\delta_y(\varphi_3) = \delta_y(\varphi_4) = 0,$$
(12)

where $\delta_y(\varphi_3)$ is the vertical spring deflection at the crankshaft angle φ_3 and $\delta_y(\varphi_4)$ is the vertical spring deflection at the crankshaft angle φ_4.

The springs are connected to the crank, the spring deflection can be written as function of the crank rotation, se equations (9) and (10) become:

$$k_x \cdot r \cdot \sin(\varphi) = F_x \,, \quad (13)$$

$$k_y \cdot r \cdot \cos(\varphi) = F_y \,, \quad (14)$$

According to equation (5), F_x has its minimum and maximum values at $\varphi_3 = \pi/2$ and $\varphi_4 = 3\pi/2$. Therefore the spring reaction achieves the minimum and maximum value at φ_3 and φ_4. Thus:

$$k_x \cdot r \cdot \sin\left(\frac{\pi}{2}\right) = -F_x\left(\frac{\pi}{2}\right), \quad (15)$$

$$k_x \cdot r \cdot \sin\left(\frac{3\pi}{2}\right) = -F_x\left(\frac{3\pi}{2}\right), \quad (16)$$

where $F_x(\pi/2)$ and $F_x(3\pi/2)$ are the horizontal forces when the crankshaft angle is at φ_3 and φ_4.

The horizontal spring has its minimum and maximum force defined according to equations (15) and (16)

The vertical spring must also be constrained. Unlike the horizontal spring, it has two different components, F_{yp} and F_{ycr}. Therefore each component has its zero value at different values of the crankshaft angle. F_y has zero value at φ_5 and φ_6. The maximum and minimum value of F_y is achieved at φ_1 and φ_2. Equivalent with the horizontal spring, the vertical spring is also constrained.

Figure 2 shows a simplified system with the critical crankshaft angle values.

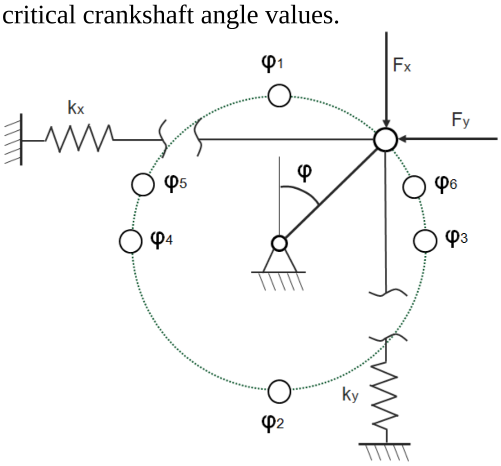

Figure 2 Simplified Model of the Spring Balanced Slider-Crank Mechanism

The spring behaviour is defined by its rate which is expressed in force vs. spring deflection. The excitations are relative to the crankshaft angle, φ. In order compare the development of the excitation and the spring reaction, both forces must be defined relative to displacement. Thus both excitations will be displayed relative to sin (φ) and cos (φ).

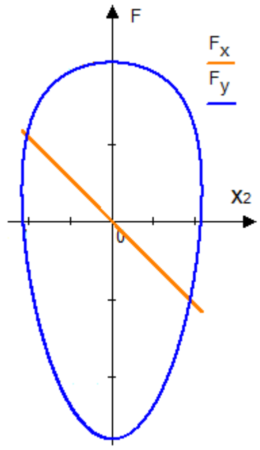

Figure 3 Excitations Relative to Horizontal Displacement

Figure 3 illustrates a linear behavior of F_x relative to horizontal displacement. Therefore the development of F_x can be replaced with a spring that has a linear rate. The spring rate can be defined as:

$$k_x = \frac{-m_{cr} \cdot \omega^2 \cdot r \cdot \sin(\varphi)}{r \cdot \sin(\varphi)} \,. \quad (17)$$

Figure 4 illustrates a non-linear nature of F_y relative to vertical displacement. Therefore the horizontal spring force F_y cannot be achieved with a spring that has a linear rate. A spring with progressive rate must be used.

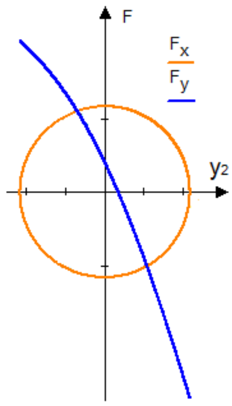

Figure 4 Excitations Relative to Vertical Displacement

The F_y curve is defined by equation (8). It is also known that:

$$k_y(\varphi_5) \cdot r \cdot \sin(0) = k_y(\varphi_6) \cdot r \cdot \sin(\pi) = 0 \,. \quad (18)$$

Solving equation (8) for φ_1 and φ_2 gives the minimum and maximum spring reactions. The solutions of equation (8) φ_5 and φ_6 indicate the crankshaft angle at which the spring reaction is null.

Thus the ideal spring reaction force is known in four points, φ_1, φ_2, φ_5 and φ_6.

A progressive spring is characterized by the fact that its spring rate is dependant to its deflection. Therefore the spring rate increases if the spring is compressed from its free length and decreases is the spring is expanded.

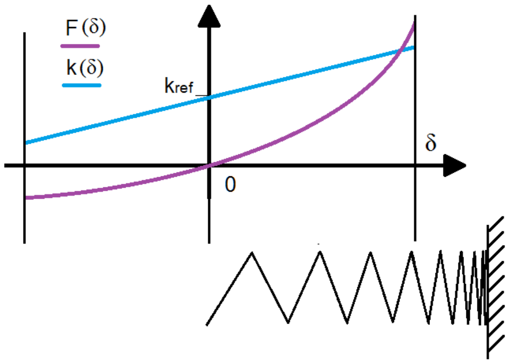

Figure 5 Typical Progressive Spring Curve

The spring rate is defined as:

$$k(\delta) = k_{ref} + c \cdot \delta \,, \quad (19)$$

where k_{ref} is the spring rate when the spring is in a relaxed state and *c* is the spring rate increment.

The force of the progressive spring can be calculated as:

$$F_{sy}(\delta) = k_{ref} \cdot \delta + c \cdot \delta^2 \,, \quad (20)$$

where F_{sy} is the force of the vertical progressive spring.

The solutions of equation (8) indicate the crankshaft angles at which $F_{sy} = 0$. In order to have complete balance, the progressive spring should be in relaxed state when the crankshaft angle is at φ_5 and φ_6.

Considering that $F_{sy}(0)=Fy(\varphi_5)=Fy(\varphi_6)=0$ the vertical spring deflection can be calculated as:

$$\delta = r \cdot \cos(\varphi) - r \cdot \cos(\varphi_5) \quad (21)$$

The limit values of Fs_y are known and also the slope at φ_5. The curve coordinates are determined relative to φ_5.

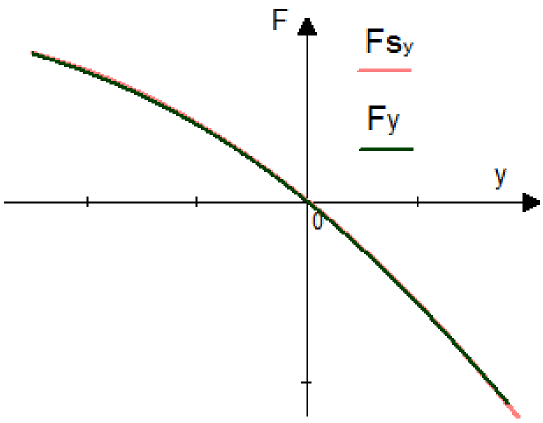

Figure 6 Vertical Excitation and Vertical Spring Reaction

As seen in Figure 5 F_{sy} overlaps with F_y. Both curves are displayed offset on the y axis. The origin of the system has been moved with r·cos (φ_5) from the axis of the crankshaft.

IV. MULTIBODY MODEL AND SIMULATION

Values are given for k_y, k_y, l_x and l_y. The physical parameters of the multibody model are taken from a commercial internal combustion engine used for electric current generators.

Table 1: Parameters of Multibody Model

Parameter		Value
Piston assembly mass	m_p	0.412 kg
Upper crank reduced mass	m_{cr1}	0.231 kg
Lower crank reduced mass	m_{cr2}	0.380 kg
Crank radius	r	43.2 mm
Crank length	l_c	144 mm
Engine speed	ω	120π rad/s
Horizontal spring rate	k_x	42 kN/m
Horizontal spring preload	F_{px}	9 kN
Vertical spring rate (reference)	k_y	150.7 kN/m
Spring rate increment	c	$1.66 \cdot 10^6$ N/m^2
Vertical spring offset		5 mm

An initial simulation was done without the balancing springs. The shaking forces where saved to be further compared with.

The vertical spring must be positioned in a way that it is compressed when the crank is in top dead centre and expanded when the crank is in bottom dead centre. Considering the hypothetical nature of this paper, the spring support will be positioned above the piston. In a realistic environment such a design raises design challenges.

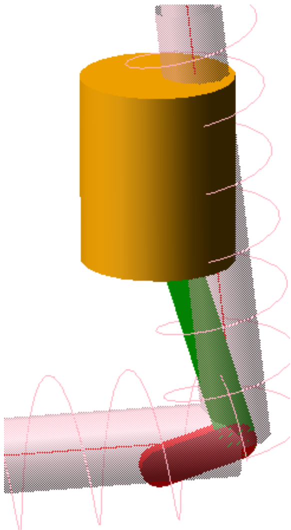

Figure 7 Multibody Model of Spring Balanced Slider-Crank Mechanism

The model was simulated with parameters according to the Table 1. The resulting shaking forces are shown in Figure 7 and Figure 8.

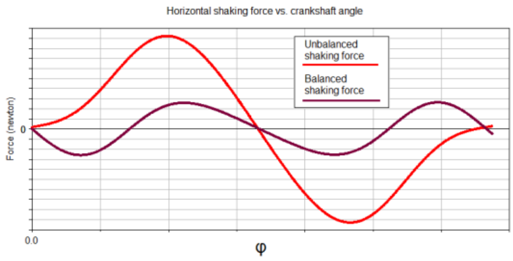

Figure 8 Balanced and Unbalanced Horizontal Shaking Force.

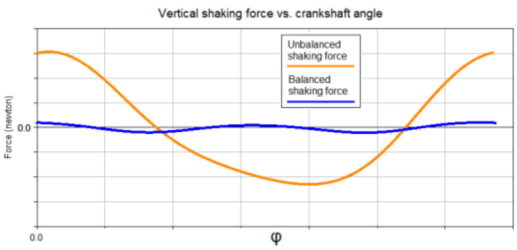

Figure 9 Balanced and Unbalanced Vertical Shaking Force.

V. CONCLUSIONS

In this paper the balancing of the slider-crank mechanism with springs is proposed. According to the multibody simulation results the next statements can be concluded:

- The slider-crank mechanism can be balanced with springs.
- The specific piston excitation of the slider-crank mechanism can be balanced with a progressive spring (Figure 8).
- The lateral shaking forces of the spring balanced model have been reduced by 71 percent (Figure 7).
- The vertical shaking forces of the spring balanced model have been reduced by 92 percent (Figure 8).

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